The Distributional Impacts of Minimum Wage Increases when Both Labor Supply and Labor Demand are Endogenous*

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Abstract

We develop and estimate a one-shot search model with endogenous entry, and therefore zero expected profits, by firms and endogenous labor supply by workers. Positive employment effects from an increase in the minimum wage can result as the employment level depends upon both the number of searching firms and the number of searching workers. Welfare implications are similar to the classical analysis: workers who most want the minimum wage jobs are hurt by the increase in the minimum wage with workers who were marginally interested in minimum wage jobs benefiting. We estimate the model using data on teenagers from the CPS and show that small changes in the employment level are masking large changes in labor supply and labor demand. Teenagers from wealthy, well educated families see their employment probabilities increase as the positive labor supply effects outweigh the negative labor demand effects. In contrast, teenagers from poor, less educated families have lower employment probabilities as they are pushed out of the market by their wealthier counterparts.

Keywords: Minimum wages, search, unemployment

JEL J6, J3

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1 Introduction

The classical analysis of the minimum wage operates from a simple labor supply and demand framework with the minimum wage serving as a price floor. When the labor market is competitive, a binding price floor leads to employment being determined solely by labor demand. With employment determined solely by labor demand, an increase in the minimum wage must lead to a decrease in employment.

Empirical work by Card and Krueger (1994, 1995) has called the classical model into question. Their research suggests that an increase in the minimum wage may even have small positive employment effects. While there has been considerable controversy regarding their findings,\(^1\) the evidence for strong negative employment effects from an increase in the minimum wage is surprisingly weak.\(^2\)

The lack of strong negative employment effects from increasing the minimum wage has led some policy-makers to support minimum wage increases as a means of combating poverty. With no employment losses, increasing the minimum wage serves to transfer money from rich firms to poor workers. In this paper we show that changes in the employment level from a minimum wage increase may be masking much larger changes in labor supply and labor demand. Further, these larger changes imply employment losses for groups that most wanted the minimum wage jobs in the first place.

To demonstrate these effects, we develop a two-sided search model with endogenous labor supply and labor demand that can exhibit positive employment effects from an increase in the minimum wage. In the classical analysis, the number of searching workers has no effect on the number of matches. In a more general search model, the number of matches increases with the number of searching workers. Hence, increasing the minimum wage may induce search which can lead to higher employment levels even with the number of firms falling. However, these positive employment effects also lead to lower probabilities of matching at the individual level. As in Luttmer (1998) and Luttmer and Glaeser (2003), in expectation, those with the lowest reservation wages are hurt most by the increase in the minimum wage when workers

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\(^1\) See in particular Neumark and Wascher (2000) and the reply by Card and Krueger (2000).

\(^2\) Kennan (1995) suggests that the reason minimum wage effects have been difficult to quantify is that changes in the minimum wage are quite small relative to the cyclical variation and serial correlation in teenage employment.
are randomly allocated to jobs.\textsuperscript{3}

Our search model can therefore generate zero or positive employment effects while also having firms earn zero expected profits both before and after the minimum wage increase. The search model shows that the effect of a minimum wage increase may appear small because the variable used to measure this effect—the employment level—does not adequately capture the churning of the labor market. Individuals induced to enter the labor market result in more matches and may not lower the employment level. However, the new matches push out those who originally wanted minimum wage jobs. Therefore, there are possibly large negative welfare effects from a minimum wage increase, even if the employment level stays constant or increases.\textsuperscript{4}

We show that the model developed in the theoretical section is estimable. Estimates of the model yield three sets of parameters: 1) the parameters of the wage generating process, 2) the parameters of the firm’s zero profit condition (labor demand), and 3) the parameters of the search decision (labor supply). Although we do not observe firm behavior, we show that the firm’s zero profit condition can be written as a function of the probability of a worker finding a match. The three sets of parameters can then be estimated from data on wages, employment, and search choices respectively.

We use a twelve-year band (1989 to 2000) of 16 to 19 year old white teenagers from the basic monthly outgoing rotation CPS files. We further focus our analysis on teenagers who are enrolled in school and whose primary is residence is with their parents. Minimum wages then vary across states and over time. We find that the employment elasticity with respect to a minimum wage increase is virtually zero for this group of teenagers. However, this is masking large increases in the likelihood of searching coupled with large decreases in the probability of finding a job conditional on search.

\textsuperscript{3}Indeed, Glaeser (2002)’s work on rent control suggest that non-competitive prices may lead to discrimination. If workers who have low reservation values also have characteristics that are unappealing but not related to productivity then the minimum wage provides even more scope for misallocation.

\textsuperscript{4}An alternative search model by van den Berg (2003) shows welfare gains from an increase in the minimum wage in a model where firms are heterogeneous. Without a minimum wage, the model has two equilibria: one where low productivity firms survive and wages are low and one where there are only high productivity firms and wages are high. The second equilibrium is shown to dominate the first, with a minimum wage potentially serving to eliminate the bad equilibrium.
Positive employment effects from a minimum wage increase then exist for sub-groups of teenagers. In particular, those who come from wealthy, highly educated families see their employment probabilities increase because they are now more likely to search for the newly desirable minimum wage job. In contrast, those who come from poorer, less educated families see their employment probabilities fall. These teenagers were more likely to search in the first place and hence the decrease in labor demand outweighs the increase in labor supply. These results are consistent with the reduced form results of Lang and Kahn (1998) and Neumark and Wascher (1995) who also show that the effects of minimum wage increases on the composition of the workforce may make raising the minimum wage unattractive. Lang and Kahn find that raising the minimum wage leads to a shift in the fast-food workforce from adults to teenagers, while Neumark and Wascher’s results suggest that minimum wage increases lead to a shift in the teenage workforce from those who have completed their schooling to those whose value of school is relatively high.

Estimation of structural search models have a rich history in labor economics. While there is much variation in the types of search models estimated, all generally rely upon infinitely lived agents in a steady-state equilibrium with reservation values determined in part by the continued value of search. This paper builds upon Flinn (2002, 2005), which examine the welfare implications of minimum wage increases in a search model where firms and workers split a match-specific surplus. In Flinn (2005), some versions of the model have the number of firms and workers determined endogenously. Our model has search follow a one-shot

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5Lang and Kahn also develop a theoretical model where minimum wage increases can lead to positive employment effects while still lowering welfare, though their model does not lend itself well to structural estimation.

6See Eckstein and van den Berg (2005) for a review

7Eckstein and Wolpin (1990) estimate a version of Albrecht and Axel (1984) where wage dispersion occurs because of heterogeneity in the reservation values of workers. Heterogeneity also exists in the productivity of firms with the marginal firm earning zero profits. van den Berg and Ridder (1998) estimate a version of the Burdett and Mortensen (1998) where now identical firms and workers can still lead to a heterogeneous distribution of wages. However, this heterogeneity comes at a cost as the number of firms and workers is exogenous and hence all firms earn positive profits.

8The initial version of Flinn (2005) was in 1997.

9Endogeneity of the number of firms and workers was absent from versions of Flinn (2005) that predated the first draft of this paper. Flinn (2005) incorporated some the techniques used here and was unable to reject the hypothesis that the contact rates were endogenous. However, this occurred because the parameters generating
game and we therefore lose some of the richness of having reservation values depend upon the
continued value of search. However, the infinite horizon model in a steady state equilibrium is
clearly not appropriate for teenagers enrolled in school during non-summer months. Further,
the one-shot model allows a much richer specification of labor supply and labor demand than
the infinite horizon models and allows us to consider equilibria across states and time.

The rest of the paper proceeds as follows. Section 2 reviews the classical model and how
it does and does not relate to the matching model. Section 3 develops the two-sided search
model, with welfare and employment analysis in section 4. Section 5 describes the data that
we use to estimate the model. The translation from the theoretical model to the structural
econometric model is presented in section 6. Section 7 contains the estimation results. Section
8 shows the results of policy simulations, and section 9 concludes.

2 The Classical Model

The classical analysis of the effects of a minimum wage can be found in most introductory
economics textbooks. However, by first examining the classical model it is possible to see why
our model is able to generate positive employment effects from an increase in the minimum
wage while in the classical model is not. Further the welfare implications of our model will
turn out to be very similar to those of the classical model.

Figure 1 shows the implications of an increase in the minimum wage in the classical model,
from $W^*$ to $W$. Employment here falls from $Q^*$ to $Q$. Note that the employment level only
depends upon labor demand. How elastic, or inelastic, labor supply may be has no effect on
the employment level. This is the primary difference between the classical model and matching
models. Matching models rely on a ‘matching function’ which takes the number of searching
worker and the number of searching firms and produces an employment level. Assuming one
vacancy per firm, the matching function in the classical model is the minimum of the number
of searching firms and the number of searching workers. The minimum must be the number
of searching firms when there is a binding minimum wage. However, other matching functions
the contact rates were only weakly identified: the optimal minimum wage without endogenous contact rates
was more than double the optimal minimum wage with endogenous contact rates. Because we treat states
as closed labor markets rather than the whole U.S. as one labor market, much more variation in the data is
available to identify the entry and exit behavior of firms and workers.
that depend upon both the number of searching firms and the number of searching workers can produce increases in the employment level because the increased labor supply may more than compensate for the decreased labor demand.

The classical model requires an additional assumption as to how jobs are assigned because there is an excess supply of workers. In Figure 1 we have assumed that the probability of employment is the same across searching workers. This probability of finding a minimum wage job would be given by $Q/\overline{Q}$ where $\overline{Q}$ is the number of individuals interested in working at the minimum wage. The area between the labor supply curve and the curve that kinks at $\overline{Q}$ gives the expected surplus over the reservation wage of the workers, $R$. Note that the expected surplus is smaller with the minimum wage increase for all workers below $Q_c$, when compared to their original surplus under the market clearing wage, defined as the area between $W^*$ and the labor supply curve. These are the workers who were most interested in being employed and would be willing to trade a lower wage for a higher probability of employment.

The matching model described below has very similar welfare implications to the classical model. If there are losers because of a minimum wage increase, it will be those individuals who were most interested in being employed. Winners are then those individuals who would either not be interested or only marginally interested in being employed at the market clearing wage.

3 The Matching Model

In this section we present a two-sided search model designed to highlight the effects of a minimum wage increase in the low wage market. All proofs are in the appendix. There are $N$ individuals available to search and individuals live for one period. Individuals are differentiated in their reservation values for not working. The $i$th individual has reservation value $R_i$, where $R_i$ is drawn from the cumulative distribution function $F(R)$ and has support $[0, \infty)$. This reservation value can be leisure or any outside option for the individual. For instance, we may assume that $R_i$ is the value of schooling for teenagers, with the treatment effect of education varying across the population. Adept students expect to acquire more

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10 For other theoretical models of job search with short time horizons see Pissarides (1992), Arcidiacono (2003), and Ahn and Arcidiacono (2004).
Figure 1: Classical Employment Losses From a Minimum Wage Increase
human capital in school and will therefore consider the minimum wage job as a less attractive option.

Denote $p_i$ as the probability that the $i$th worker would find an acceptable match conditional on searching. Denote $K$ as the search cost which is uniform across individuals and is paid whether an individual matches with a firm or not. Individuals are risk neutral and the expected value of searching, $V_i$, is given by:

$$V_i = p_i[E(W|W > R_i) - R_i] - K,$$

where $W$ is the wage. The number of searching workers, $N$, is endogenous and individuals search when $V_i > 0$.

The number of firms, $J$, is endogenous. All firms are identical and therefore have identical probabilities of a successful match with a worker, $q$. Production from a match is given by $S$ and firms pay a search cost, $C$, whether or not they find a match. Firms enter until all firms have zero expected profits. Expected profits are then given by:

$$qE(\max\{S - W, 0\}) - C = 0$$

as firms will reject matches where $S < W$.

We assume that the surplus of a match is given by:

$$S_{ij} = \bar{S} + \epsilon$$

where $\bar{S}$ is the average match value. $\epsilon$ is then a match-specific component with zero mean, is drawn from the cumulative distribution function $G(\epsilon)$, and has support $[\epsilon, \tau]$.

We now specify the wage-generating process. Matched pairs split $S_{ij}$ according to generalized Nash bargaining, with the caveat that a successful match pays at least the minimum wage, $W$. The worker’s bargaining power is set at $\beta$, $\beta \in (0, 1)$. Wages are then given by:

$$W_{ij} = \max\{\beta(\bar{S} + \epsilon_{ij}), W\}.$$ 

The splitting of the surplus in this manner can generate the spike observed at the minimum wage in the data. All matches where the worker’s share of the surplus would normally be

\footnote{Heterogeneity in the values of the search cost has no effect on the qualitative results.}

\footnote{See Flinn (2005) for a similar specification.}
below the minimum wage will earn the same wage even if their match-specific components differ.

Note that Nash bargaining in the wage generating process does not include the worker’s reservation value. This may seem incomplete as the usual bargaining framework defines surplus as the total value of the match minus the worker’s outside option. However, reservation values here are very different than in the standard search literature. In this model, reservation values are not tied to the labor market in ways that are easily verifiable through such factors as unemployment rates. Rather, reservation values here refer to the differentiated values of leisure or education. We do not believe either of these factors influence wages in the low wage labor market.\footnote{If workers differed systematically in their reservation values but not in their productivities, groups of workers with low reservation values would be more highly sought after than their high reservation counterparts. Hence, if teenagers from rich families have higher reservation values than their low income counterparts, teenagers from low income families should have lower unemployment rates, a feature that we do not observe in the data.}

A successful match must ensure that both the firm and the worker prefer $W_{ij}$ over not matching. Note that, without further assumptions, it may be possible for a worker to reject a match even though the surplus value being divided between the firm and the worker is greater than the worker’s reservation wage as $R$ does not affect the splitting of the surplus. Having $R$ affect whether matches are accepted or not significantly complicates the model.

Let $\epsilon_A$ give the expected value of $\epsilon$ conditional on a match being acceptable and let $\pi_A$ be the corresponding probability of an acceptable match conditional on matching. We make the following assumption which ensures that all matches are acceptable to workers:

$$\pi_A \beta (\epsilon_A - \xi) < K \text{ for all } \{\pi_A, \epsilon_A\}$$

This assumption means that the match-specific component is relatively unimportant compared to the cost of searching, an assumption that may be more reasonable in the market for low wage workers than in other markets. If the search costs are sufficiently high relative to the match-specific component, conditional on searching, all matches will be accepted. In the appendix, we show that this assumption does indeed generate only successful matches.

**Lemma 1** Regardless of the reservation value, a worker who finds it optimal to search will accept any match.
Note that having all matches be acceptable to workers conditional on searching does not rule out firms rejecting matches in the presence of a minimum wage. In particular, for all values of $\epsilon'$ such that $\epsilon' < W - \bar{S}$ the firm will reject the match.

To close the model we need to specify the matching function and the corresponding probabilities of finding a match. Although many matching functions allow for positive employment effects from an increase in the minimum wage, we use a Cobb-Douglas matching function as in Pissarides (1992) to illustrate the result because of its prevalence in the literature\textsuperscript{14} and also because we use it in the empirical section. The number of matches is then given by:

$$x = \min(AJ^\alpha N^{1-\alpha}, J, N),$$

where $\alpha \in (0,1)$ and $A$ is a normalizing constant. All workers and firms have the same probability of finding a match, $p = \frac{x}{N}$ and $q = \frac{x}{J}$. Proposition 1 then establishes that an equilibrium for this model exists.

**Proposition 1** Given equations (1) - (7), assumption A.1, \{F(R), G(\epsilon), W, K, C, \alpha, \bar{S}, and $N$\}, there exists an equilibrium in $N$ and $J$.

### 4 Implications of the Model

The model described above has a number of implications for a minimum wage increase. In this section we describe how a minimum wage increase affects the probability of matching and conditions under which a minimum wage increase positively affects the employment level. We further show conditions under which a minimum wage hike increases welfare for all searching workers and show which workers are hurt when these conditions are not met. We focus our attention on cases when $W < S + \epsilon$. In this case, firms will accept any match.

We first show that a minimum wage hike will always lower the probability of an individual obtaining a minimum wage job, even if the employment level increases.

**Proposition 2** $\frac{dp}{dW} < 0$, regardless of the signs of $\frac{dN}{dW}$, $\frac{dJ}{dW}$, and $\frac{dx}{dW}$.

\textsuperscript{14}Indeed, most of the empirical literature is in agreement that there is a stable aggregate matching function of the Cobb-Douglas form and constant returns to scale in unemployment and job vacancies. See Petrongolo and Pissarides (2001) for a review.
The intuition comes from examining the expected zero profit condition for firms which can be written as:

$$A \left( \frac{N}{J} \right)^{\alpha} (S - E(W)) = 0$$

In particular, increasing the minimum wage lowers profits conditional on matching, the term in parenthesis in equation (6). if the costs of the firm increase, the probability of finding a match for the firm must increase in order for the expected zero profit condition to hold. Since an increase in the match of the probability of the firm means an increase in the ratio of workers to firms, $N/J$, and the match probabilities of workers fall with an increase of $N/J$, we have the result. This holds whether or not the employment level has increased.

Although the probability of finding a job always falls with an increase in the minimum wage, the effect on the employment level is ambiguous. The next proposition show that it is possible to simultaneously have an increase in the minimum wage, a decline in the probability of employment, and an increase in the level of employment. Proposition 3 outlines conditions on the labor demand and supply elasticities under which positive employment effects due to a minimum wage hike are possible.

**Proposition 3** \( \frac{\partial x}{\partial W} \geq 0 \) if \( \alpha \varepsilon_{LD} + (1 - \alpha) \varepsilon_{LS} \geq 0 \), where \( \varepsilon_{LD} \) is the elasticity of labor demand and \( \varepsilon_{LS} \) is the elasticity of labor supply.

Proposition 3 explicitly demonstrates that the direction of growth of employment is jointly dependent on the elasticities of labor supply and demand. Furthermore, since both elasticities depend on $J$ and $N$, which are endogenous, as well as $W$, the model can exhibit positive or negative employment effects. This is because an increase in $W$ will generally pull $J$ and $N$ in opposite directions, which then leads to labor demand and supply elasticities being pulled in opposite directions. This dual effect on the employment level helps to explain not only why different studies have found positive and negative employment effects, but also why the magnitude of the effects has been so small. Since $J$ and $N$ are moving in opposite directions, $\alpha$, or the measure of sensitivity of the matching function to a relative increase in $J/N$, helps to determine which effect is larger. As $W$ increases, $\varepsilon_{LD}$ and $\varepsilon_{LS}$ continue to offset each other, which translates to a small movement in the employment level.

The proposition above gave conditions which relied upon endogenous variables. We next demonstrate sufficient conditions in terms of the exogenous parameters of the model to gen-
erate positive employment effects. However, we cannot do this without explicitly defining the cumulative distribution function for the reservation values. We focus on the case where \( R \) is distributed uniform \([0, \bar{R}]\), though conditions under which the exogenous parameters generate positive employment effects with a minimum wage hike are easy to show for many possible distributions of the reservation values.

**Proposition 4** If \( 1 - \frac{\alpha}{1-\alpha} \frac{E(W)}{S-E(W)} > 0 \) then \( \frac{dx}{dW} > 0 \).

Although proposition 4 is only an existence condition, it is instructive to analyze what the expression reveals about the employment effect. The expression is more likely to be positive if \( E(W) \) and \( \alpha \) are small. A small \( E(W) \) means less of the per-match surplus is given up to the worker. Less firms then pull out of the market in response to a minimum wage hike, which helps to increase the employment level. A small value for \( \alpha \) means that the matching function is less sensitive to changes in \( J \) and more sensitive to changes in \( N \). Since \( N/J \) must increase as the minimum wage increases, positive employment effects become more likely when the employment level depends more on the elasticity of labor supply rather than the elasticity of labor demand.

Proposition 4 is very similar in spirit to the ‘efficient equilibrium’ condition of Hosios (1990). For an ‘efficient equilibrium,’ elasticity of the matching function with respect to the number of searching workers must be equal to the worker’s share of the surplus generated from the match. Our \( 1 - \alpha \) is the elasticity of the matching function with respect to the number of searching workers, and \( \frac{E(W)}{S} \) is the worker share of the surplus. The proposition allows us to see that as long as the elasticity is greater than the worker share, a marginal increase in the worker share will induce more entry by workers and a higher level of employment overall. Hence, as in Flinn (2005), adjusting the minimum wage can serve as a blunt instrument for moving the equilibrium towards efficiency. However, this does not necessarily equate to a welfare increase for all workers. The increase in expected wage and the decrease in the probability of employment pull worker welfare in different directions. These forces affect workers with different reservation values differently.

While it is possible in the model to have an increase in the minimum wage benefit all participants in expectation,\(^{15}\) the conditions for a minimum wage to improve welfare for all workers are the same.

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\(^{15}\)The same implications hold in the classical model. If the drop in the probability of finding a job is small
participants are not the same as those for maximizing employment. Denote \( E_1(W) \) and \( p_1 \) as the expected wage and probability of finding a match before the minimum wage increase. Denote \( E_2(W) \) and \( p_2 \) as the corresponding values after the minimum wage increase. With reservation values for workers bounded below by zero, all workers are made better off in expectation by a minimum wage increase if:

\[
p_1 E_1(W) < p_2 E_2(W).
\]

Proposition 5 shows the necessary and sufficient condition for this to be the case.

**Proposition 5** All workers benefit from a marginal increase in the minimum wage if and only if

\[
1 - \frac{(1-\alpha)}{\alpha} \frac{E(W)}{S-E(W)} > 0
\]

Conditions in which proposition 4 is satisfied (low \( \alpha \) and low worker share of surplus) are not equivalent to conditions under which proposition 5 is likely to be satisfied (high \( \alpha \) and low worker share of surplus). Consider again the zero profit condition given in equation (6). Given an expected wage, there is only one value for \( N/J \) that will satisfy the zero profit condition of the firms. Since in the zero profit condition \( N/J \) is raised to the power \( \alpha \), a small \( \alpha \) means large changes are necessary for \( N/J \) in order for the zero profit condition to be met. This is why the \( \alpha/(1-\alpha) \) term in proposition 5 enters inversely here.

If the conditions for proposition 5 are not met, then some workers are made worse off by the increase in the minimum wage. In particular, as in the classical model discussed in section 2, it is those workers who most want the minimum wage jobs, those with the lowest reservation values, who are hurt by the increase. A teenager from a rich, two-parent household inherently values a minimum wage job differently compared to a teenager from a poor, single-parent household. A poor teenager will not have as many outside options or be as financially secure as the rich teenager.

A minimum wage policy that measures its success by the employment level misses the important distributional distortion caused by the increase. By increasing the minimum wage, more workers with higher reservation values enter the labor market. While these new workers are undoubtedly experiencing a welfare increase, workers who were already searching may be worse off, because these are the workers who were most willing to accept a lower paying job for a higher probability of employment.

\[\text{relative to the increase in the wage, even low reservation workers are made better off.}\]
5 Data

We now describe the data used in the empirical analysis. We use a twelve year band of the basic monthly outgoing rotation survey files of the Current Population Survey (CPS) from 1989 to 2000. These twelve years cover four federal minimum wage changes as well as sixteen states\footnote{There are actually eighteen states that paced ahead of the federal minimum wage, but we exclude Alaska and Hawaii from analysis as well as the District of Columbia.} which changed their state minimum wage to outpace the federal wage. The range of observed minimum wages in the twelve years run from $3.35 to $6.50. Our analysis covers white males who are between sixteen and nineteen years of age inclusive. We further restrict our study to those whose primary residence is with their parent(s), who attended school in the last week, and we only use data from non-summer months.\footnote{We exclude June, July, and August.} Hence, the sample becomes more selected as individuals age: nineteen year olds who are still in school are a much more selected group than sixteen year olds who are still in school.\footnote{Indeed, our final sample includes over twice as many sixteen year olds as nineteen year olds.} From the CPS, we collect hourly wage, whether the individual is searching for work or not, whether the searching worker is employed or not, as well as a number of demographic variables that may affect an individual’s reservation wage.

Table 1 presents descriptive statistics for three groups of teenagers: the population, job searchers, and those who are employed. Since search is one-shot game, job searchers refer to the sum of those who are unemployed and those who are currently working. Observations with employed individuals earning less than the minimum wage minus twenty-five cents were dropped, as well as individuals who reported earning more than $15 per hour.\footnote{Less than 0.5\% of employed workers were cut for making too much, while less than 6\% of employed workers were cut for making too little.} As in Flinn (2005), we keep those earning less than a minimum wage but within twenty-five cents\footnote{Within twenty-five cents refers to the nominal wage. After the sample selection, all wages and incomes are adjusted to 2000 dollars.} because of measurement error in reported wages. These observations are treated as earning exactly at the minimum. One key variable to the analysis is the prime age male unemployment rate. This unemployment rate is calculated at the state-quarter level using CPS data for all males aged 30 to 39. This variable is assumed to affect job search of teenagers only through the expected
wage and the probability of employment, having no effect on search costs or reservation values.

Table 1 shows that those who search are more likely to be older and those who are employed conditional on searching are likely to be even older. However, besides age, the descriptive statistics show very few differences in search and employment across parental income and education. This is in part due to the sample selection. Older teenagers are more likely to work yet older teenagers from poor and less-educated families are significantly less likely to be enrolled in school and consider their primary residence as with their parents.

Table 2 then gives descriptive statistics by age. The descriptive statistics in Table 2 confirm that those individuals who are still in school and whose primary residence is with their parents are wealthier and come from more educated families. With older teenagers more likely to be found in the labor force, the effect of family income and education may be canceling out in Table 1: higher parental income may make participation in the workforce less likely but the selected nature of the sample means that wealthier individuals may be more likely to participate when we do not condition on age. Older teenagers are more likely to participate in the labor market, less likely to have their wages bind at the minimum, and have higher expected earnings than their younger counterparts. Due to sample selection, those who have a household head who is unemployed are more likely to be younger. In contrast, those who have a household head who is employed but does not report weekly income are more likely to be older, suggesting that not reporting parental income is associated with higher parental earnings.
Table 1: Descriptive Statistics†

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Mean</th>
<th>Search</th>
<th>Mean</th>
<th>Employed</th>
</tr>
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<tbody>
<tr>
<td>Age</td>
<td>17.17</td>
<td>17.32</td>
<td>17.38</td>
<td></td>
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<tr>
<td></td>
<td>(1.042)</td>
<td>(1.031)</td>
<td>(1.023)</td>
<td></td>
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<tr>
<td>Parental Weekly Income‡ (000’s)</td>
<td>1.023</td>
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<td>1.027</td>
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<tr>
<td></td>
<td>(0.641)</td>
<td>(0.601)</td>
<td>(0.603)</td>
<td></td>
<td></td>
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<tr>
<td>Head Working, No Reported Income§</td>
<td>0.182</td>
<td>0.182</td>
<td>0.171</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Head Unemployed</td>
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<td>0.049</td>
<td>0.046</td>
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<tr>
<td>Head Other</td>
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<td>0.042</td>
<td>0.037</td>
<td></td>
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<tr>
<td>Head Education HS or less</td>
<td>0.452</td>
<td>0.450</td>
<td>0.442</td>
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</tr>
<tr>
<td>Some College</td>
<td>0.261</td>
<td>0.286</td>
<td>0.290</td>
<td></td>
<td></td>
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<tr>
<td>College Graduate</td>
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<td>0.166</td>
<td>0.169</td>
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<td>Prime Age Male Unemployment Rate</td>
<td>0.036</td>
<td>0.035</td>
<td>0.035</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pr(Search)</td>
<td>0.444</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pr(Employed</td>
<td>Search)</td>
<td></td>
<td>0.770</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pr(Minimum Wage Binds</td>
<td>Employed)</td>
<td></td>
<td></td>
<td>0.216</td>
<td></td>
</tr>
<tr>
<td>E(Wage</td>
<td>Employed)</td>
<td></td>
<td></td>
<td>6.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1.26)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>56456</td>
<td>25067</td>
<td>19045</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

†Sample includes only individuals whose primary residence was with at least one parent and who attended school in the last week.
‡Conditional on household head working and reporting an income.
§Takes on a value of one if the household head worked but did not report an income.
Table 2: Descriptive Statistics by Age†

<table>
<thead>
<tr>
<th>Variable</th>
<th>Age=16</th>
<th>Age=17</th>
<th>Age=18</th>
<th>Age=19</th>
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<tbody>
<tr>
<td>Parental Weekly Income‡ (000’s)</td>
<td>0.980</td>
<td>0.998</td>
<td>1.056</td>
<td>1.124</td>
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<tr>
<td></td>
<td>(0.618)</td>
<td>(0.631)</td>
<td>(0.658)</td>
<td>(0.673)</td>
</tr>
<tr>
<td>Head Working, No Reported Income§</td>
<td>0.177</td>
<td>0.175</td>
<td>0.187</td>
<td>0.203</td>
</tr>
<tr>
<td>Head Unemployed</td>
<td>0.057</td>
<td>0.053</td>
<td>0.049</td>
<td>0.048</td>
</tr>
<tr>
<td>Head Other</td>
<td>0.070</td>
<td>0.066</td>
<td>0.058</td>
<td>0.050</td>
</tr>
<tr>
<td>Head Education HS or less</td>
<td>0.479</td>
<td>0.466</td>
<td>0.433</td>
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</tr>
<tr>
<td>Some College</td>
<td>0.260</td>
<td>0.264</td>
<td>0.260</td>
<td>0.255</td>
</tr>
<tr>
<td>College Graduate</td>
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<td>0.164</td>
<td>0.184</td>
<td>0.210</td>
</tr>
<tr>
<td>Post-College</td>
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<td>0.106</td>
<td>0.123</td>
<td>0.148</td>
</tr>
<tr>
<td>Single Parent</td>
<td>0.188</td>
<td>0.188</td>
<td>0.175</td>
<td>0.161</td>
</tr>
<tr>
<td>Prime Age Male Unemployment Rate</td>
<td>0.036</td>
<td>0.036</td>
<td>0.036</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>Pr(\text{Search})</td>
<td>0.347</td>
<td>0.476</td>
<td>0.499</td>
<td>0.516</td>
</tr>
<tr>
<td>Pr(\text{Employed}</td>
<td>\text{Search})</td>
<td>0.691</td>
<td>0.781</td>
<td>0.809</td>
</tr>
<tr>
<td>Pr(\text{Minimum Wage Binds}</td>
<td>\text{Employed})</td>
<td>0.283</td>
<td>0.236</td>
<td>0.180</td>
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<tr>
<td>E(\text{Wage}</td>
<td>\text{Employed})</td>
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<td>5.99</td>
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</tr>
<tr>
<td></td>
<td>(0.92)</td>
<td>(1.00)</td>
<td>(1.36)</td>
<td>(1.65)</td>
</tr>
<tr>
<td>Observations</td>
<td>18701</td>
<td>17367</td>
<td>12420</td>
<td>7968</td>
</tr>
</tbody>
</table>

†Sample includes only individuals whose primary residence was with at least one parent and who attended school in the last week.
‡Conditional on household head working and reporting an income.
§Takes on a value of one if the household head worked but did not report an income.
Table 3: State Minimum Wage from 1989 to 2000.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
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<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.75</td>
<td>5.15</td>
<td>5.75</td>
<td>5.75</td>
<td>5.75</td>
<td>5.75</td>
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<tr>
<td>Connecticut</td>
<td>4.25</td>
<td>4.25</td>
<td>4.27</td>
<td>4.27</td>
<td>4.27</td>
<td>4.27</td>
<td>4.77</td>
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<td>5.18</td>
<td>5.65</td>
<td>6.15</td>
<td>6.15</td>
</tr>
<tr>
<td>Delaware</td>
<td>3.35</td>
<td>3.80</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.75</td>
<td>5.15</td>
<td>5.15</td>
<td>5.65</td>
<td>5.15</td>
<td>5.15</td>
</tr>
<tr>
<td>Iowa</td>
<td>3.35</td>
<td>3.85</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.75</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
</tr>
<tr>
<td>Massachusetts</td>
<td>3.75</td>
<td>3.75</td>
<td>3.75</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.75</td>
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<td>5.25</td>
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<td>6.00</td>
<td>6.00</td>
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<tr>
<td>Maine</td>
<td>3.75</td>
<td>3.85</td>
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<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.75</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
</tr>
<tr>
<td>Minnesota</td>
<td>3.85</td>
<td>3.95</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.75</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
</tr>
<tr>
<td>New Hampshire</td>
<td>3.65</td>
<td>3.80</td>
<td>3.85</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.75</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
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</tr>
<tr>
<td>New Jersey</td>
<td>3.35</td>
<td>3.80</td>
<td>3.80</td>
<td>5.05</td>
<td>5.05</td>
<td>5.05</td>
<td>5.05</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
</tr>
<tr>
<td>New York</td>
<td>3.35</td>
<td>3.80</td>
<td>3.80</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.75</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
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</tr>
<tr>
<td>Oregon</td>
<td>3.85</td>
<td>4.25</td>
<td>4.75</td>
<td>4.75</td>
<td>4.75</td>
<td>4.75</td>
<td>5.50</td>
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<td>6.00</td>
<td>6.50</td>
<td>6.50</td>
<td>6.50</td>
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<tr>
<td>Pennsylvania</td>
<td>3.70</td>
<td>3.80</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.75</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
</tr>
<tr>
<td>Rhode Island</td>
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<td>4.25</td>
<td>4.45</td>
<td>4.45</td>
<td>4.45</td>
<td>4.45</td>
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<td>5.15</td>
<td>5.15</td>
<td>5.65</td>
<td>6.15</td>
</tr>
<tr>
<td>Vermont</td>
<td>3.75</td>
<td>3.85</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.50</td>
<td>4.75</td>
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<td>5.25</td>
<td>5.75</td>
<td>5.75</td>
<td>5.75</td>
</tr>
<tr>
<td>Wisconsin</td>
<td>3.65</td>
<td>3.80</td>
<td>3.80</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.75</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
</tr>
<tr>
<td>Other States</td>
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<td>3.80</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.75</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
<td>5.15</td>
</tr>
</tbody>
</table>

† Minimum wage change on 1/1 or 1/2. • Minimum wage change on 3/1. † Minimum wage change on 4/1. ‡ Minimum wage change on 5/1. 
△ Minimum wage change on 7/1 or 7/2. ◄ Minimum wage change on 8/1. * Minimum wage change on 9/1. ○ Minimum wage change on 10/1.
We also use the Monthly Labor Review to collect minimum wage at the state/month level. That is, from 1989 to 2000, we observe the minimum wage in each state, each month. Table 3 presents the minimum wage in each state, each month within the range of the collected CPS data. These minimum wages are nominal values. In the analysis, the wages and incomes are inflated to 2000 dollars.

6 Parameterizing the Model

In this section we show how to estimate the structural model. Estimation has three components. First, for those individuals who successfully match we observe wages. Second, we need to estimate the parameters of the zero profit condition. Although we do not observe the probability of a firm finding a match, we are able to rewrite the zero profit condition as a function of the individual’s probability of finding a match. Finally, we observe decisions by individuals as to whether to search. We can use these decisions to estimate the supply side parameters.

6.1 Parameterizing Wages

Before specifying the distribution of wages, we first must specify the source of the wages: the surplus of the match, $S_{ij}$. We assume that $\ln(S_{ij})$ is given by:

$$\ln(S_{ij}) = X_i \theta + \epsilon_{ij}$$

(7)

where $X_i$ are characteristics of individual $i$'s market and $\theta$ is the set of parameters to be estimated. Because the left tail of the wage distribution is so important to this analysis, we do not make the standard assumption of log-normality on the $\epsilon$'s. Rather, we mix over four log-normal distributions allowing both the means and the variances of these distributions to vary. The probability of a draw coming from the $r$th distribution is then given by $\pi_r$ where the $r$th distribution is distributed $\mathcal{N}(\mu_r, \sigma_r)$.

Since the wage generating process is given by: $W_{ij} = \min\{\beta S, W\}$, without a minimum

---

21 As discussed in the data section, a market is defined at the age, state, quarter, and year level.

22 See Koning, van den Berg, and Ridder (2000) for an alternative approach to allowing for a flexible error distribution.
wage log wages would be given by:

\[ \ln(W_{ij}) = X_i \theta + \ln(\beta) + \epsilon_{ij} \]  

(8)

In the presence of a minimum wage the wage distribution is then distributed truncated log-normal with censoring at the minimum wage. The truncation occurs when the match value is so low that the firm rejects the match. This occurs whenever \( W > S_{ij} \). There are then three relevant regions for the quality of the match:

\[
\begin{align*}
\beta S_{ij} > W & \Rightarrow \{ W_{ij} = \beta S_{ij} \} \\
S_{ij} \geq W > \beta S_{ij} & \Rightarrow \{ W_{ij} = W \} \\
W > S_{ij} & \Rightarrow \{ \text{No match} \}
\end{align*}
\]

We then observe successful matches for those who are employed either at or above the minimum wage. We begin our estimation assuming that all matches are successful— the third region is empty. Assuming that all matches are successful allows us to estimate the parameters of the wage generating process separately from the parameters of the zero profit condition. State fixed effects are then indistinguishable from the Nash bargaining parameter in the wage regression. With the Nash bargaining parameter estimated from the firm’s zero expected profit condition, we can then test whether the third region is indeed empty.

Let \( N_{11} \) and \( N_{12} \) indicate the number of individuals who have wage observations above and at the minimum wage respectively. The likelihood for these observations then follows:

\[
\mathcal{L}_1 = \left( \prod_{i=1}^{N_{11}} \sum_{r=1}^{4} \pi_r \phi \left( \frac{W_i - X_i \theta - \ln(\beta) - \mu_r}{\sigma_r} \right) / \sigma_r \right) \times \left( \prod_{i=1}^{N_{12}} \sum_{r=1}^{4} \pi_r \Phi \left( \frac{\ln(W) - X_i \theta - \ln(\beta) - \mu_r}{\sigma_r} \right) \right)
\]

where \( \Phi \) and \( \phi \) are the cdf and pdf of the standard normal distribution.

6.2 Parameterizing Firms

Although we have no information on the firm, we can infer the parameters of the profit function by rewriting the zero profit condition as a function of the individual’s probability of finding a

---

Note this specification is quite similar to Meyer and Wise (1983a, 1983b) except resulting from the structural model and allows for a more flexible specification of the error distribution.
match. To see this, note that the probability of finding a match for firms and workers is given
by:
\[ q = A \left( \frac{N}{J} \right)^{1-\alpha} \quad p = A \left( \frac{J}{N} \right)^{\alpha} \]

implying that we can write \( q \) as:
\[ q = A^{\frac{1}{\alpha}} p^{\frac{\alpha-1}{\alpha}} \]

Substituting for \( q \) as a function of \( p \) in the zero profit condition yields:
\[ A^{\frac{1}{\alpha}} p^{\frac{\alpha-1}{\alpha}} E(\max\{S - W, 0\}) - C = 0 \]

Solving for \( p \) yields:
\[ p = \delta E(\max\{S - W, 0\})^{\frac{\alpha}{1-\alpha}} \tag{9} \]

where:
\[ \delta = C^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} \]

This zero profit condition is satisfied for every economy. That is, zero profits hold by age, state, quarter and year.

Given the assumed mixed log-normal distribution of \( S \) and the parameters of the wage-generating process, we can calculate \( E(\max\{S - W, 0\}) \), the expected surplus from matching. This surplus can be broken down into three parts: 1) when the match value is high enough such that the minimum wage does not bind, \( \tilde{S}_1 \), 2) when the match value is such that the minimum wage binds, \( \tilde{S}_2 \), and 3) when the match value is so low that the firm rejects the match. The last of these parts yields an expected surplus of zero. Since we estimate the wage distribution assuming all matches are successful, we have a natural test of this assumption from the zero expected profit function. In particular, we test whether the Nash bargaining parameter is low enough such that this region has no observations. With \( \tilde{S}_3 \) equalling zero, \( \tilde{S}_1 \) and \( \tilde{S}_2 \) are given by:
\[
\tilde{S}_1 = \sum_{r=1}^{2} \pi_r \left[ \exp(X \theta + \ln(1 - \beta) + \mu_r + \sigma_r^2/2) \Phi \left( \frac{\sigma_r^2 - \ln(W) + X \theta + \ln(\beta) + \mu_r}{\sigma_r} \right) \right]
\]
\[
\tilde{S}_2 = \sum_{r=1}^{2} \pi_r \left[ \exp(X \theta + \mu_r + \sigma_r^2/2) B_r \right.
\]
\[
- \left( \Phi \left( \frac{\ln(W) - X \theta - \mu_r - \ln(\beta)}{\sigma_r} \right) - \Phi \left( \frac{\ln(W) - X \theta - \mu_r}{\sigma_r} \right) \right) W \]
\]
where $B_r$ is given by:

$$B_r = \left( \Phi \left( \frac{\sigma_r^2 - \ln(W) + X\theta + \mu_r}{\sigma_r} \right) - \Phi \left( \frac{\sigma_r^2 - \ln(W) + X\theta + \ln(\beta) + \mu_r}{\sigma_r} \right) \right)$$

We then define $\tilde{S}$ such that:

$$\tilde{S} = E(\max\{S - W, 0\}) = \tilde{S}_1 + \tilde{S}_2 \quad (10)$$

and the probability of finding a match is given by:

$$p = \delta \tilde{S}^{1-\alpha} \quad (11)$$

However, in the data we do not observe whether an individual is matched with a firm, $p$, but only observe $p$ times the probability that the match is successful, $p\psi$, where $\psi$ is given by:

$$\psi = 1 - \Phi \left( \frac{\ln W - X\theta}{\sigma} \right)$$

Positive search outcomes for workers are then Bernoulli draws from $p\psi$. The likelihood function is then given by:

$$L_2 = \prod_{i=1}^{N_2} \left( p\psi \tilde{S}_1^{\frac{\alpha}{1-\alpha}} \right)^{m_i=1} \left( 1 - p\psi \tilde{S}_1^{\frac{\alpha}{1-\alpha}} \right)^{m_i=0}$$

where $N_2$ is the number of searching workers and $m_i$ indicates whether or not the $i$th worker was matched.

We then allow the $\beta$’s to vary by state and age. Identification of the $\beta$’s comes from the relationship between observed wages and the probability of employment in the data. If a particular state has a high expected wage, this can either be because of a strong economy or because worker bargaining power is high in the state. If a high expected wage translates into a high probability of finding a job, then we have evidence of the former. However, if we see a high expected wage and a high unemployment rate this must be because worker bargaining power is high.

### 6.3 Parameterizing the Individual

We now turn to the decision by individuals as to whether or not to search. Recall that an individual searches if:

$$p(E(W) - R_i) - K_i > 0.$$
where search costs may now be heterogeneous as well. With the estimates from the previous two stages it is possible to calculate expected wages and the probability of employment for each individual. We now need to parameterize the reservation values. In particular, we parameterize \( R_i \) such that all workers have positive reservation values:

\[
R_i = \exp(Z_i \gamma_1 + \epsilon_i)
\]

\( Z_i \) is then a vector of demographic characteristics which affect the individual’s outside option, the \( \gamma_1 \)'s are the coefficients to be estimated, and \( \epsilon_i \) is the unobserved portion of the reservation value. We also allow the search costs to vary where:

\[
K_i = Z_i \gamma_2
\]

and all search costs are constrained to be positive.

Individuals who come from high income families may have high reservation values, making search less likely. However, these same individuals may also have lower search costs. What separately identifies search costs from reservation values is how individuals react to the probability of finding a job. In particular, those with low search costs but high reservation values will be more willing to trade off higher expected wages conditional on matching for lower probabilities of employment. In contrast, those with high search costs but low reservation values prefer lower wages coupled with higher match probabilities.

Substituting in and solving for \( \epsilon_i \) shows that an individual will search when:

\[
\epsilon_i < \ln \left( \frac{E(W) - Z_i \gamma_2}{p} \right) - Z_i \gamma_1
\]

We assume that the \( \epsilon \)'s are distributed \( N(0, \sigma^2) \). Note that because of the log any coefficient on \( E(W) \) will be factored into the intercept term of the reservation values. Since we do not observe the \( \epsilon \)'s, the likelihood function is then given by:

\[
\mathcal{L}_3 = \prod_{i=1}^{N_3} \Phi \left( \frac{(1/\sigma) \ln \left( E(W) - \frac{Z_i \gamma_2}{p} \right) - Z_i \gamma_1}{\sigma} \right)^{s_i=1} \times \left( 1 - \Phi \left( \frac{(1/\sigma) \ln \left( E(W) - \frac{Z_i \gamma_2}{p} \right) - Z_i \gamma_1}{\sigma} \right) \right)^{s_i=0}
\]

where \( N_3 \) is the total number of potential searchers, \( s_i \) is an indicator for whether the \( i \)th individual chose to search, and \( \Phi \) is the standard normal cdf. In the standard probit, all
coefficients are relative the variance term. Here we can actually estimate \( \sigma \) as there is no other natural interpretation for the coefficient on the expression inside the log. The \( \gamma^* \)'s are then the \( \gamma \)'s divided by the standard deviation of the \( \epsilon \)'s, \( \sigma \).

7 Results

Having specified the estimation strategy, we now turn to the results. The first stage of the estimation involves estimating the parameters of the wage generating process. These estimates are given in Table 4. In addition to the reported parameters, we also included state, year, and quarter fixed effects. The coefficient on the prime age male unemployment rate is negative and significant. This will be important for the analysis of searching as this is our exclusion restriction: the adult unemployment rate only affects search through the expected wage and the probability of finding a match. Mixing over four log-normals shows that higher log wages are associated with higher variances: the highest mean is associated with a variance that is four times that of the variance associated with the lowest mean.

The estimates of the wage generating process are then taken as given in the estimation of the firms zero profit condition. The parameters that are estimated from the zero profit condition are the \( \beta \)'s, the Nash bargaining parameters which are allowed to vary by state, \( \alpha \), which measures how sensitive the number of matches are relative to the number of searching firms, and a conglomerate parameter, \( \delta \), which is a function of the search costs and the efficiency of the matching function. Estimates are reported in Table 5. The relative weight of firms to workers in determining the probability of matching, \( \alpha \), is estimated at close to 0.5— a result that is remarkably similar to the macroeconomics literature (see Peterongolo and Pissarides (2001)). The estimated \( \beta \)'s vary between 0.75 and 0.69, with the highest value being from West Virginia and the lowest Missouri. Besides the maximum and minimum, all other \( \beta \)'s are shown in the appendix. With estimated \( \beta \)'s and the parameters of the wage generating process we can test whether there are matches that would be rejected because of a low surplus value. Even for the largest \( \beta \), the probability of a match being rejected is less that 0.001.

Table 6 shows what variation in the data is identifying the \( \beta \)'s by displaying \( \beta \) along with the average probability of a searching worker finding a match and the expected wage
### Table 4: Parameters of the Wage Generating Process†

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime Age Male Unemployment Rate</td>
<td>-0.366</td>
<td>0.086</td>
</tr>
<tr>
<td>Age=17</td>
<td>0.010</td>
<td>0.002</td>
</tr>
<tr>
<td>Age=18</td>
<td>0.026</td>
<td>0.002</td>
</tr>
<tr>
<td>Age=19</td>
<td>0.045</td>
<td>0.003</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>1.597</td>
<td>0.009</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.044</td>
<td>0.002</td>
</tr>
<tr>
<td>$\pi_1$</td>
<td>0.348</td>
<td></td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>1.814</td>
<td>0.014</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.099</td>
<td>0.008</td>
</tr>
<tr>
<td>$\pi_2$</td>
<td>0.286</td>
<td></td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>1.696</td>
<td>0.010</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>0.046</td>
<td>0.003</td>
</tr>
<tr>
<td>$\pi_3$</td>
<td>0.246</td>
<td></td>
</tr>
<tr>
<td>$\mu_4$</td>
<td>2.036</td>
<td>0.020</td>
</tr>
<tr>
<td>$\sigma_4$</td>
<td>0.197</td>
<td>0.006</td>
</tr>
<tr>
<td>$\pi_4$</td>
<td>0.120</td>
<td></td>
</tr>
</tbody>
</table>

†Estimated on 19,132 employed white male teenagers. Estimation also included state, year, and quarter fixed effects.
Table 5: Estimates of the Firm’s Zero Profit Condition

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C^{\frac{\alpha}{\alpha}} A^{-1}$</td>
<td>0.3349</td>
<td>0.1168</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.4789</td>
<td>0.0211</td>
</tr>
<tr>
<td>$\beta_{max}$ (West Virginia)</td>
<td>0.7448</td>
<td>0.0660</td>
</tr>
<tr>
<td>$\beta_{min}$ (Missouri)</td>
<td>0.6959</td>
<td>0.0718</td>
</tr>
</tbody>
</table>

†Estimated on 25,067 white male teenagers who were either employed or looking for a job. Estimates of the other $\beta$’s are in the appendix.

Table 6: Identification of $\beta$

<table>
<thead>
<tr>
<th>State</th>
<th>$\beta$</th>
<th>Pr (Match)</th>
<th>E(W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>West Virginia</td>
<td>0.745</td>
<td>0.637</td>
<td>6.04</td>
</tr>
<tr>
<td>Louisiana</td>
<td>0.705</td>
<td>0.771</td>
<td>6.04</td>
</tr>
<tr>
<td>California</td>
<td>0.736</td>
<td>0.719</td>
<td>6.59</td>
</tr>
<tr>
<td>Colorado</td>
<td>0.696</td>
<td>0.836</td>
<td>6.25</td>
</tr>
</tbody>
</table>

conditional on matching. The table shows the relationships between these three variables for two states with high $\beta$’s (West Virginia and California) and two states with low $\beta$’s (Louisiana and Missouri). Expected wages in Louisiana and West Virginia are virtually identical. Yet, the unemployment rate in West Virginia is 1.5 times Louisiana’s unemployment rate. This suggest that the economy is better in Louisiana than in West Virginia but in West Virginia workers take a larger share of the surplus. Similarly, wages are significantly higher in California than in Missouri while the probability of finding employment in California is significantly lower than the probability of finding employment in Missouri.

With the estimates of the log wage regression and the parameters of the zero profit condition, we calculate the probability of matching and the expected wages conditional on matching. We then use these estimates to estimate the value of search with the results presented in Table
7. The last number in the table, $1/\sigma$, is crucial in estimating the wage elasticity. If the number is small, participation is driven primarily by unobserved reservation values. High values, in contrast, mean that individuals are very responsive to conditions in the labor market. Calculating the labor force participation elasticity yields a number around 3, a number that is substantially higher than the participation elasticity of married women. However, we would expect higher elasticities for teenagers whose primary residence is at home and who attend school as these individuals will rarely be working out of necessity.

Reservation values and search costs are also reported in Table 7. In virtually all cases, a characteristic that leads to a higher reservation value also leads to a lower search cost. This makes sense. Those who have access to technologies that might lower search costs (computers, contacts, etc.) also are likely to be provided with more income from their parents. Higher parental incomes\(^{24}\) are associated with lower search costs and higher reservation values. Using either the estimates from the reservation values or from the search costs shows that those who have a household head who works but does not report an income have by far the highest parental incomes while those who have an unemployed household head have parents whose weekly income is close to zero. As the education level of the parent increases so too does the teenager’s reservation value. Similar to parental income, increasing parental education lowers search costs. However, the effect of parental income on search costs is higher than that of education relative to the corresponding effects on reservation values.

8 Elasticities

With the estimates of the model in hand, we now see how the minimum wage affects the probability of search, the probability of obtaining employment conditional on search, and the unconditional probability of employment. The elasticities of these three variables with respect to increasing the minimum wage are given in Table 8.

The table shows that with a minimum wage increase the probability of searching increases. However, this is counteracted by a decrease in the probability of finding a job conditional on searching. The overall employment elasticity is a modest -0.01. However, this is masking large changes in labor supply and demand that are effectively canceling out. Namely, the

\(^{24}\)All individuals with no reported income were assigned a parental income of 1.
Table 7: Estimates of the Search Parameters†

<table>
<thead>
<tr>
<th>Variable</th>
<th>Reservation Values</th>
<th>Search Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ln(Parental Income)‡</strong></td>
<td>0.054</td>
<td>-0.111</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.029)</td>
</tr>
<tr>
<td><strong>Missing Parental Income§</strong></td>
<td>0.579</td>
<td>-1.038</td>
</tr>
<tr>
<td></td>
<td>(0.151)</td>
<td>(0.240)</td>
</tr>
<tr>
<td><strong>Household Head Unemployed</strong></td>
<td>-0.011</td>
<td>-0.039</td>
</tr>
<tr>
<td></td>
<td>(0.095)</td>
<td>(0.128)</td>
</tr>
<tr>
<td><strong>Household Head Other</strong></td>
<td>-0.614</td>
<td>-0.524</td>
</tr>
<tr>
<td></td>
<td>(0.169)</td>
<td>(0.214)</td>
</tr>
<tr>
<td><strong>Household Head Some College</strong></td>
<td>0.182</td>
<td>-0.333</td>
</tr>
<tr>
<td></td>
<td>(0.050)</td>
<td>(0.087)</td>
</tr>
<tr>
<td><strong>Household Head College</strong></td>
<td>0.447</td>
<td>-0.5369</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.124)</td>
</tr>
<tr>
<td><strong>Household Head Post-College</strong></td>
<td>0.619</td>
<td>-0.664</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.150)</td>
</tr>
<tr>
<td><strong>Age=17</strong></td>
<td>-0.342</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.335)</td>
</tr>
<tr>
<td><strong>Age=18</strong></td>
<td>0.005</td>
<td>-0.496</td>
</tr>
<tr>
<td></td>
<td>(0.059)</td>
<td>(0.260)</td>
</tr>
<tr>
<td><strong>Age=19</strong></td>
<td>0.036</td>
<td>-0.666</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>(0.238)</td>
</tr>
<tr>
<td><strong>1/σ</strong></td>
<td>3.165</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.774)</td>
<td></td>
</tr>
</tbody>
</table>

†Estimated on 56,456 white male teenagers who attended school last week and whose primary residence is with their parents. Estimation of reservation values included state, year, and quarter fixed effects.
‡ All individuals who did not have a working household head or whose household head was working but did not report an income were assigned a zero for log income.
§ Takes on a value of one if the household head worked but did not report an income.
search elasticity with respect to the minimum wage is 0.21 while the match elasticity, how the probability of employment conditional on search changes with and increase in the minimum wage, is -0.23.

The changes in employment and search are not uniform across the population: over forty percent of teenagers see an increase in their overall employment probabilities due to their increased probability of search. The last four rows of this table show that these increased probability of employment is concentrated among those individuals who have household heads with a substantial amount of education. Indeed, over ninety percent of teenagers whose household heads have attended more than four years of school after high school see their employment probabilities increase. The reason for this increase is clear from the first column: these individuals are more responsive to the increase in the minimum wage, in part because they were less likely to search in the first place but also because these individuals are more willing to trade off a lower probability of employment for a higher expected wage conditional on employment.

To further examine who is more likely to be employed after a minimum wage increase, we examine the characteristics of those who have positive and negative employment elasticities. These results are given in Table 9. Here we see that those who see their probabilities of employment increase were less likely to search before the minimum wage increase. The next
Table 9: Means by Positive and Negative Employment Effects

<table>
<thead>
<tr>
<th></th>
<th>Negative Employment Effects</th>
<th>Positive Employment Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr(Search Before)</td>
<td>0.4912</td>
<td>0.3815</td>
</tr>
<tr>
<td>Weekly Parental Income† ($000’s)</td>
<td>0.8496</td>
<td>1.3043</td>
</tr>
<tr>
<td>Missing Parental Income‡</td>
<td>0.1156</td>
<td>0.2703</td>
</tr>
<tr>
<td>Household Head Unemployed</td>
<td>0.0783</td>
<td>0.0186</td>
</tr>
<tr>
<td>Single Parent</td>
<td>0.2143</td>
<td>0.1372</td>
</tr>
</tbody>
</table>

†Conditional on household head working and reporting an income.
‡Takes on a value of one if the household head worked but did not report an income.

sets of rows show that, similar to education, those who come from wealthier backgrounds and two parent families are much more likely to experience positive employment effects than their low income, single parent counterparts. Indeed, having an unemployed head makes it over four times as likely that the teenager will have a lower probability of being employed as a result of an increase in the minimum wage.

Although our methodologies are very different, these results echo the concerns raised by Lang and Kahn (1998) and Neumark and Wascher (1996) on the composition effects of minimum wage increases. The former find that increases in the minimum wage lead to substitution from adults to teenagers while the latter find substitution effects from those who are out of school to those who are in school. Here we find that substitution effects also occur among those who are still attending school, with those coming from wealthier and more educated families being the primary beneficiaries.

9 Conclusion

This paper has developed a theory of two-sided search to explain the puzzling absence of a large impact on employment levels when the minimum wage is increased. A minimum wage hike may induce some agents who were not looking for minimum wage work to enter the labor
force, which increases the pool of searching workers. At the same time, firms may be induced to decrease the number of jobs available because of the decreased expected surplus from each generated match.

In the classical framework, the exit by firms would dictate a decrease in employment. However, more general matching functions such as those commonly used in the search literature can generate positive employment effects from an increase in the minimum wage. In particular, if employment depends upon both the number of searching workers and the number of searching firms, the increase in the number of searching workers may more than offset the decrease in the number of searching firms. Even if positive employment effects result from a minimum wage hike, however, the probability of any individual worker finding a job has fallen. With employment probabilities falling, if any individuals are hurt from the minimum wage hike it will be those individuals who want the minimum wage jobs the most.

Estimating the structural model was made feasible by translating the firm’s zero profit condition into a function of the probability of a searching worker finding a match. The estimates of the model allow us to decompose the employment effects into their labor supply and labor demand components. Consistent with the theory, we find very small employment effects that are masking much larger changes in labor supply and labor demand.

However, while the employment effects cancel out in the population of teenagers we considered, they do not cancel out for certain sub-groups. Namely, those who have wealthier and more highly educated parents are highly likely to see positive employment effects. In contrast, those who are poorer, have less educated parents, and come from single parent households see their employment probabilities fall.

While this study has focused on teenagers enrolled in school, the potential effects of these teenagers on the market for adults in the low wage labor market are large. The small employment effects found in the previous literature may be masking much larger effects for those who have completed their schooling and find themselves in the low wage labor market. This group is likely to be searching for work regardless of the minimum wage and may be pushed out of the labor market by teenagers induced to search because of the higher minimum wage.
References


32


Appendix

Proof of Lemma 1

We show that, given A.1, if a worker searches, he accepts all matches. Note that if a worker would accept all matches without a minimum wage, he will also accept all matches when a minimum wage exists. Hence, it is sufficient to show that a worker will accept all matches without a minimum wage. Let $\epsilon_A$ be the expected value of $\epsilon$ given some cutoff value $\epsilon'$. Let $\pi_A$ be the probability that $\epsilon > \epsilon'$.

Suppose there exists a $\epsilon_A$ such that the worker finds it optimal to search and accept all matches above $S + \epsilon_A$:

$$p\pi_A(\beta(S + \epsilon_A) - R) - K > 0$$  \hspace{1cm} (12)
but that for all $\epsilon'$ such that $\epsilon' < \epsilon_A$ imply that $\beta(\bar{S} + \epsilon') < R$. Suppose $\xi < \epsilon_A$. Then for (12) to hold,

$$p\pi_A(\beta(\bar{S} + \epsilon_A) - R) - K > p\pi_A(\beta(\bar{S} + \xi) - R)$$

$$p\pi_A\beta(\epsilon_A - \xi) > K$$

$$\pi_A\beta(\epsilon_A - \xi) > K$$

where the last inequality holds because $0 \leq p \leq 1$. But the last inequality also violates A.1 and we have a contradiction. QED.

**Proof of Proposition 1**

Note that conditional on any $N \in [0, \bar{N}]$, as $J \to \infty$, $q \to 0$. There then exists a $\bar{J}$ such that for all $N$ if $J' > \bar{J}$, profits are negative. Since the partial derivative of $\pi$ is negative with respect to $J$,

$$\frac{\partial \pi}{\partial J} = -\frac{q\alpha(\bar{S} - E(W))}{J} < 0$$

We know that for each value of $N$ there is at most one value of $J$ such that $\pi = 0$.

Similarly, define $V$ as the search value. Since $\partial V/\partial N$ is also negative,

$$\frac{\partial V}{\partial N} = -\frac{p(1 - \alpha)(E(W) - R)}{N} < 0$$

We know that for each $J$ there is at most one value of $N$ such that $V = 0$.

We can then define the following mappings:

$$f_1 = \begin{cases} 
\pi(J, N) & \text{for } J \in [0, \bar{J}], N \in [0, \bar{N}] \\
\max\{\pi(0, N), 0\} & \text{for } J = 0, N \in [0, \bar{N}] \\
\min\{V(J, \bar{N}), 0\} & \text{for } J \in [0, \bar{J}], N = \bar{N} \\
V(J, N) & \text{for } J \in [0, \bar{J}], N \in (0, \bar{N}) \\
\max\{V(J, 0), 0\} & \text{for } J \in [0, \bar{J}], N = 0
\end{cases}$$

$$f_2 = \begin{cases} 
\pi(J, N) & \text{for } J \in [0, \bar{J}], N \in [0, \bar{N}] \\
\max\{\pi(0, N), 0\} & \text{for } J = 0, N \in [0, \bar{N}] \\
\min\{V(J, \bar{N}), 0\} & \text{for } J \in [0, \bar{J}], N = \bar{N} \\
V(J, N) & \text{for } J \in [0, \bar{J}], N \in (0, \bar{N}) \\
\max\{V(J, 0), 0\} & \text{for } J \in [0, \bar{J}], N = 0
\end{cases}$$

Then for each value of $N$, there exists a unique value of $J \in [0, \bar{J}]$ that satisfy $f_1 = 0$. Further, since $\pi$ is continuous in $N$, this unique value is a continuous function of $N$. Similarly, for each $J$, there is a unique $N \in [0, \bar{N}]$ satisfying $f_2$ which is continuous in $J$. We can then use functions to define a continuous vector valued function mapping from $[0, \bar{J}] \times [0, \bar{N}]$ into
itself. Then by Brouwer’s fixed point theorem there exists a doublet \( \{J^*, N^*\} \) where \( f_1 = 0 \) and \( f_2 = 0 \). QED.

**Proof of Proposition 2**

Consider the equilibrium before the minimum wage increase where expected wages are given by \( E_1(W) \) and the probability of a firm matching is given by \( q_1 \). The firm’s expected zero profit condition is:

\[
q_1(S - E_1(W)) - C = 0
\]

Note that \( E_1(W) \) is weakly increasing in the minimum wage. Note that if a minimum wage increase does not affect the expected wage the probability of matching for the firm will remain the same. However, if \( E_2(W) > E_1(W) \) then \( q_2 > q_1 \) for the zero profit condition to still bind. Note further that the probabilities of firms and workers matching is given by:

\[
q = A \left( \frac{N}{J} \right)^\alpha \\
p = A \left( \frac{J}{N} \right)^{1-\alpha}
\]

The expression for the firm implies that \( \frac{N}{J} \) must increase for the zero profit condition to bind. But if this fraction increases then \( p \) must fall. QED.

**Proof of Proposition 3**

Differentiating the matching function with respect to the minimum wage yields:

\[
\frac{dx}{dW} = \alpha q \frac{dJ}{dW} + (1-\alpha) p \frac{dN}{dW}
\]

Rewrite as:

\[
\frac{dx}{dW} = x \left( \alpha \frac{dJ}{dW} + (1-\alpha) \frac{dN}{dW} \right)
\]

\[
= x \left( \alpha \frac{dJ}{dW} + (1-\alpha) \frac{dN}{dW} \right)
\]

\[
= \frac{x}{W} \left( \alpha \frac{dJ}{dW} + (1-\alpha) \frac{dN}{dW} \right)
\]

\[
= \frac{x}{W} (\alpha \varepsilon_{LD} + (1-\alpha) \varepsilon_{LS})
\]

Therefore, for the employment effect to be positive \( \left( \frac{dx}{dW} > 0 \right) \), it must be that \( (\alpha \varepsilon_{LD} + (1-\alpha) \varepsilon_{LS}) > 0 \), where \( \varepsilon_{LD} \) is the elasticity of labor demand and \( \varepsilon_{LS} \) is the elasticity of labor supply. QED
Proof of Proposition 4

In order to obtain conditions under which employment is increasing in the minimum wage, we first need to know how $J$ and $N$ respond to changes in the minimum wage. The zero profit condition and the worker indifference condition are as follows:

$$F_1 = q(S - E(W)) - C$$
$$F_2 = (1 - p)R^* + p \cdot E(W) - K - R^*$$

Rewrite $F_2$ such that:

$$F_2 = p(E(W) - R^*) - K$$

where $E(W) = E(\max\{\beta(S + \epsilon_{ij}), W\})$. Note that $\frac{\partial E(W)}{\partial W} = \frac{\partial E(\max\{\beta(S + \epsilon_{ij}), W\})}{\partial W}$, which equals 1 with some probability and 0 with some probability. Therefore, we define $\frac{\partial E(W)}{\partial W} = \xi$, where $0 \leq \xi \leq 1$.

The Jacobian of the system of equations is then:

$$B = \begin{pmatrix} \frac{\partial F_1}{\partial J} & \frac{\partial F_1}{\partial N} \\ \frac{\partial F_2}{\partial J} & \frac{\partial F_2}{\partial N} \end{pmatrix}$$

and,

$$\frac{\partial F_1}{\partial J} = \frac{(\alpha - 1)q(S - E(W))}{J} < 0$$
$$\frac{\partial F_1}{\partial N} = \frac{(1 - \alpha)q(S - E(W))}{N} > 0$$
$$\frac{\partial F_2}{\partial J} = \frac{\alpha p(E(W) - R^*)}{J} > 0$$
$$\frac{\partial F_2}{\partial N} = -\frac{\alpha p(E(W) - R^*)}{N} - p\frac{\partial R^*}{\partial N} < 0$$
$$\frac{\partial F_1}{\partial W} = -q\xi < 0$$
$$\frac{\partial F_2}{\partial W} = p\xi > 0$$

By the implicit function theorem:

$$\begin{pmatrix} \frac{\partial J}{\partial W} \\ \frac{\partial N}{\partial W} \end{pmatrix} = -B^{-1}\begin{pmatrix} \frac{\partial F_1}{\partial W} \\ \frac{\partial F_2}{\partial W} \end{pmatrix}$$
where

\[
B^{-1} = \frac{1}{\text{Det}(B)} \begin{pmatrix}
\frac{\partial F_2}{\partial N} & -\frac{\partial F_1}{\partial J} \\
-\frac{\partial F_1}{\partial J} & \frac{\partial F_2}{\partial J}
\end{pmatrix}
\]

and \(\text{Det}(B)\) can be written as:

\[
\text{Det}(B) = \frac{\partial F_1}{\partial J} \frac{\partial F_2}{\partial N} - \frac{\partial F_1}{\partial J} \frac{\partial F_2}{\partial N} = -\left( \frac{\alpha p(E(W) - R^*)}{N} - \frac{\alpha p R^*}{\partial N} \right) \times \left( 1 - \alpha \right) S - E(W) < 0
\]

\[
\frac{dN}{dW} = \frac{\frac{\partial F_2}{\partial J} \frac{\partial F_1}{\partial N} - \frac{\partial F_1}{\partial J} \frac{\partial F_2}{\partial N}}{\text{Det}(B)}
\]

\[
= \frac{\xi}{J} \left( 1 - \frac{\alpha}{1 - \alpha} \frac{E(W) - R^*}{S - E(W)} \right)
\]

and,

\[
\frac{dJ}{dW} = \frac{\frac{\partial F_2}{\partial J} \frac{\partial F_1}{\partial W} - \frac{\partial F_1}{\partial J} \frac{\partial F_2}{\partial W}}{\text{Det}(B)}
\]

\[
= \frac{J}{N} \frac{\xi}{\frac{\partial R^*}{\partial N}} \left( 1 - \frac{\alpha}{1 - \alpha} \frac{S - E(W)}{S - E(W)} \right) - \frac{\frac{1}{N} \frac{\partial R^*}{\partial N}}{(1 - \alpha)(S - E(W))}
\]

\[
= \frac{J}{N} \frac{\xi}{dW} \left( 1 - \alpha \frac{S - E(W)}{S - E(W)} \right) - \frac{\frac{1}{N} \frac{\partial R^*}{\partial N}}{(1 - \alpha)(S - E(W))}
\]

With the information on \(\frac{dJ}{dW}\) and \(\frac{dN}{dW}\), we can now substitute in for the labor demand and supply elasticities.

\[
\alpha \varepsilon_{LD} + (1 - \alpha) \varepsilon_{LS} = \frac{\alpha}{dW} \frac{dJ}{dW} W + (1 - \alpha) \frac{dN}{dW} W - \frac{\alpha}{N} \frac{dN}{dW} 
\]

\[
= \frac{W}{J} \left[ \alpha \left( \frac{1}{N} \frac{dN}{dW} - \frac{\xi}{(1 - \alpha)(S - E(W))} \right) \right] + (1 - \alpha) \frac{1}{N} \frac{dN}{dW}
\]

\[
= \frac{W}{J} \left[ -\frac{\alpha}{1 - \alpha} \frac{\xi}{S - E(W)} + \frac{1}{N} \frac{dN}{dW} \right]
\]

\[
= \varepsilon_{LS} - \frac{\alpha}{1 - \alpha} \frac{\xi W}{S - E(W)}
\]

\[
= \frac{\xi W}{N} \frac{\partial R^*}{\partial N} \left( 1 - \alpha \frac{E(W) - R^*}{S - E(W)} \right) - \frac{\alpha}{1 - \alpha} \frac{\xi W}{S - E(W)}
\]
Given that $R$ has support $[0, R]$, simplifying, we have:

$$\alpha \varepsilon_{LD} + (1 - \alpha) \varepsilon_{LS} = \frac{\xi W}{N} \cdot \frac{\partial R^*}{\partial N} \left[ 1 - \frac{\alpha}{1 - \alpha} \frac{E(W)}{S - E(W)} \right] + \frac{\alpha \xi W}{N} \cdot \frac{R^* - \partial R^*}{\partial N}$$

Since $R$ has uniform distribution, $R^* = \frac{N}{N_R}$ and $\frac{\partial R^*}{\partial N} = \frac{1}{N_R}$. We set $N = \bar{N}$ to get a sufficiency condition, and simplify to get:

$$1 - \frac{\alpha}{1 - \alpha} \frac{E(W)}{S - E(W)} > 0$$

QED

**Proof of Proposition 5**

In order for all workers to benefit from an increase in the minimum wage it is sufficient to show that the workers with the lowest reservation values, zero, are made better off by the increase. The value of search for these workers can be written as:

$$V = A \left( \frac{N}{J} \right)^{\alpha - 1} E(W) - K$$

Note that the zero profit condition for firms can be written as:

$$A \left( \frac{N}{J} \right)^{\alpha} (S - E(W)) - C = 0$$

and that both of these conditions depend on $N$ and $J$ only through the ratio $N/J$. Further, the zero profit condition for the firm is an identity. Differentiating profits with respect to an increase in the minimum wage yields:

$$A \left( \frac{N}{J} \right)^{\alpha} \left( (S - E(W)) \left( \frac{N}{J} \right)^{-1} \left( d \frac{N}{J} \right) dW - \frac{dE(W)}{dW} \right) = 0$$

Solving for $d(N/J)/dW$ yields:

$$d \left( \frac{N}{J} \right) dW = \frac{N}{(S - E(W)) J} \frac{dE(W)}{dW}$$

We now have all components necessary to sign $dV/dW$ for those with a reservation value of zero. Differentiating $V$ with respect to $W$ yields:

$$E(W) A (\alpha - 1) \left( \frac{N}{J} \right)^{\alpha - 2} \frac{d \left( \frac{N}{J} \right)}{dW} + A \left( \frac{N}{J} \right)^{\alpha - 1} \frac{dE(W)}{dW}$$

substituting in for $d(N/J)/dW$ and rewriting yields:

$$pdE(W) \left[ 1 - \frac{(1 - \alpha) E(W)}{\alpha (S - E(W))} \right]$$

Since $dE(W)/dW > 0$, we have the result. QED
Table 10: Estimates of $\beta$

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<th>State</th>
<th>Coefficient</th>
<th>Std. Error</th>
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