INTELLECTUAL PROPERTY RIGHTS, FOREIGN DIRECT INVESTMENT AND INDUSTRIAL DEVELOPMENT*

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This article develops a North–South product cycle model in which innovation, imitation and the flow of foreign direct investment (FDI) are all endogenously determined. In the model, a strengthening of intellectual property right (IPR) protection in the South reduces the rate of imitation and it increases the flow of FDI. Indeed, the increase in FDI more than offsets the decline in the extent of production undertaken by Southern imitators so that the South’s share of the global basket of goods increases. Furthermore, while multinationals charge higher prices than Southern imitators, real wages of Southern workers increase while those of Northern workers fall.

How does the strengthening of intellectual property right (IPR) protection by developing countries impact their industrial development? How does it affect their ability to attract foreign direct investment (FDI)? Does it increase the rate of innovation in the global economy? These and related questions have been at the heart of an ongoing debate that was brought into sharp relief during the negotiations preceding the ratification of the WTO’s Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPS) in 1995. Opposition to stronger IPR regimes in developing countries rests on two general arguments. First, there is concern that consumer welfare may be adversely impacted by enhancing the monopoly powers of innovators. Second, there is fear that stronger IPR protection in developing countries will hamper their ability to absorb foreign technologies without having any appreciable effect on Northern innovation.1 On the other side, TRIPS supporters argue that stronger IPRs world-wide will not only increase incentives for innovation but also foster industrial development in developing countries by encouraging multinationals.

1 For example, a critic of stronger IPR enforcement in developing countries may argue that the rapid postwar industrialisation in East Asian countries such as Japan and South Korea was achieved under relatively weak IPR regimes and that a premature imposition of a strong IPR regime could retard the industrial development of today’s developing countries. See Maskus (2000), who notes these arguments, and the overview and evidence presented in Ordover (1991) and Maskus and McDaniel (1999). On South Korea, see Westphal et al., (1985). For criticisms of stronger IPRs which stress static welfare losses, see McCalman (2001) and Chaudhuri et al. (2006).
to shift production there.\textsuperscript{2} In this study, we seek to illuminate this important debate by developing a North–South product cycle model in which Northern innovation, Southern imitation and the North–South flow of FDI respond endogenously to changes in the degree of Southern IPR protection available to Northern firms. Building on Grossman and Helpman (1991\textit{b}), the model provides a unified framework for assessing some of the key arguments for and against stronger IPR regimes in developing countries.

The theoretical product cycle literature on the effects of Southern IPR protection has been built on two types of growth models analysed in great detail in Grossman and Helpman (1991\textit{a}) – the variety expansion model and the quality ladders model. Important contributions to this literature were subsequently made by Helpman (1993) and Lai (1998) both of which utilised the variety expansion model and Glass and Saggi (2002) who adopted the quality ladders approach. This research established that the effects of increased IPR protection in the South on the Northern rate of innovation depend very much on whether production shifts to the South via imitation of Northern firms or via North–South FDI. Furthermore, Helpman (1993) forcefully drove home the point that while stronger Southern IPR protection can indeed increase the pace of Northern innovation, such a policy change does not necessarily benefit the South since it reallocates production in favour of Northern firms whose prices tend to be higher than those of Southern ones. Thus, international production shifting matters not just for the nature and the extent of innovation but also welfare. Two important features of our model help to shed further light on these arguments. First, like Lai (1998), the rate of innovation and the flow of North–South FDI respond endogenously to changes in the degree of Southern IPR protection. Second, like Grossman and Helpman (1991\textit{b}), imitation is treated as a costly activity and the Southern rate of imitation is endogenously determined.\textsuperscript{3}

Making both imitation and FDI endogenous helps to push forward the literature on North–South product cycle models of international trade. Furthermore, since imitation is a costly activity in the real world, analyses that treat it as exogenous fail to capture how changes in the Southern IPR regime alter the allocation of Southern resources among imitation and production. In addition to realism, an important reason for treating imitation as an endogenous activity is that North–South product cycle models with exogenous imitation have yielded remarkably different conclusions regarding the relationship between imitation and innovation from those that have treated it as endogenous. In a model with endogenous imitation and innovation, Grossman and Helpman (1991\textit{b}) uncovered a positive relationship between the two activities while Lai (1998) found that when the rate of imitation is exogenously given and Northern firms can undertake FDI in the South, the relationship between them is negative.\textsuperscript{4}

\textsuperscript{2} See Paul Romer (1993) for an insightful discussion of how and why FDI can contribute to the economic development of poor countries by helping bridge the ‘idea gap’ that they face with respect to developed countries.

\textsuperscript{3} Helpman (1993) noted that ‘...imitation is an economic activity much the same as innovation; it requires resources and it responds to economic incentives...’.

\textsuperscript{4} It is worth noting here that results also depend upon the type of innovation being considered: the quality ladders model of Glass and Saggi (2002) and Glass and Wu (2007) behave rather differently from the variety expansion models analysed in our article.

In our model, a strengthening of IPR protection in the South reduces the incentive of Southern firms to imitate Northern multinationals. This decline in imitation risk has two important consequences for production and innovation. First, the South becomes a more attractive production location for Northern firms. Second, since all Northern firms have the option to shift production to the South, an increase in the value of multinational firms increases Northern incentives for innovation. Furthermore, we find that the intra-regional reallocation of Southern production (from Southern imitators to Northern multinationals) that results from a strengthening of Southern IPR protection is dominated by the accompanying inter-regional reallocation of production: in other words, the South’s share of the global basket of goods increases with a strengthening of Southern IPR protection.

Our analysis also provides some interesting insights with respect to the effects of Southern IPR protection on prices and wages in the two regions. First, by making the South a more attractive location for production and thereby shifting aggregate labour demand from the North to the South, a strengthening of Southern IPR protection lowers the North’s relative wage.5 Second, since Northern multinationals charge lower prices relative to firms that produce in the North, the increase in FDI helps to lower prices. However, this beneficial effect on prices is partially offset by the intra-regional reallocation of Southern production from local imitators to multinationals since a typical multinational charges a higher price than a Southern imitator. Due to the nature of pricing behaviour under Dixit and Stiglitz (1977) preferences (prices are mark-ups over marginal costs), these changes in prices and nominal wages translate into clear-cut effects on real wages in the two regions: while Northern real wages decline due to stronger Southern IPR protection, Southern real wages increase. More specifically, the purchasing power of Southern workers in terms of Northern goods increases whereas their ability to purchase goods produced by Southern imitators and multinationals remains unaffected.

As noted earlier, a key argument in favour of weak IPR protection in the South is that Southern imitation lowers prices. Since Southern imitators price below Northern multinationals, this channel is also operative in our model. However, this argument ignores the labour market effects that accompany the increase in international production shifting induced by stronger IPR protection in the South. By contrast, in our model, a strengthening of Southern IPR protection raises real wages of Southern workers.6

Our model abstracts from Southern innovation. While this assumption is a good approximation for the case of many small developing countries, it is on weaker grounds insofar as some of the larger developing countries are concerned. If the South has the ability to innovate, Southern IPR reform has the potential to increase local incentives for innovation. Chen and Puttitanum (2005) provide a two-sector oligopolistic model in which the optimal level of IPR protection in the

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5 This result contrasts with those of Krugman (1979) and Grossman and Helpman (1991b) who found a negative relationship between Southern imitation and the North’s relative wage. Our result differs because imitation targets multinationals in our model whereas it targets Northern producers in theirs.

6 The real wage effects captured by our model would not arise in partial equilibrium models that ignore the labour market effects of IPR reforms. Furthermore, such effects should only be expected to arise when IPR reforms are undertaken on an economy-wide basis as opposed to being focused on a few sectors.

South balances the trade-off between encouraging imitation of advanced Northern technologies and providing incentives for local innovation. While introducing Southern innovation is beyond the scope of the present study, it is worth noting that the considerations highlighted by Chen and Puttitanum (2005) are likely to strengthen the argument in favour of Southern IPR reform – in our model, such reform confers some benefits on the South even though it is assumed to lack the ability to innovate.\(^7\)

While we endogenise the production location decision of a Northern firm, we do not consider the question of internalisation – that is, in our model, all technology transfer to the South occurs via FDI and arms length arrangements such as licensing are not considered. Antràs (2005) develops a North–South product cycle model in which the incompleteness of international contracts determines the choice between arms length technology transfer and FDI. His analysis shows that the effects of changes in the rate of technological standardisation on the North–South relative wage are quite different from those of changes in the rate at which new goods appear. This suggests that the effects of Southern IPR protection on wages in the two regions might also vary with the type of technological change being considered.\(^8\)

The relationship between FDI and IPR protection has received significant empirical scrutiny in the literature.\(^9\) As the survey by Park (2008) notes, as far as US data is concerned, there appears to be a clear positive relationship between the degree of IPR enforcement in developing countries and investment by US firms – see, for example, Lee and Mansfield (1996) and Nunnenkamp and Spatz (2004). Results derived from non-US data portray a more mixed picture: while Mayer and Pfister (2001) find a negative effect of stronger patent rights on location decisions of French multinationals, Javorcik (2004a) finds that stronger patent rights in Eastern Europe and former Soviet Union states have a positive effect on FDI in high-technology sectors. The most recent and perhaps the most relevant empirical study for our purposes is that by Branstetter et al. (2011). They investigate the impact of IPR reform on multinational production by analysing the responses of US multinationals to a series of well-documented IPR reforms by 16 countries in the 1980s and 1990s. Consistent with our model, they find that US-based multinationals expand the scale of their activities in reforming countries after IPR reform. They also analyse UN industry-level data from reforming countries and show that industry-level value added increases after reforms, particularly in technology-intensive industries.\(^10\)

\(^7\) He and Maskus (forthcoming) have shown that when the South invests in innovation and there exists a ‘backward’ spillover from the South to the North, there can be a U-shaped relationship between North–South FDI and the risk of imitation. When Southern innovation is not possible and the risk of imitation is exogenously given, like us, He and Maskus (forthcoming) find a negative relationship between FDI and imitation.\(^8\) Since we do not model internalisation, our model does not include the process of ongoing standardisation that plays a crucial role in the incomplete contracts framework of Antràs (2005).\(^9\) For a nuanced and detailed discussion of this literature, see Maskus (2000).\(^10\) Following Feenstra and Rose (2000), they also construct for each reforming country an annual count of ‘initial export episodes’ – the number of 10-digit commodities for which recorded US imports from a given country exceed zero for the first time. This serves as a rough indicator of the net rate at which production shifts to the reforming countries, capturing changes in multinational production as well as indigenous imitation. This net rate of production shifting increases sharply after IPR reform, suggesting that any decline in indigenous imitation is more than offset by the increase in the range of goods produced by multinational affiliates.
The rest of the article is organised as follows. Section 1 presents the model. Sections 2 describes the effects of a strengthening of Southern IPR protection. In Section 3, we provide an extensive discussion of the robustness of our main results. Section 4 concludes. There is also a Mathematical Appendix.

1. Model

Consider a world comprised of two regions: North and South. Labour is the only factor of production and region \(i\)'s labour endowment equals \(L_i, i = N, S\). As in Grossman and Helpman (1991b) preferences are identical in the two regions and a representative consumer chooses instantaneous expenditure \(E(t)\) to maximise utility at time \(t\):

\[
U = \int_t^\infty e^{-\rho(t-\tau)} \log D(\tau) d\tau
\]

subject to the intertemporal budget constraint

\[
\int_t^\infty e^{-r(t-\tau)} E(\tau) d\tau = \int_t^\infty e^{-r(t-\tau)} I(\tau) d\tau + A(t) \quad \text{for all } t,
\]

where \(\rho\) denotes the rate of time preference; \(r\) the nominal interest rate; \(I(\tau)\) instantaneous income and \(A(t)\) the current value of assets. The instantaneous utility \(D(\tau)\) is given by

\[
D = \left[ \int_0^n x(j) d\tau \right]^{1/2},
\]

where \(x(j)\) denotes the consumption of good \(j\); \(n\) the number of goods available and \(0 < \alpha < 1\).

As is well known, under the above assumptions, the consumer’s optimisation problem can be broken down into two stages. First, he chooses how to allocate a given spending level across all available goods. Second, he chooses the optimal time path of spending. The instantaneous utility function \(D(\tau)\) implies that the elasticity of substitution between any two goods is constant and equals \(\varepsilon = 1/(1-\alpha)\) and demand for good \(j\) (given expenditure \(E\)) is given by

\[
x(j) = \frac{Ep(j)^{-\varepsilon}}{P^{1-\varepsilon}},
\]

where \(p(j)\) denotes the price of good \(j\) and \(P\) a price index such that

\[
P = \left[ \int_0^n p(j)^{1-\varepsilon} d\tau \right]^{1/(1-\varepsilon)}.
\]

Furthermore, as is well known, under the two-stage procedure the optimal spending rule is given by

\[
\frac{\dot{E}}{E} = r - \rho,
\]
that is, nominal consumption spending grows at a rate equal to the difference between the interest rate and the subjective rate of time preference.

1.1. Product Market

Three types of firms produce goods: Northern firms (N), Northern multinationals (M) and Southern imitators (S). Denote firms by $J$ where $J = N, M$, or $S$. Northern firms can either produce in the North or the South. A Northern firm needs one worker to produce a unit of output in the North, whereas $\theta \geq 1$ workers per unit of output are needed in the South. Intuitively, this is due to the costs of coordinating decisions over large distances and operating in unfamiliar foreign environments. Indeed, the theory of the multinational enterprise argues that such firms rely on ‘ownership’ advantages derived from technological assets and/or brand names in order to offset the disadvantages they face relative to local firms (Markusen, 1995).

Given the constant elasticity demand functions, it is straightforward to show that prices of Northern firms are mark-ups over their marginal costs:

$$p_N^N = \frac{w_N^N}{\alpha} \quad \text{and} \quad p_M^M = \theta \frac{w_S^S}{\alpha}.$$  \hspace{2cm} (7)

Southern firms can produce only those goods that they have successfully imitated and they need one worker to produce one unit of output. If successful in imitating a multinational, a Southern firm charges its optimal monopoly price

$$p_S^S = \frac{w_S^S}{\alpha}.$$  \hspace{2cm} (8)

Note that this price can be sustained if and only if it lies below the multinational’s marginal cost $\theta w_S^S$:

$$\frac{w_S^S}{\alpha} < \theta w_S^S \Leftrightarrow \theta \alpha > 1.$$  \hspace{2cm} (9)

In what follows, we assume $\theta \alpha > 1$.$^{11}$

Let $x^J$ denote the output level of firm $J$ where $J = N, M$, or $S$. We know from the demand functions that

$$\frac{x(i)}{x(j)} = \frac{p_i^e}{p_j^e}. \hspace{2cm} (7)$$

Using the pricing equations for the three types of products, we have

$$\frac{x_S^S}{x_M^M} = \theta^e, \hspace{2cm} (8)$$

and

$^{11}$ When $\theta \alpha < 1$, a Southern imitator limit prices the Northern firm whose product it has copied by setting its price equal to the Northern firm’s marginal cost $\theta w_N^N$. An earlier version of the article also analysed the case where Southern firms limit price. The analysis of this limit pricing case yields no additional insights regarding the main questions of interest and we omit its discussion in order to conserve space.
Flow profit of a Northern producer is given by

\[ \pi^N = (p^N - w^N)x^N = \frac{(1 - \alpha)w^N x^N}{\alpha}. \]  

(10)

Similarly, a multinational’s flow profit equals

\[ \pi^M = (p^M - \theta w^S)x^M = \frac{\theta(1 - \alpha)w^S x^M}{\alpha}, \]  

(11)

while that of a Southern firm equals

\[ \pi^S = (p^S - w^S)x^S = \frac{(1 - \alpha)w^S x^S}{\alpha}. \]  

(12)

1.2. FDI and Imitation

Of the \( n \) goods that exist, \( n_N \) are produced in the North, \( n_M \) are produced in the South by Northern multinationals and \( n_I \) are produced by Southern imitators. Let \( n_S \equiv n_I + n_M \) denote all goods produced in the South. In what follows, we think of the level of Southern industrial development as the Southern share of global manufacturing; that is, the ratio of goods produced in the South to the number of goods that exist at a point in time. Since this measure of industrial development explicitly includes the activities of affiliates of Northern multinationals, the advance of Southern industrial development in our model depends on the rate of FDI.

Let the rate of imitation \( \mu \) be defined by

\[ \mu \equiv \frac{\dot{n}_I}{n_M}, \]  

(13)

that is, \( \mu \) denotes the rate of increase of the stock of imitated goods relative to the total number of goods produced by Northern multinationals. Since both multinationals and Southern imitators produce in the South, imitation simply transfers ownership of a good (and the associated flow of profits) from the hands of a multinational to a Southern imitator.

The rate of North–South FDI is defined by

\[ \phi \equiv \frac{\dot{n}_S}{n_N}, \]  

(14)

where \( n_N \) denotes the number of goods produced in the North. In other words, at each instant, the total stock of goods produced in the South increases by \( \phi n_N \). Note that this measures the inflow of North–South FDI because imitation only targets Northern multinationals and does not, by itself, lead to North–South production shifting.
Like Grossman and Helpman (1991b) and Lai (1998), we study a steady state equilibrium in which prices, nominal spending, and all product categories grow at the same rate $g$:

$$
g \equiv \frac{\dot{n}}{n} = \frac{\dot{n}_N}{n_N} = \frac{\dot{n}_I}{n_I} = \frac{\dot{n}_M}{n_M} = \frac{\dot{n}_S}{n_S} = \frac{\dot{E}}{E}. \quad (15)
$$

Equations (6), (14) and (15) imply that in steady state the interest rate equals the sum of the subjective discount rate and the growth rate:

$$
r = \rho + g.
$$

Furthermore, the steady state allocation of products across the two regions satisfies

$$
\frac{n_N}{n} = \frac{g}{g + \phi} \quad \text{and} \quad \frac{n_S}{n_M} = \frac{\phi}{g}.
$$

Similarly, the ratio of multinationals to their two types of competitors equals

$$
\frac{n_M}{n_N} = \frac{\phi}{g + \mu} \quad \text{and} \quad \frac{n_M}{n_I} = \frac{\mu}{g}.
$$

As in Grossman and Helpman (1991b), the lifetime value of a Southern producer (i.e. the reward earned by a successful imitator) equals

$$
v^S = \frac{\pi^S}{\rho + g}.
$$

Note from above that since future products creates competition for existing products, an increase in the rate of innovation ($g$) reduces the life-time value of a Southern firm. Similarly, the lifetime steady-state value of a Northern firm that opts to produce in the North equals:

$$
v^N = \frac{\pi^N}{\rho + g}.
$$

While it is cheaper to produce in the South (as we show below, the Southern relative wage is lower in equilibrium), shifting production to the South invites the risk of imitation and the value of a Northern multinational firm equals

$$
v^M = \frac{\pi^M}{\rho + \mu + g}.
$$

As is clear, in calculating the value of a multinational firm, the flow profit $\pi^M$ is discounted not just by the effective interest rate (which equals $\rho + g$) but also by the rate of imitation $\mu$. In our model, imitation targets only Northern multinationals and the risk faced by Northern firms that refrain from shifting production to the South has been normalised to zero.\footnote{In Section 3.1, we discuss in detail why such a formulation of Southern imitation is sensible in the context of our model.}

In reality, Northern firms that do not undertake FDI can also have their technologies imitated but the risk of imitation they face is probably...
lower than that of multinational firms that produce in the South. As is known from the work of Mansfield (1994), Lee and Mansfield (1996) and Maskus (2000), multinational firms indeed internalise the risk of imitation that they face due to weak IPR protection in host countries.

As is clear, our modelling of the FDI decision is rather simple: it abstracts from the usually studied trade-off between exporting at a higher marginal cost to the Southern market versus bearing the fixed cost of building another plant there to serve the local market. We abstract from these considerations mainly for tractability and for focusing on the aggregate response of international production shifting to changes in imitation risk induced by IPR reform. In the real world, one observes ‘horizontal’ multinationals producing the same good in different locations. By design, such multinationals do not arise in our model.\textsuperscript{13} To see briefly why we adopt this route, note that if a multinational were to split output across countries the question of imitation risk becomes more complicated. Should the risk of imitation depend positively upon the share of output produced by a multinational in the South? Or should it be independent of it in that any level of production shifting leads to the same risk? The underlying logic of our model would be more consistent with the former approach but adopting it complicates the model substantially since the risk of imitation faced by a firm would then depend on its allocation of production across the two regions. On the other hand, if the second formulation is adopted, our model does not have much to add to the already rich literature explaining the existence of horizontal multinationals. As a result, we have chosen the simpler formulation that allows us to focus on the main questions motivating our analysis.

1.3. North–South Relative Wage

Since all Northern firms have the option of becoming multinationals, we must have \( v^N = v^M \) which implies

\[
\frac{\pi^M}{\pi^N} = 1 + \frac{\mu}{\rho + g}.
\]

Note immediately from above that if the risk of imitation is positive (i.e. \( \mu > 0 \)) then we must have \( \pi^M > \pi^N \). This is intuitive: since any Northern firm is free to become a multinational, the flow profit earned by a multinational must be higher in order to compensate for the risk of imitation faced (only) by multinationals.\textsuperscript{14}

From the definition of profit we have

\[
\frac{\pi^M}{\pi^N} = \frac{\theta w^S x^M}{w^N x^N} = \left( \frac{\theta w^S}{w^N} \right)^{1-\epsilon}
\]

\textsuperscript{13} Note that the transfer of technology from the parent firm that undertakes the R&D in the North to its affiliate in the South can be viewed as giving rise to a ‘vertical multinational’ and all the multinationals in our model are of this type. We thank an anonymous referee for drawing our attention to the fact that our model abstracts from horizontal FDI, an important type of FDI in the real world. See Markusen (1995) for an insightful survey of theories explaining the emergence of such multinationals.

\textsuperscript{14} Indeed, since prices of Northern firms and multinationals are marked up over their respective marginal costs by the same amount (i.e. \( 1/\epsilon \)) the relative value of sales of a typical multinational must exceed that of a Northern firm: \((p^M x^M)/(p^N x^N) = 1 + \mu/(\rho + g)\).
The last two equations allow us to write the Northern relative wage \((w^R)\) as a function of the rate of innovation and imitation as well as some of the exogenous parameters of the model:

\[
w^R \equiv \frac{w^N}{w^S} = \theta \left( 1 + \frac{\mu}{\rho + g} \right)^{1-1}
\]

As is clear, the relative wage in the North increases with the production disadvantage faced by Northern multinationals \((\theta)\) as well as with the Southern rate of imitation \((\mu)\) since both of these factors discourage Northern firms from relocating production to the South. This reluctance to shift production to the South increases the relative demand for Northern labour and therefore North’s relative wage.

As we noted earlier, this result differs from that of Grossman and Helpman (1991b) and is line with Lai (1998). Why do these models yield such different results regarding the determinants of the North–South relative wage? In Grossman and Helpman (1991b), Southern imitation of firms producing in the North serves as the channel through which international reallocation of production (and therefore labour demand) occurs. By contrast, in our model as well as in Lai (1998) Southern imitation targets multinational firms and North–South FDI is the channel of international reallocation of production. In our model, by lowering the risk of imitation, a strengthening of Southern IPR protection increases the incentive for FDI and the demand for Southern labour while it reduces demand for Northern labour. In Grossman and Helpman (1991b), the opposite happens: as imitation declines, more production stays in the North and less of it occurs in the South. Hence the North–South relative wage behaves rather differently across these models.

### 1.4. Free Entry

Free entry into innovation implies that the value of a Northern firm must exactly equal the cost of innovation:

\[
v^N = \frac{w^N a_N}{n} \leftrightarrow \pi^N = \frac{w^N a_N}{n},
\]

where \(a_N\) is the unit labour requirement in innovation and \((w^N a_N)/n\) measures the up-front cost of product development. This formulation assumes that the cost of designing new products falls with the number of products \((n)\) that have been invented. In other words, knowledge spillovers from innovation sustain further innovation. This assumption is standard in the literature (Romer, 1990; Grossman and Helpman, 1991a,b) and in its absence growth cannot be sustained in our variety expansion model with fixed resources. This is because the flow profit of a successful innovator declines with the number of products invented and incentives for innovation disappear in the long run if the cost of innovation does not also fall with an increase in the number of products.

Substituting from (19) into (22) gives the output level of a Northern firm

\[
x^N = \frac{a_N x (\rho + g)}{n(1 - x)}.
\]
Let the unit labour requirement in imitation be $a_I$ and the cost function for imitation be given by

$$q_I = \frac{ka_I w^S}{n_S}, \quad (24)$$

where $n_S = n_I + n_M$ denotes the number of products produced in the South and $k \geq 1$ is an index of the degree of IPR protection in the South. The idea underlying this formulation is that as IPR protection is strengthened in the South (i.e. as $k$ increases), imitation becomes a more costly activity for Southern firms because evading local enforcement of IPRs becomes more difficult.\(^{15}\) Note also that the above cost function for imitation assumes that the cost of imitation declines with the number of goods produced in the South ($n^S$). Since both multinationals and local imitators produce in the South, the idea underlying this formulation is that both local imitation and Northern FDI generate knowledge spillovers for the South.\(^{16}\) As is the case of innovation, the cost of imitation must decline over time in order to sustain imitation in the long run because as the number of products in the world economy expand, the flow profit of a successful imitator falls. In Section 3.2, we discuss a scenario where spillovers from past innovations are incomplete.

Free entry into imitation implies that the reward from imitation should equal its cost:

$$v^S = \frac{ka_I w^S}{n_S} \iff \frac{\pi^S}{\rho + g} = \frac{ka_I w^S}{n_S}. \quad (25)$$

Substituting from (18) into the above equation and using (8) gives the sales levels of a Southern imitator and a Northern multinational:

$$x^S = \frac{\alpha}{1 - \alpha} \frac{ka_I (\rho + g)}{n^S (\theta - 1)} \quad \text{and} \quad x^M = \frac{ka_I (\rho + g)}{n^S (\theta - 1) \theta^e}. \quad (26)$$

Finally, from (22) and (25) we have

$$\frac{n}{n_S} \frac{ka_I \nu^N w^S}{a_N \nu^S \nu^N} = 1 \iff \frac{n}{n_S} \frac{ka_I x^N}{a_N x^S} = 1. \quad (27)$$

Using (9) and (21) the above equation becomes

$$\frac{n_S a_N}{n \ a_I} \frac{\theta^e}{1 - \alpha} \left(1 + \frac{\mu}{\rho + g}\right) = 1. \quad (28)$$

Substituting from (16) and (17) into the above equation gives us the ‘first equilibrium condition’ in terms of three endogenous variables $g$, $\phi$ and $\mu$ and exogenous parameters of the model:

$$\frac{\phi}{\phi + g \ ka_I} \frac{\theta^e}{1 - \alpha} \left(1 + \frac{\mu}{\rho + g}\right) = 1. \quad (29)$$

\(^{15}\) Later in the article we briefly discuss the case where Southern IPR protection determines the degree to which Southern imitators can capture their product market profits post-imitation.

\(^{16}\) This formulation of localised knowledge spillovers for the South is consistent with our modelling of FDI wherein we posit that only Northern firms producing in the South face the risk of imitation by local firms.

Intuitively, this condition follows from the assumption of free entry into imitation and innovation and it ensures that neither activity leads to excess profits for firms that are successful in such activities.

1.5. Resource Constraints

The other two equilibrium conditions are derived from the resource constraints in the two regions. In the North, labour is allocated between innovation and production:

\[
\frac{a_N}{n} \dot{n} + n_N x_N = L_N. \tag{30}
\]

Substituting into the above resource constraint from the market measure (16), (17) and (23) yields the ‘second equilibrium condition’:

\[
L_d^N = a_N g + \frac{g}{g + \phi} a_N \pi (\rho + g) \frac{1}{1 - \alpha} = L_N. \tag{31}
\]

Southern labour is allocated to imitation and production by multinationals and local firms:

\[
\frac{k a_I}{n_S} \dot{n}_I + \theta n_M x_M + n_I x_S = L_S. \tag{32}
\]

Substituting into the above resource constraint from (16), (17) and (26) gives the ‘third equilibrium condition’:

\[
L_d^S = k a_I g \mu + \frac{g}{g + \mu} a_I \pi (\rho + g) \frac{1}{1 - \alpha} \frac{\mu}{g + \mu} = L_S. \tag{33}
\]

Observe immediately that the above equation can also be written as

\[
\frac{a_I g \mu}{g + \mu} + \frac{g}{g + \mu} a_I \pi (\rho + g) \frac{1}{1 - \alpha} \frac{\mu}{g + \mu} = L_S. \tag{34}
\]

In other words, from the viewpoint of the South, holding constant the rates of imitation (\(\mu\)) and innovation (\(g\)), an increase in the degree of IPR protection (\(k\)) is an effective reduction in the real resources available since all three activities that the South is engaged in – imitation, production by multinational firms and production by local imitators – require more resources as \(k\) increases. It is intuitively obvious why an increase in the cost of imitation increases the resources required to sustain a given level of imitation. But why do the two production activities undertaken in the South also become more resource intensive with an increase in the IPR index \(k\)? The intuition for this comes from the free entry condition in imitation: as the cost of imitation increases, the sales of a firm that is successful in imitation also must increase in order to maintain the zero profit condition in imitation. Finally, the sales of a multinational (\(x_M^M\)) are proportional to the sales of a Southern imitator (\(x_M^S\)) and if \(x_S^S\) increases, so must \(x_M^M\). We are now in a position to study the effects of a strengthening of Southern IPR protection.
2. Equilibrium Effects of Southern IPR Protection

We begin by establishing some crucial properties of the North–South flow of FDI.

2.1. Flow of North–South FDI

To do so, we first solve (29) for FDI flow $\phi$ in terms of the other two endogenous variables ($g$ and $\mu$). From (29) we have

$$\phi = \frac{g}{\theta a_N (A(\mu, g) \alpha_1 - 1)},$$

where

$$A(\mu, g) = \left(\frac{\rho + g}{\rho + g + \mu}\right)^{\frac{1}{\alpha}} < 1.$$

**Lemma 1.** $A(\mu, g) < 1$ and $\partial A(\mu, g)/\partial \mu < 0 < \partial A(\mu, g)/\partial g$.

Observe immediately from (35) that holding $\mu$ constant the denominator of the right-hand side increases with $g$: this is because $\mu / g$ falls with $g$ whereas $A(\mu, g)$ increases (Lemma 1). This implies the following result:

**Remark 1.** Holding constant the rate of imitation ($\mu$), factors that increase the North–South flow of FDI ($\phi$) must also increase the rate of Northern innovation ($g$).

Since both innovation and FDI are endogenous, Remark 1 simply notes that the flow of FDI and the rate of innovation are positively related in our model. In this context, it is worth noting that a large number of empirical studies have demonstrated that there is indeed a positive correlation between innovation and FDI; as Markusen (1995) notes, this finding is so pervasive that it has become a cornerstone of the modern theory of the multinational firm. Furthermore, since $A(\mu, g)$ decreases with $\mu$, we have:

**Remark 2.** Holding constant the rate of innovation ($g$), factors that decrease the Southern rate of imitation ($\mu$) must also increase the North–South flow of FDI ($\phi$).

Since our model exhibits a negative feedback between FDI and imitation and a positive feedback between FDI and innovation, it necessarily implies a negative feedback between innovation and imitation. This is an important property of the model which differentiates it from the results of Grossman and Helpman (1991b) and aligns it with those of Lai (1998).

Consider now the direct effect of Southern IPR protection on the North–South flow of FDI. From (35) directly observe that the denominator in the formula of $\phi(\mu, g)$, decreases with $k$ so that we have:

**Remark 3.** Holding constant the rates of imitation ($\mu$) and innovation ($g$), the flow of FDI ($\phi$) to the South increases with a strengthening of Southern IPR protection (i.e. an increase in $k$).
The intuition for this result comes from (28) which requires the rate of return on innovation and imitation to equal each other. Since the right-hand side of this equation always equals 1, an increase in the IPR index \( k \) must be counterbalanced by an increase in the ratio of production \( \frac{n_S}{n} = \frac{\phi}{(\phi + g)} \) that occurs in the South for the cost of imitation to not increase relative to the cost of innovation which in turn requires the flow of FDI \( \phi \) to increase with the degree of Southern IPR protection \( k \).

It is well known that multinational firms conduct a large share of global research and development (R&D). Indeed, a generation of empirical studies have documented the positive correlation between FDI flows and R&D investment (Markusen, 1995). Given this, it is worth noting from (35) that, holding constant the rate of innovation and imitation, an increase in the R&D productivity of Northern firms (as measured by an decrease in \( a_N \)) implies a faster North–South flow of FDI. We later discuss the general equilibrium response of FDI to an increase in Northern R&D productivity taking into account its effects on the rates of imitation and innovation.

2.2. Impact on Innovation and Imitation

Equations (29), (31) and (33) define the steady-state equilibrium of the model in terms of the three endogenous variables: the rate of innovation \( g \), the rate of imitation \( l \) and the rate of FDI \( \phi \). All of the effects of increased IPR protection in the South (i.e. an increase in \( k \)) are derived from the effects on these endogenous variables.

Assuming the rate of imitation is exogenously given, Lai (1998) has shown that a decline in this rate increases Northern innovation and the rate of production shifting to the South. A crucial question is whether this important result holds when both imitation and innovation are endogenous and the underlying exogenous variable is the degree of IPR protection (i.e. parameter \( k \)). Using the equation for the equilibrium flow of FDI and the two resource constraints, we can derive a system of two equations in two unknowns that helps to provide a graphical illustration of the consequences of stronger IPR protection in the South.

First note from (33) that the Southern labour market constraint is independent of the flow of FDI \( \phi \). Also, recall that \( L_S^d \) measures aggregate labour demand in the South (given by the LHS of (33)). Direct calculations yield

\[
\frac{\partial L_S^d}{\partial \mu} = \frac{ka_l[\theta^e(g + z \rho) - \theta(\rho + g)]}{(\mu + g)^2(1 - \rho)} > 0,
\]

because \( \theta^e > \theta \) and \( z < 1 \) – that is, holding constant the rate of innovation \( g \), aggregate labour demand in the South increases with the rate of imitation \( \mu \). Similarly, holding constant the rate of imitation, demand for Southern labour increases with the rate of innovation:

\[
\frac{\partial L_S^d}{\partial g} = \frac{ka_l[\mu(\mu - z \rho)\theta^e + \theta \rho \mu + 2 \theta g \mu + \theta g^2]}{(\mu + g)^2(1 - \rho)} > 0,
\]

where we have assumed that \( \mu > z \rho \).

Thus, the Southern labour market constraint (i.e. the SS curve) is downward sloping in the \((g, \mu)\) space.
\[
\sigma^S(g, \mu) \equiv \frac{\partial L^S}{\partial g} \bigg|_{L^S = L^S} = \frac{\partial L^S(\mu, g)}{\partial g} / \frac{\partial L^S(\mu, g)}{\partial \mu} < 0.
\]

In other words, since the South has only a fixed amount of labour resources, an increase in the Southern rate of imitation \(l\) implies that the rate of innovation \(g\) that can be supported by the global economy must be lower.

Also, we have
\[
\frac{\partial L^N}{\partial \mu} = \frac{k\alpha l(\rho + g)A(\mu, g)}{(\rho + \mu + g)(1 - \alpha)\theta^f} > 0,
\]
that is, the higher the rate of imitation \(\mu\), the higher the demand for Northern labour. The logic for this is as follows. Since FDI is endogenously determined, a higher rate of imitation makes FDI less attractive to Northern firms. For a fixed rate of innovation, the demand for Northern workers is inversely related to the flow of FDI.

Next consider how an increase in the rate of innovation effects aggregate labour demand in the North. Recall that demand for Northern labour comes from innovation \((L^N_n = a_N g\)) and from production \((L^N_p = n_N x^N)\). It is obvious that an increase in \(g\) raises labour demand in innovation \((L^N_n)\). On the production side, labour demand can be written as
\[
L^N_p = \frac{n_N a_N x(\rho + g)}{n(1 - \alpha)} \quad \text{where} \quad \frac{n_N}{n} = \frac{g}{\phi + g}
\]
which immediately implies that if \(n_N/n\) were to increase in \(g\), then it must be that \(L^N_p\) (and therefore aggregate labour demand) in the North increases in \(g\). Further note from above that if \(\phi\) were independent of \(g\), it would immediately follow that \(n_N/n\) increases in \(g\). This thought experiment is useful for highlighting the role of the flow of FDI in our model: if the flow of FDI flow were invariant to the rate of innovation, labour demand in the North would necessarily increase with the rate of innovation. However, Remark 1 notes that the flow of FDI and the rate of innovation are positively related. This raises the possibility that \(n_N/n\) might decrease with \(g\). Intuitively, such a situation could arise since the elasticity of the flow of FDI with respect to the rate of innovation exceeds unity in our model. Despite this, we show in the Appendix that labour demand in the North necessarily increases with the rate of innovation:
\[
\frac{\partial L^N}{\partial g} > 0.
\]

As a result, like the Southern labour market constraint, the Northern labour market constraint (i.e. the \(NN\) curve) is also downward sloping in the \((g, \mu)\) space:
\[
\sigma^N(g, \mu) \equiv \frac{\partial L^N}{\partial g} \bigg|_{L^N} = \frac{\partial L^N}{\partial g} / \frac{\partial L^N}{\partial \mu} < 0.
\]

It is worth emphasising the role FDI plays in this context: in the absence of FDI, in a variety expansion product cycle model such as Grossman and Helpman (1991b), the Northern market labour constraint is actually upward sloping in the \((g, \mu)\) space. This is because when imitation is the only channel via which production is reallocated internationally, an increase in the rate of imitation frees up Northern labour for use in innovation thereby generating a positive feedback between imitation and innovation.
By contrast, in our model imitation targets production by multinationals and by slowing down FDI, an increase in the rate of imitation actually pulls Northern resources out of innovation and into production.

For a unique steady-state equilibrium to exist, the SS curve and the NN curve must have a unique intersection in the \((g, \mu)\) space. We have already noted that both curves are downward sloping. Neither curve intersects the vertical axis and we show in the appendix that under minor conditions, the horizontal intercept \((g^*)\) of the SS curve is larger than that \((g^N)\) of the NN curve.

Given these properties of the two curves, any intersection of the two curves will be unique if the NN curve is steeper than the SS curve: i.e. \(\sigma' \equiv |\sigma^N|/|\sigma^S| > 1\). We can show that \(\sigma' > 1\) iff \(a_R = a_N/a_I\) exceeds some threshold \(a^*_R\), where \(a^*_R\) is a function of exogenous parameters and the rates of imitation and innovation. Furthermore, as \(\rho\) approaches zero, \(a^*_R\) can be shown to be decreasing in the rate of imitation \(\mu\). In other words, for \(\rho\) close to zero, the required threshold \(a^*_R\) is the highest (and therefore the most difficult to meet) at \(\mu = 0\). Next, it can be shown that at \(\mu = \rho = 0\), \(a^*_R\) decreases in \(\theta\) and at the lowest feasible value of \(\theta\) (which is \(1/\alpha\)), the condition \(a_R > a^*_R\) is necessarily satisfied for all feasible \(\alpha\). Thus, we proceed with the scenario where the NN curve is steeper than the SS curve and the two curves have a unique intersection that pins down the steady-state equilibrium of the global economy.

Intuitively, in the \((g, \mu)\) space the NN curve ought to be relatively steeper than the SS curve because the rate of innovation is determined primarily by the size of the Northern economy (since only the North innovates) while the rate of imitation is determined primarily by the size of the Southern one (since only the South imitates). Of course, the North–South flow of FDI is what links the two economies and their resource constraints to each other.

As was already noted, holding constant the rates of imitation \((\mu)\) and innovation \((g)\), an increase in the degree of Southern IPR protection \((k)\) increases labour demand in the South in all three activities (i.e. local imitation, production by Southern firms and production by multinationals).\(^{17}\) This is equivalent to an inward shift in the Southern labour market constraint in the \((g, \mu)\) space. Further note that holding constant \(g\) and \(\mu\), an increase in \(k\) effects the Northern labour market constraint via its effect on the flow of FDI \(\phi\). Given that the flow of FDI increases in the Southern IPR index \(k\), it follows that labour demand in the North \(L^N(\mu, g)\) (i.e. the left-hand side of (31)) decreases with \(k\). The effect of a strengthening of IPR protection in the South on equilibrium rates of imitation and innovation can now be derived. As IPR protection in the South increases, the Southern labour market constraint (i.e. the SS curve) shifts down while the Northern constraint (i.e. the NN curve) shifts up. These shifts in the two constraints deliver one of our key results:

**Proposition 1.** A strengthening of IPR protection in the South decreases the Southern rate of imitation \((\mu)\) while it increases the Northern rate of innovation \((g)\): \(d\mu/dk \lessgtr 0 < dg/dk\).

\(^{17}\) It is worth noting here that since preferences are of the Dixit–Stiglitz type, the output of a typical multinational relative to a typical imitation is fixed and this ratio is given in equation (8). Furthermore, all else equal, the stronger the degree of IPR protection, the higher must be the equilibrium output level of a typical imitator for there to be equality between cost of imitation and the value of an imitating firm. Finally, by assumption, the degree of IPR protection directly affects the cost of imitation and therefore the aggregate resource requirement in imitation.

Since Proposition 1 is our central result, it is worth noting that it applies to marginal changes in the degree of Southern IPR protection $k$ as opposed to large, discrete reforms that were called for by the TRIPS agreement. However, in the real world, any IPR reform is only as good as the degree of local enforcement. Since IPR enforcement is likely to improve gradually, it is reasonable to focus on the effects of marginal changes in $k$.\(^{18}\)

Figure 1 drawn in the $(g, \mu)$ space illustrates Proposition 1.

In Figure 1, point $A$ denotes the initial steady-state equilibrium. Now suppose that Southern IPR protection is strengthened (i.e. $k$ increases). This policy change implies an inward shift in the Southern resource constraint and an outward shift in the Northern constraint. Why the Southern constraint shifts has already been explained: all three activities in the South become more resource intensive and this effectively reduces the resource base. The Northern constraint shifts out because of the FDI response: as the flow of North–South FDI increases, more Northern resources become available for innovation. The outward shift in the Northern constraint is relatively smaller because the North is affected via a single, indirect channel (i.e. through the response of North–South flow of FDI) whereas the effect on the South is a more direct one and it occurs via all three activities that take place there. As shown in Figure 1, these shifts in the two resource constraints caused by a strengthening of IPR protection in the South imply that in the new steady state equilibrium $B$ the Southern rate of imitation is significantly lower than that at $A$ while the Northern rate of innovation is higher.\(^{19}\) We should emphasise that the properties of the model noted in Remarks 1 and 2 are quite crucial since these

\(^{18}\) It is worth noting that the estimated response of US multinationals to IPR reform documented in Branstetter et al. (2006) and Branstetter et al. (2011) suggests a gradual shift in their investment, technology transfer and R&D in reforming countries. These studies found that there was no sharp change in the year of reform but, rather, a gradual change that was phased in over many years.

\(^{19}\) In a two-country model where both countries invest in labour saving innovation, Taylor (1994) finds that the global innovation and technology transfer are both higher when countries offer the same degree of IPR protection to innovating firms regardless of their national origin relative to when they offer such protection to only their own firms.
establish a positive feedback between FDI and innovation and a negative feedback
between these two variables and the rate of imitation. As long as a strengthening of
Southern IPR protection discourages imitation, its positive effects on innovation and
FDI are implied by Remarks 1 and 2.

Now briefly consider the case where a Southern imitator’s flow profit from imitation
equal
\[ \pi_s^I = (1 - k)\pi_s^N = (1 - k)(\theta - 1)u^S x^S \]
where \( k \) determined the degree of IPR protection and \( 0 \leq k \leq 1 \). Under such a formulation, the Northern labour market
equilibrium condition is unaltered whereas the other two equilibrium conditions are
slightly modified. In equation (29) we simply need to replace \( 1/k \) by \((1-k)\) whereas in
equation (33) the same substitution is needed in the second and third terms of the
LHS; in the first term of the same equation, \( k \) needs to be simply replaced by 1. It is
straightforward to show that results obtained under our cost-based formulation of IPR
protection continue to hold under this profit–tax formulation.

Finally, we note how an improvement in R&D productivity (i.e. a decrease in \( a_N \))
affects the North–South flow of FDI as well as the global allocation of production, once
the effects on innovation and imitation are taken into account. First note that a
decrease in \( a_N \) has no direct effect on the SS constraint whereas the effect on the NN
constraint is essentially the same as that an increase in the Northern labour supply –
that is in Figure 1, the NN curve shifts out. This immediately implies that with an
increase in Northern R&D productivity, the rate of imitation decreases whereas the rate
of innovation increases. Relying on arguments similar to those used to derive the effects
of Southern IPR protection, we directly state the following:

**Proposition 2.** With an increase in the R&D productivity of Northern firms (i.e. a decrease
in \( a_N \)), the rate of innovation, the North–South flow of FDI, the share of Southern production in
the hands of Northern multinationals and the sales of multinationals relative to other firms, all
increase whereas the rate of imitation decreases.

2.3. Allocation of Global Production

An important objective of this article was to understand how a strengthening of IPR
protection in the South alters the distribution of production across the two regions as
well as between Northern multinationals and Southern imitators. How Southern IPR
protection affects the global allocation of production depends on its effects on
Southern imitation, Northern innovation, and the North–South flow of FDI. To see the
effect of an increase in \( k \) on the international allocation of production, note that
equation (28) can be written as
\[
\frac{\theta_s a_N}{k a_I} \left( 1 + \frac{\mu}{\rho + g} \right)^{\frac{1}{\varepsilon - 1}} \frac{n_S}{n} = 1,
\]
where \( \theta_s = [z^{1-\varepsilon}(\theta - 1)]/(1-\varepsilon) \) does not depend upon the degree of Southern IPR
protection \( k \). As \( k \) increases, \( (\theta_s a_N)/(k a_I) \) decreases as does the term in the square
parentheses (since the rate of imitation \( \mu \) falls while the rate of innovation \( g \) increases).
Since the right-hand side always equals 1, this implies that \( n_S/n \) must increase with an
increase in the degree of Southern IPR protection \( k \).
Proposition 3 (International Allocation of Production). A strengthening of Southern IPR protection increases the South’s share of the total basket of goods produced in the global economy: \( \frac{d(n_S/n)}{dk} > 0. \)

Another way of restating Proposition 3 is that the North’s share of the global basket of goods \((n_N/n)\) must decrease with \(k\):

\[
\frac{d(n_N/n)}{dk} < 0 \quad \text{where} \quad \frac{n_N}{n} = \frac{1}{1 + x} \quad \text{and} \quad x = \frac{\phi}{g} \left( 1 + \frac{\mu}{g} \right).
\]

Since \(\mu/g\) falls with \(k\), it must be that \(\phi/g\) rises with \(k\) or else \(n_N/n\) cannot decrease with \(k\). This finding sheds light on the relative impact of a strengthening of Southern IPR protection on the Northern rate of innovation and the North–South flow of FDI since it says that while both the flow of FDI and the rate of innovation increase with a strengthening of Southern IPR protection, the positive effect on the flow of FDI is relatively stronger:

\[
\frac{d\left( \frac{\phi}{g} \right)}{dk} > 0.
\]

Given that \(n_I/n_M = \mu/g\) decreases with \(k\), we can state the following result regarding the allocation of production within the South between multinationals and Southern firms:

Proposition 4 (Intra-regional Allocation of Production). A strengthening of Southern IPR protection increases the share of Southern production undertaken by Northern multinational firms: \( \frac{d(n_M/n_S)}{dk} > 0. \)

It is straightforward to show that the total value of multinational sales relative to those of Southern imitators has the following simple expression:

\[
\frac{n_M p^M x^M}{n_I p^I x^I} = \frac{\phi}{g} \left( 1 + \frac{\mu}{\rho + g} \right).
\]

Since the rate of innovation \((g)\) increases while the rate of imitation \((\mu)\) falls with an increase in the degree of Southern IPR protection, we can state the following result:

Corollary 1. A strengthening of Southern IPR protection leads to an increase in the aggregate sales of multinational firms relative to those of Southern imitators.

Now consider a comparison of multinational sales relative to those of firms producing in the North:

\[
\frac{n_M p^M x^M}{n_N p^N x^N} = \frac{\phi}{g} \left( \frac{\theta w^S}{w^N} \right)^{1-\varepsilon} = \frac{\phi}{g} \left( 1 + \frac{\mu}{\rho + g} \right).
\]
Since \( n_M/n_N = \phi/g \), (36) implies that a typical multinational must have higher relative sales compared to a Northern firm (i.e. the ratio \( p^M x^M/p^N x^N \) must exceed 1). Intuitively, since imitation only targets multinational firms, for a typical multinational to earn the same rate of return as a Northern firm producing in the North, the multinational must have a higher relative profit flow. However, with a decline in the rate of imitation, this relative profit flow actually has to shrink in order to ensure multinationals and Northern firms earn the same rate of return. This yields:

**Corollary 2.** A strengthening of Southern IPR protection decreases the sales of a typical multinational firm relative to those of a Northern firm.

In this context, one further subtlety that arises from general equilibrium considerations is worth noting: an decrease in the rate of imitation \( \mu \) increases the relative Southern wage and therefore the cost of production of multinationals relative to Northern firms. However, since prices of both types of firms are mark-ups over their respective marginal costs, this cost increase has a proportional effect on prices of multinationals relative to those of Northern firms. In other words, by increasing the South’s relative wage, IPR reform increases the prices charged by multinationals relative to those of Northern firms and this translates into lower relative sales for a typical multinational.

### 2.4. Real Wages and the Price Index

What are the effects of a strengthening of IPR protection in the South on real wages in the two regions? By definition, the real wage effects of such a policy change depends upon nominal wages in the two regions and the prices of goods produced by three types of firm: firms located in the North, multinationals producing in the South and Southern imitators. Recall that

\[
\begin{align*}
p^N &= \frac{w^N}{\alpha}; & p^M &= \frac{\theta w^S}{\alpha}; & p^S &= \theta w^S,
\end{align*}
\]

which allows us to write Northern real wages in terms of the three types of goods:

\[
\begin{align*}
\frac{w^N}{p^N} &= \alpha; & \frac{w^N}{p^M} &= \frac{\alpha}{\theta} w^R; & \frac{w^N}{p^S} &= \frac{1}{\theta} w^R.
\end{align*}
\]

In other words, the Northern real wage in terms of goods produced by Northern firms is unaffected by Southern IPR protection whereas in terms of the other two goods, it moves in the same direction as the Northern relative wage \( w^R \). We already know that Northern relative wage \( w^R \) decreases as a result a strengthening of Southern IPR protection since the rate of imitation \( \mu \) falls while the rate of innovation \( g \) increases with such a policy change. This decline in the Northern relative wage \( w^R \) implies the following:

**Proposition 5.** A strengthening of Southern IPR protection decreases real wages in the North.

An important implication of Propositions 1 and 5 is that from the perspective of the North, stronger Southern IPR enforcement in our model generates a classic trade-off
between a static welfare loss and a dynamic welfare gain: the static loss being the
decrease in real wages (or in its terms of trade since the relative price of Northern
exports is determined by the relative wage) and the dynamic gain being the increase in
the rate of innovation. What is noteworthy, however, is that the trade-off in the North
results from changes in the IPR policy of the South.

Consider now the effect on Southern real wages. We have

\[
\frac{w^S}{p^S} = \frac{1}{\theta}; \quad \frac{w^S}{p^N} = \frac{\alpha}{w^R}; \quad \text{and} \quad \frac{w^S}{p^M} = \frac{\alpha}{\theta}.
\]

In other words, the only effect on Southern real wages of a change in its IPR policy is in
terms of goods produced in the North. However, since \( w^R \) decreases with an increase in
\( g \) and a decrease in \( \mu \), we can now state the following:

**Proposition 6.** A strengthening of Southern IPR protection increases real wages in the South.\(^{21}\)

The general equilibrium nature of the above result deserves emphasis. The typical
argument in favour of weaker IPR protection in the South is that Southern imitation
lowers prices and therefore benefits consumers. Since prices of Southern imitators are
lower than those of Northern multinationals, this channel is operative in our model as
well. However, the standard argument ignores the labour market effects of interna-
tional production shifting that results from a reduction in the rate of imitation. In our
model, a strengthening of Southern IPR protection leads to a higher Southern relative
wage since the resulting decline in imitation risk makes the South a more attractive
location for Northern multinationals. Indeed, changes in prices are dominated by the
change in the Southern relative wage so that the purchasing power of Southern workers
in terms of goods produced in the North increases whereas there is no change in their
ability to purchase goods produced in the South. Thus, somewhat surprisingly and
perhaps controversially, we find that a strengthening of IPR protection confers both a
static benefit and a dynamic benefit on the South: real wages of its workers increase, as
does the Northern rate of innovation.

As is clear from the discussion above, the real wage effects captured here would not
arise in partial equilibrium models that ignore the labour market effects of IPRs.
However, this point should not be over-emphasised. In our model, IPR reform in the
South affects all goods produced in the world economy. In the real world, this is
unlikely. A multi-sector model in which IPR reform affects only those sectors that invest
in imitation may not necessarily yield the same results as Propositions 5 and 6. In
particular, if Southern IPR reform only affects a few of the sectors, it may not neces-
sarily allocate aggregate labour demand in favour of the South thereby increasing its
relative wage.\(^{22}\)

\(^{21}\) As we discuss in greater detail below, it bears keeping in mind that despite an increase in real wages,
Southern welfare does not necessarily increase because the flow of utility equals the log of real spending
\(( \log u = \log E - \log P )\) and a reduction in profits of Southern imitators lowers Southern income and can
adversely impact Southern spending.

\(^{22}\) We should note, however, that the conflict created by technology transfer between the welfare of
Northern and Southern workers that is a feature of our model also appears to arise in Cheng et al.'s (2005)
North–South Ricardian model of trade and FDI where the global economy produces a continuum of goods.
Another qualification of our wage results deserves mention. Suppose both the North and the South have a multi-sector economy where one of the sectors is perfectly competitive in both the labour market and the product market (call it ‘traditional’) with no ongoing innovation while the other sector comprises of differentiated goods of the type considered here. Then, if under free trade both countries are diversified (i.e. produce the traditional good as well as some of the differentiated goods) the North–South relative wage would be pinned down by the North–South relative productivity in the traditional sector, much like in a classical Ricardian model. Under such a situation, IPR reform in the multi-sector model will fail to have an impact on the North–South relative wage so long as both regions remain diversified in production.

An important assumption underlying the wage results reported in Propositions 5 and 6 is that Northern firms that do not shift production to the South are immune from the risk of imitation. As we show in Section 4.1, when Northern firms can be imitated regardless of their location of production, whether or not these results continue to hold depends on how the imitation risk facing multinationals responds relative to that facing Northern producers when Southern IPR protection is strengthened.

It is useful to consider how a strengthening of Southern IPR protection affects the aggregate price index $P$. By definition,

$$P = \left[ \int_0^1 p(j)^{1-\varepsilon} \, dj \right]^{1/\varepsilon},$$

which can be rewritten as

$$P = \left[ n_M (p^M)^{1-\varepsilon} + n_I (p^S)^{1-\varepsilon} + n_N (p^N)^{1-\varepsilon} \right]^{1/\varepsilon},$$

which is the same as

$$P = n^{1/\varepsilon} \left[ n_M (p^M)^{1-\varepsilon} + n_I (p^S)^{1-\varepsilon} + n_N (p^N)^{1-\varepsilon} \right]^{1/\varepsilon}.$$

While goods produced by multinationals are cheaper than those produced by Northern firms ($p^M < p^N$), it is the Southern imitators that produce the cheapest goods ($p^S < p^M$). Recall that $n_I/n_M = \mu/g$ decreases with the degree of Southern IPR protection ($k$) since imitation slows down while innovation increases. This implies that $n_I/n_M = n_M/n = \mu/g$ decreases with $k$, that is the share of global production that is in the hands of multinational firms increases. Furthermore, recall from Proposition 3 that a strengthening of Southern IPR protection shifts production away from the North and towards the South (international reallocation). Since $p^M < p^S < p^N$, the international reallocation of production from North to the South helps lower prices. However, since $p^M > p^S$, the intra-regional reallocation of Southern production in favour of Northern multinationals and away from Southern imitators tends to increase prices. This implies that if the international reallocation of production is substantial, Southern imitation has the potential to partially benefit Northern consumers by lowering the aggregate price index $P$. Indeed, this is the key reason why Helpman (1993) finds that some amount of imitation is in the interest of the North.
However, in our model, since FDI also offers the potential for lowering prices, imitation is not as crucial for welfare purposes. This is worth explaining in some detail. Unlike us, Helpman (1993) assumes that the risk of imitation applies equally to Northern firms and multinationals. As a result, multinationals and Northern producers can coexist in equilibrium only if the two regions have equal wages.\(^{23}\) Under such wage equalisation, FDI offers no reduction in costs of production and therefore has no price effects. By contrast, in our model, both FDI and imitation imply cost savings and the allocation of production across regions as well as within the South have implications for the aggregate price index.

A complete welfare analysis along the lines of Helpman (1993) is beyond the scope of this article due to the model’s underlying complexity. To see why, consider the viewpoint of the South. First, as has already been noted, changes in Southern IPR protection have conflicting price effects in our model and the overall price index can move in either direction: international production shifting lowers prices while inter-regional production shifting raises them. Second, since Southern imitation yields profits, aggregate income (and therefore expenditure) in the South depends both on wages as well as profits, which moves in opposite directions due to a strengthening of Southern IPR protection. In particular, the decline in imitation reduces Southern profit income while it increases Northern profit income. Similarly, from the North’s viewpoint, Southern IPR reform unleashes countervailing effects. Firm profitability clearly increases: not only does a typical multinational enjoy its profit stream for a longer duration, a greater share of Southern production comes to lie in the hands of multinationals. These forces tend to increase Northern income. However, this beneficial profit effect for the North is offset by the decrease in real wages of its workers. These complex set of interactions imply that our results regarding real wages are only one component of the calculus determining the global welfare impact of Southern IPR reform and should be interpreted as such.

3. Further Discussion of Results and Robustness

In this Section, we examine whether our results are robust to three important assumptions underlying our model.\(^{24}\) This discussion also helps to shed further light on the real wage results reported in Propositions 4 and 5.

3.1. Imitation of Northern Production

Our model assumes that Southern imitation targets Northern multinationals so that any Northern firm that chooses not to shift production to the South is immune from the risk of imitation. This is a strong assumption, and it contrasts sharply with the assumption made in Grossman and Helpman (1991b), where all international technology transfer occurs via imitation of Northern goods and none via FDI. Our

\(^{23}\) Our model would yield the same result if the rate of imitation facing multinationals and Northern producers were the same (i.e. \(\mu = 0\)) and multinationals did not face any frictions that hamper their ability to be as effective in production as local Southern firms (i.e. \(\theta = 1\)).

\(^{24}\) We thank the editor and two anonymous referees for comments and suggestions that led to the analysis presented in this Section.

assumption reflects the influence of recent theoretical and empirical work stressing the importance of multinationals as a conduit for international knowledge flows. However, our theoretical results do not require that Southern firms be incapable of imitating Northern firms. The structure of our model implies that so long as imitating multinationals is even slightly cheaper than imitating firms producing in the North – a highly plausible scenario – no Southern imitator would choose to imitate a Northern firm. To see why, consider the profit flow earned by a firm that successfully imitates a Northern firm. Given the nature of consumer preferences, such an imitator would charge the same price as one that is successful in imitating a multinational, that is, the optimal price $p^S$. Given that both types of imitators produce in the South, the cost of production for both would be same and so would be the profit flow. This immediately implies that the lifetime discounted flow of profits of both types of imitators (post-imitation) would be equal. But if copying multinationals is even marginally cheaper, no rational Southern entrepreneur would choose to imitate a firm producing in the North since the rate of return on targeting a multinational would be higher.

Recent empirical evidence strongly supports the view that imitating multinationals ought to be easier than imitating firms located in the North. In surveying this evidence, we start with the point that multinational managers perceive themselves to be more vulnerable to imitation when they set up facilities abroad. This is shown in the work of Mansfield (1994) and Lee and Mansfield (1996), which uses survey evidence to measure US multinational managers’ concerns about investing in countries with weak intellectual property rights. These perceptions are fully consistent with two decades of empirical research on international knowledge flows, nearly all of which shows that knowledge flows much more easily within countries than across national borders. This result was convincingly documented by Jaffe et al. (1993) and recently reaffirmed by Thomson and Fox-Kean (2005).

Precisely because knowledge flows are limited by geographical, cultural and linguistic distance, multinationals can play an important role in facilitating technology diffusion, both intentionally and unintentionally. Branstetter et al. (2006), Keller and Yeaple (2009) and Griffith et al. (2006) present evidence in favour of this view for advanced industrial economies, while Javorcik (2004b) and Blalock and Gertler (2008) present evidence from developing economies. Researchers have long suspected that the movement of workers from multinationals to local firms is an important channel of knowledge diffusion, and Gorg and Strobl (2005) and Hale and Long (2006) provide some early direct evidence in favor of this view. More recent evidence drawn from fairly rich data-sets that track the movement of individual workers across firms in developing countries also strongly supports this view; see Poole (2009) for a particularly intriguing example. Obviously, movement of workers is easier within than across countries, and this mechanism alone makes multinationals an easier target for would-be imitators.

25 In this context, it is worth noting that Antràs (2005) has argued that primary factor behind the technological development of the electronics industry in some of the East Asian countries such as Taiwan and Korea was production shifting and technology transfer by multinationals as opposed to imitation by local companies of firms producing in other countries. In particular, he notes that ‘the bulk of technology transfer is driven by the voluntary decisions of Northern firms, which choose to undertake offshore production within firm boundaries or transact with independent contractors or licensees’.

If multinationals really are more susceptible to imitation than Northern firms, then a significant strengthening of the Southern IPR protection is likely to induce more FDI, as foreign managers’ concerns about imitation are assuaged. This implication is supported by recent empirical work. Branstetter et al. (2011) find evidence of an increase in FDI by US-based multinationals after the IPR regime is strengthened in 16 countries, most of which would be considered developing countries at the time of IPR reform. Ito and Wakasugi (2009) find evidence that Japanese multinationals respond to stronger IPR by increasing measured technology transfer and investment.

The strength of this evidence notwithstanding, for theoretical completeness, we can consider here what impact imitation of Northern firms might have on our model. Since there might be products for which imitating multinationals is not necessarily cheaper than imitating Northern firms, suppose the unit labour requirement in imitating Northern firms is the same as that for imitating multinationals (i.e. is given by \(a_1\)).

Under such a scenario, both types of imitation would occur in the South. Let \(\mu^N\) denote the imitation risk facing Northern firms producing in the North and \(\mu\) that facing Northern multinationals. This implies that the lifetime value of a Northern firm would equal \(v^N = \pi^N / (\rho + g + \mu^N)\) while that of a typical multinational would be the same as before: \(v^M = \pi^M / (\rho + g + \mu)\). Since all Northern firms are free to become multinationals if they wish, we must have:

\[
v^N = v^M \iff \frac{\pi^N}{\rho + g + \mu^N} = \frac{\pi^M}{\rho + g + \mu}.
\]

Using the definition of flow profits for each type of firm and cancelling terms, we can write

\[
\frac{w^N x^N}{\rho + g + \mu^N} = \frac{\theta w^S x^M}{\rho + g + \mu},
\]

which immediately gives

\[
w^R = \theta \left( \frac{\rho + g + \mu}{\rho + g + \mu^N} \right)^{1-\theta}.
\]

Observe from the above equation that in the unlikely event where the risks of imitation facing multinationals and Northern producers are exactly equal, that is \(\mu = \mu^N\), the Northern relative wage equals the relative productivity of Northern labour in production vis-à-vis Southern labour employed by multinationals (i.e. \(w^R = \theta\)). Under such a scenario, the North–South relative wage becomes unresponsive to Southern IPR protection. This, of course, implies that our results regarding the impact of Southern IPR protection on real wages in the two regions reported in Propositions 4 and 5 would not hold for the boundary case of \(\mu = \mu^N\).

---

26 Of course, if copying Northern firms were cheaper than copying Northern multinationals and all successful imitators charge the optimal price \(p^S\), no imitation of multinationals would occur since all imitative activity in the South would target firms producing in the North. But, as we noted above, this is not a plausible scenario for a variety of reasons.
However, when $\mu \neq \mu^N$, using the above formula for $w^R$ we have,

$$\frac{d \ln w^R}{dk} = \frac{1}{\varepsilon - 1} \left[ \frac{1}{\rho + g + \mu} \frac{d(\rho + g + \mu)}{dk} - \frac{1}{\rho + g + \mu^N} \frac{d(\rho + g + \mu^N)}{dk} \right],$$

which implies that

$$\frac{d \ln w^R}{dk} < 0 \quad \text{iff} \quad g_k(\mu^N - \mu) + \mu_k(\rho + g + \mu^N) < \mu^N_k(\rho + g + \mu),$$

where

$$g_k \equiv \frac{dg}{dk}; \quad \mu_k \equiv \frac{d\mu}{dk}; \quad \text{and} \quad \mu^N_k \equiv \frac{d\mu^N}{dk}.$$

Suppose both types of imitation decline with IPR protection (i.e. $\mu^N_k < 0$ and $\mu_k < 0$) whereas innovation increases with it ($g_k > 0$). Then, if the risk faced by multinationals is no lower than that faced by Northern producers (i.e. $\mu^N \leq \mu$), the relative Northern wage necessarily decreases with Southern IPR protection if

$$\mu_k < \mu^N_k \frac{\rho + g + \mu}{\rho + g + \mu^N} \quad \text{or} \quad |\mu_k| > |\mu^N_k| \frac{\rho + g + \mu}{\rho + g + \mu^N},$$

a condition which requires that the decline in the rate of imitation targeting multinationals ($|\mu_k|$) be sufficiently larger than that targeting firms producing in the North ($|\mu^N_k|$).

### 3.2. Incomplete Knowledge Spillovers

As in Grossman and Helpman (1991b) and Romer (1990), growth is driven in our model by the assumption that past innovations lower the cost of future innovations. Recall that the cost of product development is given by $(w^N a_N)/n$ and it declines at the rate at which $n$ increases. What if such knowledge spillovers are incomplete? To address this question, let the cost of product development be given by $(w^N a_N)/(x^N n)$ where $0 < x^N \leq 1$ measures the degree to which the stock of knowledge created by past innovations helps lower the cost of current innovation. Similarly, let the cost of imitation be given by $(ka_I a_S)/(x^S n_S)$ where $0 < x^S \leq 1$.

Does the dampening of spillovers effect the qualitative nature of our results? We argue below that it does not. Solving the model under these alternative assumptions modifies our three equilibrium conditions as follows. The flow of FDI is now given by

$$\phi = \frac{g}{\lambda^N A(\mu, g) k a_I},$$

while the Northern labour market equilibrium condition becomes

$$\frac{a_N}{\lambda^N} g + \frac{g}{g + \phi \lambda^N (1 - \alpha)} A_N = L^N.$$
Finally, in the South we have
\[
\frac{k a_1 g \mu}{\lambda^S g + \mu} + \frac{k a_1 \theta x(\rho + g)}{\lambda^S \theta' (1 - \alpha) g + \mu} + \frac{k a_1 z(\rho + g)}{\lambda^S 1 - \alpha} g + \mu = L^S. \tag{40}
\]

A few points are worth noting. First, note from (38) that if the degree of spillovers is equally incomplete in both regions (i.e. $\lambda^S = \lambda^N$), the North–South flow of FDI is the same as that our core model. Second, as expected, the dampening of spillovers reduces the productivity of the two types of investment activities in our model (i.e. imitation and innovation). Indeed, this can be observed immediately from the modified labour market equilibrium conditions (39) and (40): a decrease in $\lambda^S$ is akin to a shrinkage of the Southern resource base $L^S$ while a decrease in $\lambda^N$ has the same effect on the Northern resource base $L^N$. This in turn implies that the level of imitative activity that can go on in the South and the level of innovation that can be supported in the North would be lower when spillovers are incomplete. However, it is worth noting that all of our Propositions would continue to hold since these do not depend on the absolute levels of $g$ and $\mu$ but rather on how these variables respond to changes in the underlying degree of Southern IPR protection.

### 3.3. If Imitated Goods are Sold Only in the South

For simplicity, our model assumes that all goods are sold in both markets. However, it be more realistic to assume that imitated goods are sold only in the South. How do our results change if we adopt this alternative formulation? Suppose the share of product market profits that a typical imitator collects from its sales in the Southern market is given by $\beta$ where $0 < \beta < 1$. Then, the reward to a successful imitator $\pi^S(\beta) = (p^S - w^S)\beta x^S = [(1 - \alpha)\beta w^S x^S]/\alpha$. Note that $\pi^S(\beta) = \beta \pi^S$ and since profits are linear in sales, such an export restriction scales down the reward from imitation. As might be expected, its basic effect is to reduce the incentive to imitate. However, since the mechanics of our model depend on how imitation and innovation respond to IPR protection and not on their absolute levels, the qualitative nature of our results remains unchanged. To see this clearly, first note that the relative wage equation (21) as well as equation (35) which determines the North–South flow of FDI remain unchanged. However, the equality between cost of imitation and the value of a successful imitator implies that we must have

\[
x^S(\beta) = \frac{x^S}{\beta n_s} = \frac{\alpha}{1 - \alpha} \frac{k a_1 (\rho + g)}{\beta n_s} = \frac{x^S}{\beta}.
\]

Since $\beta < 1$, the above implies that, all else equal, Southern imitators must have higher sales in the South to cover their costs of imitation if they are unable to export to the Northern market. Similarly, we have $x^M(\beta) = (x^M)/\beta$. As a result, the Southern labour market equilibrium condition now becomes
\[
\frac{k a_1 g \mu}{g + \mu} + \frac{k a_1 \theta x(\rho + g)}{\beta \theta' (1 - \alpha) g + \mu} + \frac{k a_1 z(\rho + g)}{\beta 1 - \alpha} g + \mu = L^S.
\]
Intuitively, since sales levels are now higher in the South, more of the Southern labour gets allocated to production by Southern imitators and Northern multinationals, leaving less of it available for allocation to imitation which reduces the rate of imitation relative to before. However, as is evident from the above discussion, the basic mechanics determining the effects of Southern IPR reform remain unaltered when Southern firms cannot export their imitated goods to the North.

4. Conclusion

Opinions regarding the strengthening of IPR regimes in developing countries required under the TRIPS agreement of the WTO vary remarkably across individuals and nations. While the issue is multi-faceted and complex, the following statement broadly captures the disparity in views regarding TRIPs: developing countries have tended to argue that stronger IPR regimes in their markets will have adverse effects on prices without having much of a positive impact on innovation whereas developed countries have stressed that not only innovation but also FDI flows would respond strongly to such reforms. In principle, an increase in FDI has the potential to offer two major sources of welfare gains. One, it can lower prices by shifting production to lower cost locations. Two, FDI has the potential to encourage Southern industrial development by introducing new technologies into the South. In this article, we present a general equilibrium North–South product cycle model with a degree of endogenity that allows us to assess these arguments in a unified framework.

Our major results are as follows. First, we find that a strengthening of IPR protection in the South fosters innovation whereas it discourages imitation. Second, it increases FDI to a degree that the Southern production base actually expands – that is, the decline in Southern imitative activity is more than offset by the increase in the production activity of Northern multinationals who are drawn to the South because local IPR reform renders it a more attractive production location by reducing the risk of imitation. Third, while prices of those goods that are reallocated from firms producing in the North to multinationals fall, prices of goods that are reallocated from potential imitators to Northern multinationals increase. In other words, IPR reform in the South has conflicting effects on consumer welfare when viewed solely through the price channel. However, what actually matters for consumer welfare is purchasing power. And from this viewpoint, Southern IPR reform benefits the South since it increases not only the South’s wage relative to the North but also the purchasing power of Southern consumers. By contrast, not only does the Northern relative wage decline, the real income of Northern workers also falters. It is worth emphasising that only a general equilibrium model such as ours can help to assess the full impact of the price changes that result from IPR reform since these can be offset (or be dominated) by the accompanying changes in wages. Finally, we should note that while the model’s richness makes it difficult to provide a full-fledged welfare analysis along the lines of Helpman (1993), we hope that the clarity with which the various channels that affect welfare emerge in the model sheds new light on a rather complex set of issues.

Appendix A. Mathematical Appendix

A.1. Slope of NN Curve

We already noted in the main text that $\frac{\partial L^N(\mu, g)}{\partial \mu} > 0$. Direct calculations yield

$$
\frac{\partial L^N(\mu, g)}{\partial g} = \frac{\theta^g (\rho + \mu + g) a_N - a_I A(\mu, g) [x (\rho + \mu + g) - \mu]}{(\rho + \mu + g)(1 - x)^{\theta^g}}.
$$

From where it follows that a sufficient condition for $\frac{\partial L^N(\mu, g)}{\partial g} > 0$ is that $a_N/a_I > 1 + x/\theta^g$. This is because $(\rho + g)[\theta^g a_N - x a_I A(\mu, g)] > 0$ due to the fact that $A(\mu, g) < 1$, $x < 1$, $a_N \geq a_I$ and $\theta^g > 1$. Next note that the condition $a_N/a_I > 1 + x/\theta^g$ is satisfied for all feasible parameter values: since $a_N \geq a_I$, at the lowest feasible value of $a_N$ this condition becomes $\theta^g > 1 + x$ which necessarily holds since $\theta > 1/x$.

A.2. Horizontal Intercepts

It is trivial to observe that neither curve can intersect the vertical axis since labour demand in each country approaches zero as the growth rate approaches zero. The NN curve intersects the horizontal axes at $g^n$ where

$$
g^n = \frac{\theta^g L^N(1 - x) - x \rho (a_N \theta^g - a_I)}{a_N \theta^g - x a_I}.
$$

Similarly, the SS curve intersects the horizontal axis at $g^s$ where

$$
g^s = \frac{L^S(1 - x) \theta^g - x \theta a_I \rho}{\theta a_I \rho}.
$$

From where it follows that

$$
g^s > g^n \text{ iff } L^S > \overline{L} \text{ where } \overline{L} = \frac{(L^N + \rho a_N) x \theta a_I}{\theta^g a_N - x a_I}.
$$

We assume that $L^S > \overline{L}$.

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References


