Pricing advice: The market for diagnostic information

Ashish Arora
Carnegie Mellon University, Pittsburgh, USA

Andrea Fosfuri
Universidad Carlos III de Madrid, Spain, and CEPR, London, Uk

January, 2003

Abstract
Diagnostic information helps agents to make more accurate decisions. One such decision is about investing in projects with uncertain outcomes. The value of diagnostic information is the difference in expected payoffs with and without it, and we show that such a value is non-monotonic in the ex-ante expected value of the project to be undertaken. We analyze optimal pricing schemes for selling information to buyers with unknown ex ante value. With a monopolist information seller, a striking result is that the optimal menu of contracts is remarkably simple. A pure royalty is offered to buyers whose projects have low ex-ante expected value and a pure fixed fee is offered to buyers whose projects have high ex-ante expected value. This result is robust to the presence of different types of information and to the introduction of competition in the market for diagnostic information.

* We would like to thank T. Roende, D. Soskice and participants at the EPRIS first annual meeting (London, 2000) and at the EARIE conference (Dublin, 2001) for helpful comments and suggestions on an earlier draft. The authors gratefully acknowledge financial support from the Merck Foundation (EPRIS project). Part of this research was conducted when Andrea Fosfuri was visiting the Software Industry Center at the Heinz School of Public Policy and Management (Carnegie Mellon University), whose hospitality is gratefully acknowledged. The usual disclaimer applies.
1. Introduction

It is now a commonplace that we live in the information economy. However, the value of information has been appreciated by economists well before the advent of the IT era. In the economic literature, information is modeled as a signal that changes estimates of the probabilities of the different states of nature (Arrow, 1962). Thus, an informed agent can take better decisions. In particular, information has an economic value insofar the decision maker changes his actions as a consequence of the revised probabilities.\(^1\) Similarly, Nelson (1959) models basic research as information, enabling researchers to focus on the lines of research that are more likely to succeed. Indeed, Arrow (1962) envisaged the possibility of information becoming a commodity, albeit subject to a variety of problems including moral hazard and non-convexities that have been extensively studied.\(^2\)

However, if the value of information depends on unknown (to the seller) characteristics of the potential buyer, then how should information be priced? From a theoretical perspective, this problem is interesting because the willingness to pay for information turns out to be non monotonic in the buyer’s “type”, implying that the standard “revelation mechanism” solution cannot be mechanically applied. From a practical viewpoint, this problem is relevant to the growing pressures for “unbundling” of pure information provision (i.e. the diagnostic information) from other types of related activities. For instance, recent calls for separating equity research from activities such as brokerage and investment banking are grounded in public outrage over the excessive “positive” bias in public reports issued by equity analysts and their bias against downgrading a stock. Similarly, as discussed in greater detail below, the pricing of research tools in biomedical research (such as databases and libraries of chemical compounds) has raised controversy.

We develop a simple model where an agent has to decide whether to invest or not in a project with an uncertain outcome, and can purchase information that will update his

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\(^1\) Information, in addition to affecting the estimates of the probabilities of the different states of nature, can also help identify new alternatives that were unknown a priori. Although this is an important component of the value of information, economists have tended to disregard it because of the difficulty to put a value on a priori non-existing states of nature. In this paper we will follow in this tradition and focus on information as modifying estimated probabilities of known states of the world.
beliefs about the outcome. The value of information is the difference in expected payoffs with and without it. We show that the value of information is non-monotonic in the ex-ante expected value of the project. Indeed, information is more valuable to buyers who have the highest a priori uncertainty between investing or not in the project. Buyers who have either a highly negative or a highly positive ex ante expected value will hardly change their actions as a consequence of the revised probabilities.

We then look at how a market for information works. We focus on the optimal pricing schemes for a monopolist information owner. A striking result is that the optimal menu of contracts is remarkably simple. A pure royalty is offered to buyers whose projects have low ex-ante expected value and a pure fixed fee is offered to buyers whose projects have high ex-ante expected value. A slightly modified version of this finding survives to the presence of different types of information and to the introduction of competition in the market for information.

Research tools are a good example of pure diagnostic information unbundled from execution and other services.\(^3\) Research tools are technologies that have no direct commercial value to end users, but are inputs to R&D (Schankerman and Scotchmer, 1999). In the pharmaceutical industry, many companies, have entered into the business of developing and supplying tools that aid in drug development. Prominent examples of research tools in biomedical research include databases of gene sequences and locations and details of cellular structures, which can help researchers develop new cures.

In many cases, these tools are protected as intellectual property, often covered by various patents (Eisenberg and Heller, 1998; Schankerman and Scotchmer, 1999). One feature of the commercialization practices of research tools has attracted much comment from users. Many users point with disapproval to the attempt by the providers of the research tools to charge a royalty on the revenues earned from the use of the tools. This implies that the contracts “reach through” to the drug that may be discovered with the use of the tool. Many pharmaceutical firms find this provision particularly onerous and some of

\(^2\) Since our focus here is on the mechanisms through which information can be sold we shall simply ignore these otherwise important and crucial problems.

\(^3\) In many circumstances, information is bundled with physical objects and services (i.e. diagnostic information is bundled with execution). This is the case of most of the so-called “information products”. For instance, IT consultants and management consultants are typically paid for both diagnosing the
them claim that they would rather do without the tool than agree to such terms (Eisenberg, 2000). Note that such royalties are simply payments contingent on a successful outcome, namely the development and commercialization of a drug. Thus, on the face of it, such hostility about the form of the contract is not easy to understand.

Research tools are not the only example of markets for diagnostic information. An increasing number of law firms is offering their customers pure diagnostic legal information. Most of the times the client has the option to resort to the provider of the information for the execution and other related services. However, there are firms that specialize in providing pure legal diagnostic information. For instance, The Philadelphia Bar Association LRIS, a Philadelphia-based company founded in 1948, has a business model built around the provision of diagnostic legal services and legal help. The company attorneys counsel customers on their legal rights, and, if needed, they will identify the appropriate attorney who can undertake further representation. The company typically charges a fixed-fee for its services. Other companies use more complicated pricing schemes for charging legal diagnostic advice. In addition to the fixed fee, a contingency fee agreement is sometimes employed, so that the law firm will only get paid if the client wins the case or if the claim is settled. In some cases, the diagnostic information is not charged if the client agreed to be represented by the firm.

A third example is provided by independent financial research. In the past few years, the increase in the number of self-directed investors has created demand for unbiased and independent stock market research that provides trading ideas. In particular, with the recent upsurge of online brokers that do not typically produce their research, there is an opening to be filled by independent research companies. Indeed, many investors cast a jaundiced eye at the research of the brokerage firms, placing a value on independent research companies. Wall Street Strategies is an example of such firms. The company competitive edge lies in its ability to provide effective, timely and independent stock market advice that identifies buying and selling opportunities in the market. The firm

problem and also solving it. Similarly, many accounting and audit products both scan for errors and also rectify them.

Making payments contingent on successful outcome is a natural way to provide incentives for unobserved effort when diagnosis is bundled with a service. For instance, lawyers may work harder if they only get paid when they win. However, as we show below, an information seller can also use such contingent payments to discriminate between buyers with different willingness to pay.
sells its research (information) at a fixed price that varies accordingly to the trading horizons of the investors. Institutional investors are typically charged a much higher price.\textsuperscript{5}

The literature on pricing diagnostic information is quite limited. The paper closest to ours is Chang and Jevons Lee (1994) that analyzes a framework where the information owner is a marketing consultant who sells the information to a downstream industry of producers. The consultant can perfectly price discriminate since buyers’ characteristics are common knowledge. Hence, he can always extract the full value of the information from each buyer. The optimal pricing scheme consists in the selection of a subset of buyers to which the information is sold. Since buyers interact in the product market, the fact that a buyer has the information affects negatively how much other buyers would pay for it. Admati and Pfleiderer’s work has focused on the market for financial information. See Admati and Pfleiderer (1986, 1988, 1990). They assume that the seller of information can both exploit directly the information by trading in the financial markets and by selling it to other traders. Informed traders compete among themselves and moreover there is a leakage of information through asset prices (traders can free-ride on the information of others). So, the seller might find optimal to be very restrictive in its selling strategy.\textsuperscript{6} In our paper we abstract from the interaction in the downstream market that have been explored by Admati and Pleiderer and Chang and Jevons Lee by assuming that the values of buyers’ projects are independent.\textsuperscript{7} In turn, our paper generates much greater insights on the contractual mechanisms upon which a market for information is based.

This paper is also related to the literature on the value of information. A seminal paper by Radner and Stiglitz (1984) argued that the value of information exhibits increasing

\textsuperscript{5} Markets for diagnostic information are by no means limited to these three examples. Both doctors and car mechanics perform and charge for diagnostic tests separately and independently of the subsequent execution. Marketing research providers are another example of firms specialized in producing and supplying diagnostic information.

\textsuperscript{6} Admati and Pfleiderer explore different possibilities for relaxing competition among informed traders and for avoiding the leakage of information through asset prices. When the information is sold directly, the seller might prefer to have noisy signals (i.e. selling imperfect information). An alternative solution is to sell shares of a mutual fund that has been created by exploiting the information.

\textsuperscript{7} We focus on information that is non-rival in that it can be applied in several contexts. For instance, PCR (polymerase chain reaction) is a method of rapidly creating copies of genetic material. As such, it is widely used, not only in drug discovery but also in a variety of other uses, including forensic medicine and paleontological research. Formally, we assume that the values that buyers derive from information are
marginal returns over some range. This non-concavity characteristic of information would make particularly difficult the functioning of a market for information. Moscarini and Smith (2000) argue that information is the sum of many signals whose value (i.e. the marginal value of information) is decreasing, so that non-concavity in the value of information is not longer a problem. In our paper, the purchase of information is a zero-one decision, either the buyer acquires the information or he does not. In other words, we do not allow the buyers to purchase several pieces of information. We purposely keep as simple as possible the way in which we model the value of information since we put all our emphasis on the pricing mechanisms through which information is sold. Finally, there is a quite well developed literature on technology licensing contracts to which this paper is also related. Several authors have analyzed the ‘optimal’ contractual arrangements for the exploitation of innovations and their welfare properties. Kamien and Tauman (1986) and Kamient et al. (1992) show that licensing by means of a royalty is inferior to a fixed fee policy both for a non-producer patentee and for consumers. Muto (1993) finds that a royalty might be superior to a fixed fee in a differentiated goods duopoly with Bertrand competition. Rockett (1990) shows that output royalties can be optimal when the licensor and the licensee compete in the same product market. In a framework with asymmetric information, Gallini and Wright (1990) show that unit royalties may be preferred to fixed fees when the licensor has superior precontractual information about the economic value of the technology. In contrast our paper posits that it is the patentee who lacks information about the “type” of the buyers. See also Beggs (1992).

The next section derives the value of diagnostic information. Section 3 analyzes optimal pricing in the basic model with homogenous information, whereas section 4 considers the case of differentiated information. Section 5 concludes the paper.

2. A model for diagnostic information

Buyers use the diagnostic information to make decisions about investment projects. Each buyer can only undertake one project. Projects can either be good (G) or bad (B). A G
project generates a (gross) return of $V > 0$. A B project generates a return of 0. There is an ex ante probability $q$ that a given project is of type G, and $1 - q$ that is of type B. Projects have a fixed cost, $I$, which must be paid independently of the type of the project.\(^8\) Buyers differ in the probability that the project is of type G. We assume that buyers, indexed by $q$, are distributed according to the distribution function $F(q)$. We also assume that the support of $F(q)$ is compact, $f(q)$ is the density function and that $q \in [0,1]$.\(^9\) One can think at $q$ as the likelihood that a project will be successful, which might be related to the size of the buyer, its market share or the efficiency of its organization.\(^10\) Indeed, a larger $q$ means that the project has a larger expected value. The underlying assumption is that the seller only knows the distribution of $q$.

We model diagnostic information in the following way. Diagnostic information produces a signal about the type of the project. See table 1. After observing the signal, the buyer updates his belief and takes a better investment decision. Notice that under this modelling assumption information does not increase the “real” value of the project (i.e. $V$, $q$ and $I$ are not affected by the use of information). It is in this sense that information is “diagnostic”. Although in the rest of the paper we simply use the word “information”, one has to keep in mind that we shall only talk about information of the diagnostic type, i.e. information unbundled from execution and other services.

| TABLE 1 ABOUT HERE |

Let $Pr(Y|G) = m$ and $Pr(N|B) = n$. Hence $Pr(N|G) = (1 - m)$ and $Pr(Y|B) = (1 - n)$. An $N$ signal conditional on a $G$ project is also referred to as a “false negative”, whereas a $Y$ signal conditional on a $B$ project is typically called a “false positive”. Notice that the

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\(^8\) For instance, in the case of research tools, $q$ is the ex ante probability that a given research path will lead to drug discovery, $V$ is the profit from the commercialization of the drug and $I$ is the R&D expenditure. For legal advice, $q$ is the ex ante probability that the case is won or settled, $V$ is the damage compensation established by the court or agreed in a settlement and $I$ is the legal expenses.

\(^9\) Alternatively, one can assume that the source of heterogeneity across buyers is $V$, the gross return of a successful project. In a previous version of this paper we followed this route obtaining qualitatively similar results. Details are available from the authors upon request.

\(^10\) The interpretation we shall adopt throughout the paper is that large firms are characterized by larger values of $q$, whereas small firms are captured by small values of $q$. Notice that the ex-ante expected value of a project is an increasing function of $q$. So, large value projects are associated with large values of $q$. 
“quality” of the information depends on m and n. Larger values of m and n imply a smaller probability of false negative and false positive, i.e. the signal is more reliable and the information it provides more valuable.

After observing the signal the buyer updates his beliefs. Using Baye’s rule it is easy to compute the probability that the project is of type G once we observe a Y signal:

$$\Pr(G|Y) = \frac{mq}{mq+(1-q)(1-n)}$$

and the probability that the project if of type G once we observe an N signal:

$$\Pr(G|N) = \frac{(1-m)q}{(1-m)q+(1-q)n}.$$ 

2.1. The value of information: Non-monotonicity

The value of information for a given buyer is the difference between his expected profits with the information and his expected profits without it. Differences in q translate in differences in the private benefit of information. However, not necessarily larger q benefit the most from having the information.

We need to consider two separate cases depending on whether the buyer would have invested or not without information.

Case 1: \( qV \geq 1 \), the buyer invests even without information.

The value of information is given by:

$$W(q) = qm(1-V) + (1-q)(1-n)(1-I) - (qV - I).$$

Notice that the sum of first two terms is the expected profit of investing in the presence of a Y signal. The last term is the expected profit without information. One can rewrite this expression as

$$W(q) = nl - q\left[(1-m)V + (m+n-1)I\right].$$

Notice that \( W(q) \) is equal to zero at

$$q = \frac{nl}{V(1-m) + (m+n-1)I} = q_2.$$  

Case 2: \( qV < 1 \), the buyer does not invest without information.

This interpretation works better if one assumes that the source of heterogeneity across buyers is V. See previous footnote.

\(^{11}\) It is easy to see that for any \( q \in \left[\frac{I}{V}, q_2\right], \Pr(B|N) V - I \leq 0 \), so that the buyer does not invest if he observes a N signal.
The value of information is given by \( W(q) = q m (V - I) + (1 - q)(1 - n)(-I) - 0 \). In this case the expect profit of the buyer without information is zero. One can rewrite this expression as \( W(q) = -(1 - n)I + q[mV - (m + n - 1)I] \). Notice that \( W(q) \) is equal to zero at \( q = \frac{(1 - n)I}{mV - (m + n - 1)} = q_1 \).\(^{12}\)

Figure 1 shows graphically the value of information. The intuition behind this graph is the following. For \( q \geq I/V \), \( W(q) \) represents the saving in investment costs, \( nI(1-q) \) minus the loss from a false negative (relative to the baseline, which is always invest in Case 1), \((1-m)(V-I)q\). As \( q \) rises, the cost of a false negative rises and the saving in investment cost decreases. For Case 2, \( W(q) \) represents the gain from investing (the baseline is “do not invest”), \( mq(V-I) \), minus the loss from a false positive, \((1-n)(1-q)I\). Notice that the gain is increasing in \( q \) and the cost of a false positive is decreasing in \( q \).

**FIGURE 1 ABOUT HERE**

**Proposition 1**: The value of information is non-monotonic in \( q \).

**Proof**: It is easy to see that the derivative of \( W(q) \) with respect to \( q \) is increasing for \( q < I/V \) and decreasing for \( q > I/V \). QED

**Remark 1**: The value of information is strictly increasing in both \( m \) and \( n \), decreasing in \( V \) and increasing in \( I \) if \( q > I/V \), increasing in \( V \) and decreasing in \( I \) if \( q < I/V \).

**Proof**: Omitted.

**Remark 2**: All \( q > I/V \) are more sensitive to a false negative than a false positive. All \( q < I/V \) are more sensitive to a false positive than a false negative.

\(^{12}\) It is easy to see that for any \( q \in \left[q_1, \frac{I}{V}\right] \), \( \Pr(G|I)V - I \geq 0 \), so that the buyer does invest if he observes a \( Y \) signal.
Proof. Let \( m + n = k \). For \( q > I/V \),

\[
W(q) = nI - q[(l + n - k)V + (k - 1)I].
\]

Notice that

\[
\frac{\partial W(q)}{\partial n} = I - qV \leq 0.
\]

For \( q < I/V \), the proof goes in the same direction. QED

In other words, all buyers on the right of \( I/V \) (see Figure 1) are relatively more concerned with the probability of a false negative, whereas all buyers on the left of \( I/V \) are relatively more concerned with the probability of a false positive. Indeed, for \( q > I/V \) the baseline is to invest. These buyers are going to change their actions only if they observe an N signal. Hence, they would like the N signal be as precise as possible. The precision of the N signal is larger when the probability to receive an N signal conditional on the project being good is low, i.e. when \( \Pr(N|G) = 1 - m \), the probability of a false negative, is small.

3. Pricing diagnostic information

Using this framework we will analyze the optimal mechanism for selling information. Two classical simple contracts are a fixed fee and a royalty. In the first case, the buyer pays a price for the information. In the latter case, the buyer pays an amount of money that depends on certain observable (and verifiable) actions or outcomes. We shall model the royalty as a share of project’s (gross) returns, \( V \). This might be thought of as the contingent payments used by law firms or the reach trough contracts used by providers of research tools.

Before proceeding further, it is worthwhile to stress the characteristics of both a pure fixed price and a pure royalty as mechanisms to appropriate rents. A pure fixed price means that the payment for the information does not vary with \( q \). A pure royalty implies that the payment for the information is an increasing function of \( q \). Hence, low \( q \) buyers tend to prefer a royalty scheme, whereas high \( q \) buyers tend to prefer a fixed price.

Moreover, a royalty is especially bad-suited to capture value from high \( q \) buyers since for

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A qualitatively similar alternative is to model the royalty as a share of net project’s returns, \( V - I \). All results would hold unchanged with the royalty parameter simply scaled up. If investment decisions are observable then the royalty could be an amount of money, \( T \), payable conditional on investment taking place. Notice that investment occurs with probability \( mq + (1-n)(1-q) \). Hence, for any given \( T \), each potential buyer \( q \) should pay \( (1-n)T + q(m+n-1)T \). However, this payment can be replicated with a well-designed two-part tariff, where a fixed price is combined with a royalty on \( V \), so our analysis of two-part tariffs encompasses this special royalty scheme.
these buyers the value of the information is decreasing in q. Figure 2 and figure 3 show
an arbitrary fixed fee and an arbitrary royalty respectively.

FIGURES 2 AND 3 ABOUT HERE

With this in mind, we now look at the possibility for the seller to offer a menu of
contracts. We then return to other simpler pricing schemes to make a comparison.
Let any arbitrary contract be defined by the pair \( \{\alpha, \beta\} \). A menu of contracts is a set of
such pairs. Given that every buyer has the same V and I, any contract is equivalent to a
two-part tariff. A pure fixed price contract and a pure royalty contract are therefore
defined as \( \{\alpha, 0\} \) and \( \{0, \beta\} \), respectively. An arbitrary two-part tariff is represented in
figure 4.

FIGURE 4 ABOUT HERE

**Menu of contracts:** Can the seller design contracts that are tailored to the different
buyers? In other words, can the seller offer a menu of contracts and the buyers choose the
ones that are better suited to their characteristics? How does the optimal menu of
contracts look like? In this section we shall address these issues.

We shall proceed in the following way: first, we characterize the optimal menu of
contracts and then we compute it under a special case. At this stage, it is important to
notice that the characterization of the optimal menu of contracts does not rely on any
assumption about the distribution of q.

It is useful to start by proving the following lemma:

**Lemma 1:** If \( \alpha, \beta \in \mathbb{R} \) then the optimal menu of contracts consists of the following two
contracts

\[
\begin{align*}
\alpha_1 &= -(1-n)I; \quad \beta_1 = \frac{mV - (m + n - 1)I}{mV} \\
\alpha_2 &= nI; \quad \beta_2 = -\frac{(1-m)V + (m + n - 1)I}{mV}
\end{align*}
\]

\( \{\alpha_1; \beta_1\} \) is chosen by all \( q \in \left[ q_1, \frac{I}{V} \right] \) and

\( \{\alpha_2; \beta_2\} \) is chosen by all \( q \notin \left[ q_1, \frac{I}{V} \right] \) where

\( q_1 = \min(q) \) and \( q_2 = \max(q) \).
\{\alpha_2; \beta_2\} is chosen by all \(q \in \left(\frac{I}{V}, q_2\right]\), where \(q_i = \frac{(1-n)I}{mV - (m+n-1)I}\) and
\[
q_1 = \frac{nI}{(1-m)V + (m+n-1)I}.
\]

**Proof:** First notice that \(\forall q < \frac{I}{V}, \alpha_1 + \beta_1mVq < \alpha_2 + \beta_2mVq\) and that \(\forall q \geq \frac{I}{V}, \alpha_1 + \beta_1mVq \geq \alpha_2 + \beta_2mVq\) (= only at \(q = \frac{I}{V}\)). This implies that the incentive compatibility constraints are satisfied. Also notice that \(W(q) = \alpha_1 + \beta_1mVq\), \(\forall q \in \left[q_1, \frac{I}{V}\right]\) and that \(W(q) = \alpha_2 + \beta_2mVq, \forall q \in \left[\frac{I}{V}, q_2\right]\). This implies that participation constraints are satisfied. It remains to show that this menu of contracts is optimal for the seller. Notice that this menu of contracts extracts the full value generated by the information, so there are no other menus of contracts that allow the seller to obtain larger profits. \textit{QED}

Figure 5 helps illustrate the proof of Lemma 1. Notice that any possible contract yields an amount of payments, \(R(q)\), that is linear in \(q\), i.e. \(R(q) = \alpha + \beta mVq\). This means that by appropriately choosing \(\alpha\) and \(\beta\) one can define any possible line in the \(\{R(q), q\}\) space. Moreover, the value of information in the intervals \(\left[q_1, \frac{I}{V}\right]\) and \(\left[\frac{I}{V}, q_2\right]\) is everywhere linear in \(q\). Hence, graphically, finding the optimal menu of contracts means to identify the two lines that perfectly overlap \(W(q)\) for \(q < I/V\) and for \(q \geq I/V\).

This special case has two noteworthy features: first, the optimal menu of contracts is extremely simple; second, the seller extracts the full value of information from the buyers. Whereas simplicity of the optimal menu of contracts remains, value extraction is not any longer complete when \(\alpha\) and \(\beta\) are forced to take non-negative values. In the rest of the paper we will assume that \(\alpha\) and \(\beta\) are non-negative.\textsuperscript{14}

\textsuperscript{14} Negative prices are typically not observed in practice, and would probably be vetoed by the Antitrust Authority.
Before proceeding it is useful to establish two properties of the optimal menu of contracts.

**Property 1.** Let \( \{\alpha(q), \beta(q)\} \) be the contract designed for buyer \( q \). Then, for any \( q' > q \), \( \alpha(q) \leq \alpha(q') \) and \( \beta(q) \geq \beta(q') \).

**Proof.** Incentive compatibility implies that \( \alpha(q) + \beta(q)mVq \leq \alpha(q') + \beta(q')mVq' \) and \( \alpha(q) + \beta(q)mVq' \geq \alpha(q') + \beta(q')mVq' \). Putting together these two inequalities one obtains that \( mVq[\beta(q') - \beta(q)] \geq \alpha(q) - \alpha(q') \geq mVq'[\beta(q') - \beta(q)] \). Since \( q' > q \), it must be that \( \beta(q') \leq \beta(q) \). Notice also that \( \alpha(q') - \alpha(q) \geq mVq[\beta(q) - \beta(q')] \geq 0 \). Hence, \( \alpha(q') \geq \alpha(q) \). QED

**Property 2 [Concavity].** Let \( \{\alpha(q), \beta(q)\} \) be the contract designed for buyer \( q \). Take \( q_1, q_2, q_3 \) such that \( q_1 < q_2, q_3 = \lambda q_1 + (1 - \lambda)q_2 \) with \( 0 \leq \lambda \leq 1 \). Then,

\[
\lambda[\alpha(q_1) + \beta(q_1)mVq_1] + (1 - \lambda)[\alpha(q_2) + \beta(q_2)mVq_2] \leq \alpha(q_3) + \beta(q_3)mVq_3.
\]

**Proof.** Incentive compatibility implies that \( \alpha(q_1) + \beta(q_1)mVq_1 \leq \alpha(q_3) + \beta(q_3)mVq_1 \) and \( \alpha(q_2) + \beta(q_2)mVq_2 \leq \alpha(q_3) + \beta(q_3)mVq_2 \). Multiply both sides of the inequalities by \( \lambda \) and \( 1 - \lambda \) respectively, and then sum across them to complete the proof. QED

**Proposition 2 [Characterization of the optimal menu of contracts].** If \( \alpha, \beta \in \mathbb{R}^+ \) then the optimal menu of contracts consists of the following two contracts: a pure royalty \( \{0; \beta_1\} \) and a pure fixed price \( \{\alpha_2; 0\} \). \( \{0; \beta_1\} \) is chosen by all \( q \in [q_1(\beta_1), \bar{q}] \) and \( \{\alpha_2; 0\} \) is chosen by all \( q \in (\bar{q}(\beta_1, \alpha_2), q_2(\alpha_2)) \], where \( q_1(\beta_1) = \frac{(1-n)I}{m(1-\beta_1)V-(m+n-1)I} \), \( q_2(\alpha_2) = \frac{nI - \alpha_2}{(1-m)V + (m+n-1)I} \) and \( \bar{q}(\beta_1, \alpha_2) = \frac{\alpha_2}{\beta_1 mV} \).
Proof: Assume the seller has designed a menu of contracts and call $q_1^*$ and $q_2^*$ the values of q such that for all $q < q_1^*$ and for all $q > q_2^*$ no buyer is interested in purchasing the information (given the menu).

Let $\{\alpha_1;\beta_1\}$ and $\{\alpha_2;\beta_2\}$ be the two contracts designed respectively for $q_1^*$ and $q_2^*$.

Notice that $W(q_1^*) \geq \alpha_1 + \beta_1 mVq_1^*$ and that $W(q_2^*) \geq \alpha_2 + \beta_2 mVq_2^*$. Consider any $q \in [q_1^*, q_2^*]$. Since the incentive compatibility constraints must be satisfied, the following inequality has to hold: $\alpha(q) + \beta(q) mVq \leq \min \{\alpha_1 + \beta_1 mVq; \alpha_2 + \beta_2 mVq\}$, where $\{\alpha(q); \beta(q)\}$ is the contract designed for buyer q. Otherwise $q \in (q_1^*, q_2^*)$ will choose either $\{\alpha_1;\beta_1\}$ or $\{\alpha_2;\beta_2\}$.

Call $\Pi^M = \int_{q_1^*}^{q_2^*} [\alpha(q) + \beta(q) mVq] f(q) dq$ the profit of the seller. Then, the following inequality holds $\Pi^M \leq \int_{q_1^*}^{q_2^*} \min \{\alpha_1 + \beta_1 mVq_1^*; \alpha_2 + \beta_2 mVq_2^*\} f(q) dq \leq \int_{q_1^*}^{q_2^*} \min \left\{ \frac{W(q_1^*)}{q_1^*}; \frac{W(q_2^*)}{q_2^*} \right\} f(q) dq \equiv M$. The last inequality is due to $W(q_1^*) \geq \alpha_1 + \beta_1 mVq_1^*$, which implies $\frac{W(q_1^*)}{q_1^*} \geq \frac{\alpha_1 + \beta_1 mVq_1^*}{q_1^*} \geq \alpha_1 + \beta_1 mVq_1^*$ (given that $q \geq q_1^*$), and $W(q_2^*) \geq \alpha_2 + \beta_2 mVq_2^* \geq \alpha_2 + \beta_2 mVq$ (given that $q_2^* \geq q$).

Finally, notice that the seller can extract exactly $M$ by offering the following menu of contracts $\left\{ \alpha_1 = 0; \beta_1 = \frac{W(q_1^*)}{mVq_1^*} \right\}$ and $\left\{ \alpha_2 = W(q_2^*); \beta_2 = 0 \right\}$. Since $\Pi^M \leq M$, then there does not exist any other menu of contracts for the interval $[q_1^*, q_2^*]$ such that buyer’s profits are larger.

Finally define $\left\{ \tilde{q} = \frac{\alpha_2}{\beta_2 mV} = \frac{W(q_2^*)}{W(q_1^*)} mVq_1^* \right\}$, then it is easy to see that $\left\{ 0; \beta_1 \right\}$ is chosen by all $q \in [q_1^*, \tilde{q}]$ and $\left\{ \alpha_2; 0 \right\}$ is chosen by all $q \in (\tilde{q}, q_2^*)$. Finally, notice that $q_1^* = q_1(\beta_1)$ and $q_2^* = q_2(\alpha_2)$. QED

Figure 6 illustrates the optimal menu of contracts.
After characterizing the optimal menu of contracts, it is relatively simple to compute the optimal value of $\beta_1$ and $\alpha_2$. Note that $q_1(\beta_1) = \frac{(1-n)I}{m(1-\beta_1)V - (m+n-1)I}$.

$q_2(\alpha_2) = \frac{nI - \alpha_2}{(1-m)V + (m+n-1)I}$ and $\bar{q} = \frac{\alpha_2}{\beta_1 mV}$. One can write the seller’s profits as follows:

$$\Pi^M = \int_{q_1(\beta_1)} q_1 mVq \phi(q)dq + \int_{\bar{q}} q_2(\alpha_2) f(q)dq.$$ Solving for the first order conditions one can compute the optimal value of $\beta_1$ and $\alpha_2$.

**Remark 3 [Comparative statics].** Let $q$ be uniformly distributed between 0 and 1, then

$$\frac{\partial \beta_1}{\partial \gamma} > 0, \frac{\partial \alpha_2}{\partial \gamma} > 0,$$ where $\gamma$ is a scale factor that multiplies both $V$ and $I$.

*Proof.* Simulations. See the appendix.

The comparative statics with respect to $\gamma$ is easily understood if one thinks at a change in $\gamma$ as a change in the unit of measure of both $V$ and $I$. Whereas the fixed price, also measured in monetary units, should change accordingly, the royalty should not be affected at all (being a share).

### 3.1. Comparison across pricing schemes when $m=n=1$

Assuming that $m=n=1$ implies that there is no risk of either a false positive or a false negative, i.e. the information delivers a perfect signal. We also assume that $q$ is uniformly distributed between 0 and 1. Under these two assumptions it is possible to explicitly compute the optimal pure fixed price, royalty, two-part tariff and menu of contracts. In addition, one can easily compare profits and welfare. Fortunately, simulations allow us to conjecture that these results hold under any value of $m$ and $n$. 
(provided that q is uniformly distributed between 0 and 1). Some of the simulations are reported in the appendix.\textsuperscript{15} Recall that prices must be non-negative. Table 2 summarizes these calculations.

\textbf{TABLE 2 ABOUT HERE}

By comparing different expressions from Table 2 one can derive the following results:

\textbf{Result 1 [Profits].} Profits are always larger under the menu of contracts than under any other pricing scheme. However, profits under royalty are larger (smaller) than profits under fixed fee if \( V < 2I \) (\( V > 2I \)).

\textbf{Result 2 [Welfare].} Welfare is always larger under the menu of contracts than under any other pricing scheme. The comparison of welfare between pure royalty and pure fixed price is more interesting. If \( V \geq \frac{4}{3} I \) welfare is always larger under fixed price. However, for \( 2I > V \geq \frac{4}{3} I \) the seller chooses a royalty over a fixed price (profits are larger).

\textbf{Result 3 [Buyers’ preferences over pricing mechanisms].} First, large (small) q prefers a fixed price (royalty) over a royalty (fixed price). Indeed, if \( V \geq 2I \) all \( q > \frac{V - I}{2V} \) prefer the fixed price and if \( V < 2I \) all \( q > \frac{I}{2V} \) prefer the fixed price. If \( V < 2I \) all \( q > \frac{I}{2V - I} \) prefer the menu of contracts over the royalty, whereas all \( q < \frac{I}{2V - I} \) are indifferent. If \( V \geq 2I \) all \( q > \frac{V - I}{2V - I} \) prefer the menu of contract over the royalty and all \( q < \frac{V - I}{2V - I} \) prefer the royalty over the menu of contract. Finally, all \( q > \frac{V}{2V} \) prefer the fixed price over the menu of contracts. Summing up, there is no buyer that strictly prefers the menu of contracts over a pure pricing scheme.

\textsuperscript{15} The full set of simulations along with a Mathematica file are available from the authors upon request.
In words, this suggests the following: Firms with low ex ante expected value projects (small firms) tend to prefer a royalty over a fixed fee, whereas firms with high ex ante expected value projects (large firms) have opposite preferences. This seems to explain the pricing controversy over research tools in biochemical research. Indeed, as we discussed in the introduction, many large pharmaceutical firms have shown great hostility for reach through contracts. Moreover, this worry about contingent payments is also grounded on efficiency considerations. Indeed, with fixed fee and royalty as only available options, welfare is always maximized when the seller optimally chooses the fixed price over the royalty. However, whenever the royalty is optimally chosen by the seller, it is not guaranteed that welfare is maximized.

### 3.2. Competition

Assume that the information can be provided by two different sellers. What would be their pricing policies? For the sake of simplicity, consider only pure pricing policies: either royalty or fixed fee (however, the argument applies in general). If both firms use the same pricing scheme, it is straightforward to see that they will end up charging zero prices. Indeed, this is simply Bertrand competition with a homogeneous good. One could argue that firms might seek differentiation in their pricing policies in order to obtain positive profits. So, one firm might use a royalty and the other a fixed fee. However, even in this case all profits will be competed away. Indeed, the royalty seller (i.e. the seller offering information in exchange of a royalty) has always an incentive to completely undercut the fixed price seller (i.e. the seller offering information in exchange of a fixed price). Take any candidate equilibrium with positive $\alpha$ and $\beta$. In Figure 7, two solid lines represent these two parameters. Let $\bar{q}$ be the buyer indifferent between buying the information from one firm at the fixed price, $\alpha$, and buying the same information from the other firm at the royalty, $\beta$. Now, notice that this cannot be an equilibrium since by slightly reducing $\beta$, the royalty seller earns larger profits. Indeed, for any given $\alpha$ the

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16 An indifferent buyer does not exist when one firm supplies all demand. However, this cannot be an equilibrium since the other firm would earn zero profits and would have incentives to decrease its price till it attracts some positive demand.
royalty seller profits can be written as $\frac{\alpha \tilde{q}}{2} - \frac{q_1(\beta)W(q_1(\beta))}{2}$. Notice that, as long as

$$q \leq q_2(\alpha), \quad q = \frac{\alpha}{\beta mV}$$

is decreasing in $\beta$, $q_1(\beta)$ is increasing in $\beta$ and $\frac{\partial W(q_1)}{\partial q_1} > 0$.

Hence, the royalty seller will keep reducing his price till $\tilde{q} = q_2(\alpha)$. However, this cannot be optimal for the fixed price seller since he earns no profits and can always do better by reducing the price.

**FIGURE 7 ABOUT HERE**

### 4. Differentiated information

In this section we allow for the information to be differentiated, i.e. the seller has different types of information to commercialize.\(^{17}\) Our main objective is to show that the optimal menu of contracts we have derived in the previous section maintains most of its properties. Proposition 3 fully characterizes such menu of contracts. We then compute the optimal menu of contracts under some simplifying assumptions. Finally, we introduce competition among information sellers. First, and quite obvious, we show that when one allows for some degree of heterogeneity in the information supplied by the sellers, equilibrium profits are bounded away from zero. Second, we provide an example in which firms can commit to a pricing scheme (either a pure royalty or a pure fixed price) and we analyze the Nash equilibrium in the space of contracts.

In our framework two parameters are responsible for the quality of information, $m$ and $n$, so different types of information correspond to different combinations of these two parameters. We start our analysis by assuming that the seller has available for commercialization two types of information, $\{\bar{n}; \bar{m}\}$ and $\{\tilde{n}; \tilde{m}\}$ with $\bar{n} > \tilde{n}, \bar{m} > \tilde{m}$ and $\bar{n} + \bar{m} = \tilde{n} + \tilde{m} = 1 + k$. Type 1 information delivers a signal with a lower probability of a false positive and a higher probability of a false negative. Type 2 information delivers a

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\(^{17}\) An important assumption we make throughout this section is that each buyer can only buy one type of information or, put it differently, that the marginal value of having more than one type of information is zero.
signal with a lower probability of a false negative and a higher probability of a false positive. Figure 8 shows the two types of information.

FIGURE 8 ABOUT HERE

4.1. Characterization of the optimal menu of contracts

The main result of this section, reported in Proposition 3, is analogue to Proposition 2 of the previous section. Proposition 3 shows that allowing for more than one type of information does not substantially change the shape of the optimal menu of contracts. That is, high q will be charged a fixed fee and small q will be charged a pure royalty.

We start our analysis by stating a set of Lemmas, whose formal proof is contained in the appendix, which establish some intermediate results. Our objective here is to show that the seller finds always profitable to supply both types of information (vis-à-vis selling only one type), and that, given an optimal menu of contracts, there must be a unique buyer indifferent between buying any of the two types of information. We will also show that as q increases the buyers move from type 1 information to type 2 information, so that type 1 (2) information is bought by buyers with low (high) ex ante expected value projects. Given these preliminary results, Proposition 3 identifies the optimal contracts to be offered to the indifferent buyer and to the first and the last buyer actually acquiring the information (given the menu). We finally argue that these contracts define the upper bound to what the seller can extract from the buyers. Hence, there is no way for the seller to improve upon this restricted set of contracts.

Lemma 2. The seller offers both types of information.

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18 We take the two types of information as exogenously given. However, it would not be too difficult to endogenize the choice of the types of information to be produced and commercialized by the seller. One could start assuming that the cost of supplying information with $m+n=1+k$ is zero, whereas the cost of supplying information with $m+n>1+k$ is sufficiently large. Hence, the seller can trade precision between m and n, but not increase the ‘global’ precision $(m+n)$ of the information. If free to choose the seller would commercialize the two types of information that maximize his profits. In addition, k could be decided optimally by the seller given a cost $f(k)$ that goes to infinity for $k$ close to 1. Although this is an interesting exercise, it is behind the purpose of this section. Moreover, the characterization of the optimal menu of contracts is derived for any arbitrary two types of information and would not change for the two types optimally selected by the seller.
Lemma 3. Let $q_1$ and $q_2$ be respectively the lowest and the highest value of $q$ buying information, then in the optimal menu of contracts $q_1$ must buy type 1 information and $q_2$ must buy type 2 information.

Lemma 4. Let $q_1$ and $q_2$ be respectively the lowest and the highest value of $q$ buying information, then all $q \in (q_1,q_2)$ will also acquire the information.

Lemma 5. There exists a unique indifferent buyer.

Proposition 3 [Characterization of the optimal menu of contracts]. Let $\alpha(q), \beta(q) \in \mathbb{R}^+$, then the optimal menu of contracts consists of at most three contracts. No two-part tariff is used in the optimal menu of contracts, and each type of information is sold through at most two different contracts. The lowest $q$’s always buy type 1 information at a pure royalty. The highest $q$’s always buy type 2 information at a pure fixed fee.

Proof. We first show that for each type of information the seller uses at most two contracts to extract rents. Let $\{\alpha(q_1); \beta(q_1)\}$ and $\{\tilde{\alpha}(q_2); \tilde{\beta}(q_2)\}$ be the contracts meant for respectively $q_1$ and $q_2$. $\{\tilde{\alpha}(\tilde{q}); \tilde{\beta}(\tilde{q})\}$ and $\{\tilde{\alpha}(\tilde{q}); \tilde{\beta}(\tilde{q})\}$ be the two contracts for the unique indifferent buyer ($\tilde{q}$). Notice that it must hold that 
\[ W(\tilde{q}) - (\alpha(q) + \beta(q)\bar{m}Vq) = \tilde{W}(\tilde{q}) - (\tilde{\alpha}(\tilde{q}) + \tilde{\beta}(\tilde{q})\bar{m}\tilde{Vq}). \]
Call $\Pi^M \leq \int_{q_1}^{\tilde{q}} \left[ \bar{\alpha}(q) + \bar{\beta}(q) \bar{m} V_q \right] f(q) dq + \int_{\tilde{q}}^{q_2} \left[ \bar{\alpha}(q) + \bar{\beta}(q) \bar{m} V_q \right] f(q) dq$ the profit of the seller. Then, the following inequality holds

$$\Pi^M \leq \int_{q_1}^{\tilde{q}} \min_{q_1} \left\{ \bar{\alpha}(q_1) + \bar{\beta}(q_1) \bar{m} V_q; \bar{\alpha}(\tilde{q}) + \bar{\beta}(\tilde{q}) \bar{m} V_q \right\} f(q) dq +$$

$$\int_{\tilde{q}}^{q_2} \min_{q_2} \left\{ \bar{\alpha}(\tilde{q}) + \bar{\beta}(\tilde{q}) \bar{m} V_q; \bar{\alpha}(q_2) + \bar{\beta}(q_2) \bar{m} V_q \right\} f(q) dq \leq$$

$$\int_{q_1}^{\tilde{q}} \min_{q_1} \left\{ \bar{\alpha}(\tilde{q}) + \bar{\beta}(\tilde{q}) \bar{m} V_q; \bar{\alpha}(q_2) \right\} f(q) dq + \int_{\tilde{q}}^{q_2} \min_{q_2} \left\{ \bar{\beta}(\tilde{q}) \bar{m} V_q; \bar{\alpha}(q_2) \right\} f(q) dq = M$$

where, $\bar{\beta}(q) = \frac{W(q)}{m V_q}, \bar{\alpha}(q) = \frac{W(\tilde{q})}{\bar{m} V_q}, \bar{\beta}(\tilde{q}) = \frac{W(q)}{m V_q}$ and $\bar{\alpha}(q_2) = \tilde{W}(q_2)$. Notice that the last inequality has been shown to hold in the proof of Proposition 2. By offering the following menu of contracts: $\{0; \bar{\beta}(q_1)\}, \{\bar{\alpha}(\tilde{q}); 0\}, \{0; \tilde{\beta}(\tilde{q})\}$ and $\{\bar{\alpha}(q_2); 0\}$ the seller can extract exactly $M \geq \Pi^M$. So, it maximizes profits. Let $\tilde{q}' = \frac{\bar{\alpha}(\tilde{q})}{\bar{\beta}(q_1) \bar{m} V}$ and

$$\tilde{q}'' = \frac{\bar{\alpha}(q_2)}{\bar{\beta}(\tilde{q}) \bar{m} V}, \text{ all } q \in [q_1, \tilde{q}'] \text{ buy type 1 information through a royalty contract, all }$$

$q \in [\tilde{q}', \tilde{q}] \text{ buy type 1 information through a fixed fee contract, all } q \in [\tilde{q}, \tilde{q}''] \text{ buy type 2 information through a royalty contract, and all } q \in [\tilde{q}'', q_2] \text{ buy type 2 information through a fixed fee contract.}$

It remains to show that at most three contracts are used in the optimal menu of contracts.

We will show that when $\tilde{q} < \frac{1}{V}$, only one contract is used for type 2 information. A similar argument would apply for $\tilde{q} > \frac{1}{V}$ and type 1 information. Let, therefore, $\tilde{q} < \frac{1}{V}$.

We will show by contradiction that the contract $\{0; \bar{\beta}(\tilde{q})\}$ is not binding, in the sense that it will not be attractive to any $q > \tilde{q}$. Suppose that some $q > \tilde{q}$ buy information 2 through the $\{0; \tilde{\beta}(\tilde{q})\}$ contract. By incentive compatibility, it must be the case that $\tilde{\beta}(\tilde{q}) \bar{m} V_q < \bar{\alpha}(q_2)$. However, this is not a profit maximizing outcome for the seller.
can always extract higher profits by selling type 2 information to all $q > \tilde{q}$ through the $\{\tilde{\alpha}(q_2); 0\}$ contract. In other words, he can increase $\tilde{\beta}(\tilde{q})$ till the point in which no buyer of type 2 information finds the royalty contract attractive. Notice that if some buyers switch to type 1 information they end up paying a higher price (in this interval of $q$, buyers are willing to pay more for type 1 information, so if the indifferent buyer is located here it must be that the seller is charging a higher price for type 1 information).

QED

4.1. Computing the optimal menu of contracts

It turns out that the actual computation of the optimal menu of contracts is very much simplified if $\{n = 1; m = k\}, \{\tilde{n} = k; \tilde{m} = 1\}$. Also let $q$ be uniformly distributed. First of all, notice that $q_1$ (the lowest value of $q$ buying information) is equal to zero. Hence, $\left\{0; \tilde{\beta}(q_1) = \frac{V - I}{V}\right\}$ is the contract for all $q$ in the right neighborhood of $q_1$. Assume that $\tilde{q} < \frac{I}{V}$ (we will check this is indeed the case once we compute the optimal menu of contracts), then by Proposition 3, in the optimal menu of contracts there are two contracts (a royalty and a fixed fee) for type 1 information and only one (a fixed fee) for type 2 information. Let $q^*$ be the buyer indifferent between buying type 1 information under the royalty contract $\left\{0; \tilde{\beta}(q_1) = \frac{V - I}{V}\right\}$ and buying the same type of information under the fixed fee contract $\{\alpha(\tilde{q}); 0\}$. Also let $\tilde{q}$ be the indifferent buyer (indifferent between buying type 1 information and type 2 information). One can show that $q^* = \frac{\alpha(\tilde{q})}{k(V - I)}$ and $\tilde{q} = \frac{I}{V} + \frac{\tilde{\alpha}(q_2) - \alpha(\tilde{q})}{(1 - k)V}$. Also notice that $q_2 = 1 - \frac{\tilde{\alpha}(q_2)}{kI}$ where $q_2$ is the highest value of $q$ buying the information. We can now write the seller’s profits as follows:

\[19\] Notice that, under the assumption that the cost of supplying information with $m + n = 1 + k$ is zero, whereas the cost of supplying information with $m + n > 1 + k$ is sufficiently large, the seller would choose to offer exactly these two types of information.
\[ \Pi^M = \frac{k(V-I)}{2} \left[ \frac{\alpha(\tilde{q})}{k(V-I)} \right]^2 + \left[ \frac{\alpha(q_2) - \alpha(\tilde{q})}{(1-k)V} + \frac{I}{V} - \frac{\alpha(\tilde{q})}{k(V-I)} \right] \alpha(\tilde{q}) + \left[ 1 - \frac{\alpha(q_2)}{kI} - \frac{I}{V} - \frac{\alpha(q_2) - \alpha(I)}{(1-k)V} \right] \alpha(q_2) \].

Maximizing this expression with respect to \( \alpha(\tilde{q}) \) and \( \alpha(q_2) \) and solving for the first order conditions one can show that \( \alpha(\tilde{q}) = \frac{kI(V-I)}{V+k(V-I)} \) and \( \alpha(q_2) = \frac{kI(1+k)(V-I)}{2[V+k(V-I)]} \).

Also notice that \( q_2^* = \frac{I}{V+k(V-I)} \), \( \tilde{q} = \frac{I}{2[V+k(V-I)]} + \frac{I}{2V} \) and \( q_2 = \frac{1}{2} + \frac{I}{2[V+k(V-I)]} \). Finally, \( \Pi^M = \frac{kI(V-I)[(1+k)V+(1-k)I]}{4V[V+k(V-I)]} \). One can easily check that both prices (the two fixed fees) and profits are increasing in \( V \) and \( k \).

### 4.2. Competition: an example

Assume that each type of information is supplied by a different firm. Firms compete with each other for the prospective buyers. The first obvious remark is that competition does not drive prices to zero. So, as long as information is slightly differentiated firms have positive profits in equilibrium.

**Remark 4.** Under differentiated information, equilibrium prices are bounded away from zero.

**Proof.** By contradiction, let both firms changing zero price be an equilibrium. Sellers’ profits are therefore equal to zero. Let’s take any of the two firms and consider the alternative strategy of offering a very small but positive price. Notice that there are some \( q \) for which one type of information has zero value whereas the other type of information has positive value. At a sufficiently small price such buyers will acquire the latter even if the former is offered at zero price. The firm offering the information at a small positive price will have some demand and its profits will be positive. Hence, zero pricing cannot be an equilibrium. QED
It is quite complex to characterize competition in general terms. So, we will only provide an illustrating example. We focus on pure pricing policies (either royalty or fixed fee). Each firm can choose either to supply the information through a royalty or through a fixed fee. We analyze the following two-stage game: in the first stage, each firm picks up a pricing policy; in the second stage, firms compete for the prospective buyers given their chosen pricing policy. The first stage of the game gives rise to four possible scenarios: 1) both firms compete through a royalty, 2) both firms compete through a fixed fee, 3) the firm selling type 1 information charges a royalty whereas the firm selling type 2 information charges a fixed fee, 4) the firm selling type 1 information charges a fixed fee, whereas the firm selling type 2 information charges a royalty.

We can solve backwards for competition in the second stage of the game to find out which one of these four possible scenarios is an equilibrium. Intuitively, we expect that scenario 3) is the equilibrium of this game. This somehow resembles the optimal menu of contracts under a monopolist seller. In particular, lower $q$ are charged through a royalty and higher $q$ are charged through a fixed fee.

To explicitly compute the profits we will consider the following example: $V = 4, I = 2, \{\bar{n} = 1; \bar{m} = \frac{1}{2}\}, \{\tilde{n} = \frac{1}{2}; \tilde{m} = 1\}$. In addition, we assume that $q$ is uniformly distributed.

The results of this exercise are summarized in the following two tables. Table 3 shows the payoff matrix of the game and the indifferent buyer given the pricing policies chosen by the firms. Notice that all $q < \bar{q}$ buy type 1 information and all $q > \tilde{q}$ buy type 2 information. Given the indifferent buyer it is easy to compute the profits of each firm. By maximizing these profits with respect to the chosen price (either royalty or fixed fee) one obtains the first order conditions whose solution gives the optimal price for each firm.

Results are reported in Table 4. As expected the Nash equilibrium of this game is one where the firm selling type 1 information uses a royalty whereas the firm selling type 2 information uses a fixed fee.

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20 In general, it is quite complex to find an explicit solution. We have computed the payoff matrix under different parameter values obtaining always the same outcome as an equilibrium of our game. Although we believe that the basic feature of this equilibrium remains when one allows $V, I, \{\bar{n}; \bar{m}\}, \{\tilde{n}; \tilde{m}\}$ to take any arbitrary value, we have not been able to prove it formally.
5. Conclusion

Diagnostic information that simply changes the estimates of the probabilities of the different states of nature, has economic value. Indeed, the decision maker might change his actions as a consequence of the revised probabilities. However, trading information is not easy and although Arrow (1962) envisaged the possibility of information becoming a commodity, he also warned us about the difficulties that information exchange might entail. We do not certainly dispute the existence of such problems. However, as the examples of research tools, independent research, legal advice among others, have shown there are markets, albeit imperfect, for information. In this paper, we have tried to understand how these markets work and, in particular, how information is priced.

We have found that the buyer’s willingness to pay for information is non monotonic in the buyer’s “type”, implying that the standard “revelation mechanism” solution cannot be mechanically applied. However, the optimal pricing scheme for a monopolist information owner turned out to be extremely simple. A pure royalty is offered to buyers whose projects have low ex ante expected value and a pure fixed fee is offered to buyers whose projects have high ex ante expected value.

There are several possible extensions of our basic framework that we did not address in the paper. In particular, we have assumed that information does not affect the level of investment, but only if the investment is actually undertaken or not. The level of investment could be a decision variable for the buyer as well. In turn, larger investment should generate larger returns. This has important implications for the shape of the value of information, which does not need to be linear any longer. Moreover, the linearity of the contract is also broken. Indeed, under this scenario the royalty affects the returns of the investment, and, in turn, how much the buyer will invest. So, the royalty has an additional negative effect that was absent in our basic model. This extension, as other possible interesting ones, is left to further research.
REFERENCES


APPENDIX

Proof of Remark 3: Extract of simulations from Mathematica.

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</table>

ADD SIMULATIONS TO SUPPORT FINDINGS IN TABLE 2 IN THE TEXT

Proof of Lemma 2. The argument of this proof is graphically illustrated in figure 9. First recall that Remark 2 guarantees that \( \bar{W}(q) > \tilde{W}(q) \) for all \( q < 1/V \) and \( \bar{W}(q) < \tilde{W}(q) \) for all \( q > 1/V \). By contradiction, suppose that only type 1 information is offered. Then, by Proposition 2, we know that the optimal menu of contracts is: a pure royalty \( \{0; \tilde{\beta}\} \) and a pure fixed price \( \{\bar{\alpha}; 0\} \). Let \( q_2(\bar{\alpha}) > \frac{I}{V} \) be the buyer indifferent between buying type 1 information and not buying it (since \( f(q) \) has a compact support and the contracts are linear in \( q \), such indifferent buyer always exists). Now suppose that type 2 information is offered at \( \{\bar{\alpha} = \alpha_2; 0\} \). First, notice that if some buyers switch from type 1 information to type 2 information they will pay in any case an equal or a greater amount. Moreover, for all \( q > 1/V \), \( \bar{W}(q) < \tilde{W}(q) \), hence \( \tilde{W}(q_2(\bar{\alpha})) > \bar{\alpha} \). Since \( f(q) \) has a compact support, there must exist some \( q \) in the right neighborhood of \( q_2(\bar{\alpha}) \) that buy type 2 information but did not buy type 1 information. Hence, offering only type 1 information cannot be optimal. Now, suppose that only type 2 information is offered. A similar argument would apply. QED
Proof of Lemma 3. By Lemma 2 both types of information must be offered. Suppose that \( q_1 \) buys type 2 information and that there exist some \( q > q_1 \) that buy type 1 information. Let \( q_1' \) be the smallest of such \( q \) and \( \{ \bar{\alpha}(q_1); \bar{\beta}(q_1) \} \) and \( \{ \bar{\alpha}(q_1'); \bar{\beta}(q_1') \} \) be the contracts offered respectively to \( q_1 \) and \( q_1' \). Then, the following inequalities must hold:

\[
\bar{W}(q_1') - (\bar{\alpha}(q_1') + \bar{\beta}(q_1')\bar{m}Vq_1') \geq \bar{W}(q_1') - (\bar{\alpha}(q_1) + \bar{\beta}(q_1)\bar{m}Vq_1')
\]

and

\[
\bar{W}(q_1) - (\bar{\alpha}(q_1) + \bar{\beta}(q_1)\bar{m}Vq_1) \geq \bar{W}(q_1) - (\bar{\alpha}(q_1') + \bar{\beta}(q_1')\bar{m}Vq_1').
\]

The first inequality guarantees that \( q_1' \) prefers to buy type 1 information instead of choosing the contract designed for \( q_1 \) and acquiring type 2 information. The second inequality makes sure that \( q_1 \) buys type 2 information. Putting together the two inequalities and using the property that \( \partial \bar{W}(q) / \partial q < 0 \), one can show that \( \bar{\beta}(q_1') \bar{m} < \bar{\beta}(q_1)\bar{m} \) and \( \bar{\alpha}(q_1') > \bar{\alpha}(q_1) \).

Notice that all \( q \in [q_1, q_1'] \) buy type 2 information but have higher willingness to pay for type 1 information. Now consider the following contract to be offered to any \( q \in [q_1, q_1'] \) in exchange of type 1 information: \( \{ \bar{\alpha}''; \bar{\beta}'' \} \) where \( \bar{\alpha}'' = \bar{\alpha}(q_1) \) and

\[
\bar{\beta}'' = \bar{\beta}(q_1') + \frac{\bar{\alpha}(q_1') - \bar{\alpha}''}{\bar{m}Vq_1'}.
\]

Notice that for \( q = q_1' \), \( \bar{\alpha}'' + \bar{\beta}''\bar{m}Vq = \bar{\alpha}(q_1') + \bar{\beta}(q_1')\bar{m}Vq \) and that \( \bar{\beta}'' > \bar{\beta}(q_1') \) since \( \bar{\alpha}(q_1') > \bar{\alpha}'' \). This, in turn, implies that for \( q > q_1' \), \( \bar{\alpha}'' + \bar{\beta}''\bar{m}Vq > \bar{\alpha}(q_1') + \bar{\beta}(q_1')\bar{m}Vq \). Hence, the optimal contract for all \( q > q_1' \) is not affected. However, some \( q \) in the \([q_1, q_1']\) interval would jump to type 1 information paying a higher price. Indeed, incentive compatibility implies that

\[
\bar{W}(q_1') - (\bar{\alpha}'' + \bar{\beta}''\bar{m}Vq_1') \geq \bar{W}(q_1') - (\bar{\alpha}(q_1) + \bar{\beta}(q_1)\bar{m}Vq_1').
\]

Notice that

\[
\frac{\partial}{\partial q} [\bar{W}(q) - \bar{W}(q)] < 0 \quad \text{and} \quad \frac{\partial}{\partial q} [\bar{\beta''\bar{m}} - (\bar{\beta}(q_1)\bar{m})]vq > 0.
\]

Therefore,

\[21\] Technically speaking, such a \( q \) exists only if we deal with a closed set. Since the distribution of \( q \) has a compact support and any contract is linear in \( q \), this condition must be satisfied. Indeed, such a \( q \) corresponds to the buyer who is indifferent between type 1 and type 2 information.
$\overline{W}(q_i) - (\alpha' + \beta'\overline{m}Vq_i) > \tilde{W}(q_i) - (\tilde{\alpha}(q_i) + \tilde{\beta}(q_i)\tilde{m}Vq_i)$. Hence, we have a contradiction. A similar argument could be put forward for $q_2$. QED

Proof of Lemma 4. By contradiction, suppose this is not true. Then, there must exist at least a $q'$ that does not buy (either because no contract is offered to it or because the cost is greater than the value of information). Let $q' < \frac{I}{V}$ and $\{\tilde{\alpha}(q_i); \tilde{\beta}(q_i)\}$ be the contract offered to $q_i$ that, given Lemma 3, is going to buy type 1 information. Since $\alpha(q_i), \beta(q_i) \in \mathbb{R}^+$ then $\overline{W}(q') - (\alpha(q_i) + \beta(q_i)\overline{m}Vq') > 0$ since $\overline{W}(q_i) - (\alpha(q_i) + \beta(q_i)\overline{m}Vq_i) \geq 0$ and $\frac{\partial \overline{W}(q_i)}{\partial q} > \beta(q_i)\overline{m}V$ for any $q < I/V$. So, $q'$ buys the information, the seller extracts positive profits from it, and no other contract in the optimal menu of contracts needs to be changed. If $q' \geq \frac{I}{V}$ a similar argument would apply. QED

Proof of Lemma 5. Existence of at least one indifferent buyer comes from Lemma 3 and Lemma 4. To prove uniqueness, first notice that there can only be an odd number of indifferent buyers since in the right neighborhood of $q_1$ (the lowest value of $q$ buying information) type 1 information is bought and in the left neighborhood of $q_2$ (the highest value of $q$ buying information) type 2 information is bought. Assume that the seller has designed an optimal menu of contracts for both types of information, let $\overline{R}(q) = \alpha(q) + \beta(q)\overline{m}q$ and $\tilde{R}(q) = \tilde{\alpha}(q) + \tilde{\beta}(q)\tilde{m}q$ be the total payments by agent $q$ buying respectively type 1 and type 2 information. As we proved in section 3 these payments must be increasing and concave in $q$.

Let $q < \frac{I}{V}$. We shall prove that there cannot be two indifferent buyers in this interval. A similar argument would apply for $q \geq \frac{I}{V}$ and it is omitted. By contradiction, let $\tilde{q}_1$ and $\tilde{q}_2$, with $\tilde{q}_1 < \tilde{q}_2$, be the two indifferent buyers. Since in the right neighborhood of $q_1$
type 1 information is bought, then, all \( q \in (\tilde{q}_1, \tilde{q}_2) \) must buy type 2 information and, hence, the following inequality must be true: \( \tilde{W}(q) - \tilde{R}(q) \geq \tilde{W}(q) - \tilde{R}(q) \). Let \( \Delta(q) = \tilde{W}(q) - \tilde{W}(q) \). Now, we prove that this menu of contracts cannot be optimal for the seller. Consider the following menu of contracts for type 1 information (the menu of contracts for type 2 information remains unchanged): \( \tilde{R}'(q) = \tilde{R}(q) \) if either \( q \leq \tilde{q}_1 \) or \( q \geq \tilde{q}_2 \) and \( \tilde{R}'(q) = \tilde{R}(q) + \Delta(q) \) if \( q \in (\tilde{q}_1, \tilde{q}_2) \). Notice that all \( q \in (\tilde{q}_1, \tilde{q}_2) \) weakly prefer to buy type 1 information. However, they pay now a higher price: \( \tilde{R}'(q) > \tilde{R}(q) \). It remains to show that the new menu of contracts is incentive compatible. It is easy to check that \( \tilde{R}'(q) \) is concave and increasing in \( q \). Given the linearity of contracts, this menu can be implemented through contracts with increasing \( \alpha \) and decreasing \( \beta \). Since we have found an alternative menu of contracts that, being incentive compatible, yields higher profits, we have reached a contradiction. QED
TABLES AND FIGURES

Table 1: Modeling diagnostic information

<table>
<thead>
<tr>
<th></th>
<th>Good project (G)</th>
<th>Bad project (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive signal (Y)</td>
<td>Pr(Y</td>
<td>G)</td>
</tr>
<tr>
<td>Negative signal (N)</td>
<td>Pr(N</td>
<td>G)</td>
</tr>
</tbody>
</table>

Table 2: Comparison of different pricing schemes (m=n=1)

<table>
<thead>
<tr>
<th></th>
<th>Optimal pricing</th>
<th>Demand</th>
<th>Seller’s profits</th>
<th>Total welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Price</td>
<td>$\alpha = \frac{I(V - I)}{2V}$</td>
<td>$q_1 = \frac{I}{2V}$, $q_2 = \frac{1}{2} + \frac{I}{2V}$</td>
<td>$\frac{(V - I)I}{4V}$</td>
<td>$\frac{3(V - I)I}{8V}$</td>
</tr>
<tr>
<td>Royalty</td>
<td>$\beta = \frac{I}{V}$ if $V \geq 2I$</td>
<td>$q_1 = 0, q_2 = \frac{1}{2}$</td>
<td>$\frac{I}{8}$</td>
<td>$\frac{(3V - 4I)I}{8V}$</td>
</tr>
<tr>
<td></td>
<td>$\beta = \frac{V - I}{V}$ if $V &lt; 2I$</td>
<td>$q_1 = 0, q_2 = \frac{I}{V}$</td>
<td>$\frac{(V - I)I^2}{2V^2}$</td>
<td>$\frac{(V - I)I^2}{2V^2}$</td>
</tr>
<tr>
<td>Two-part tariff</td>
<td>Pure royalty if $V &lt; 2I$.</td>
<td>Pure fixed fee if $V &gt; 2I$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Menu of contracts</td>
<td>${\alpha_i = 0, \beta_i = \frac{V - I}{V}}$</td>
<td>$q_1 = 0, \tilde{q} = \frac{I}{2V - I}$</td>
<td>$\frac{(V - I)I}{2(2V - I)}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>${\alpha_i = \frac{(V - I)I}{2V - I}, \beta_2 = 0}$</td>
<td>$q_2 = \frac{V}{2V - I}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Welfare is the sum of seller’s profits and buyers’ surplus.
Table 3: Competition with differentiated information: Payoff matrix and indifferent buyer

<table>
<thead>
<tr>
<th>Firm 1 (( \alpha_1 ))</th>
<th>Firm 2</th>
<th>Royalty (( \beta_2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed price</td>
<td>( {\Pi^1(\alpha_1, \alpha_2); \Pi^2(\alpha_1, \alpha_2)} )</td>
<td>( {\Pi^1(\alpha_1, \beta_2); \Pi^2(\alpha_1, \beta_2)} )</td>
</tr>
<tr>
<td>Fixed price</td>
<td>( \tilde{q} = \frac{1 + \alpha_2 - \alpha_1}{2} )</td>
<td>( \tilde{q} = \frac{1 - \alpha_4}{2(1 - 2\beta_2)} )</td>
</tr>
<tr>
<td>Royalty (( \beta_1 ))</td>
<td>( {\Pi^1(\beta_1, \alpha_2); \Pi^2(\beta_1, \alpha_2)} )</td>
<td>( {\Pi^1(\beta_1, \beta_2); \Pi^2(\beta_1, \beta_2)} )</td>
</tr>
<tr>
<td>Royalty (( \beta_1 ))</td>
<td>( \tilde{q} = \frac{1 + \alpha_2}{2(1 + \beta_1)} )</td>
<td>( \tilde{q} = \frac{1}{2(1 + \beta_1 - 2\beta_2)} )</td>
</tr>
</tbody>
</table>

Table 4: Nash equilibrium in the space of pricing schemes

<table>
<thead>
<tr>
<th>Firm 1 (( \alpha_1 ))</th>
<th>Firm 2</th>
<th>Royalty (( \beta_2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed price</td>
<td>( \alpha_1 = \frac{1}{5}; \tilde{q} = \frac{1}{2}; q_1 = \frac{1}{5}; q_2 = \frac{4}{5} )</td>
<td>( \alpha_1 = 0.185; \beta_2 = 0.076; \tilde{q} = 0.48; )</td>
</tr>
<tr>
<td>Fixed price</td>
<td>( \Pi^1(\beta_1, \alpha_2) = \Pi^2(\beta_1, \alpha_2) = \frac{3}{50} = 0.06 )</td>
<td>( q_1 = 0.185; q_2 = 0.767 )</td>
</tr>
<tr>
<td>Royalty (( \beta_1 ))</td>
<td>( \beta_1 = \frac{1}{2}; \alpha_2 = \frac{1}{4}; \tilde{q} = \frac{5}{12}; q_1 = 0; q_2 = \frac{3}{4} )</td>
<td>( \Pi^1(\alpha_1, \beta_2) = 0.0547; )</td>
</tr>
<tr>
<td>Royalty (( \beta_1 ))</td>
<td>( \Pi^2(\alpha_1, \beta_2) = 0.0543 )</td>
<td>( \Pi^2(\beta_1, \beta_2) = 0.0543 )</td>
</tr>
<tr>
<td>Royalty (( \beta_1 ))</td>
<td></td>
<td>( \beta_1 = \frac{1}{2}; \beta_2 = 0.106; \tilde{q} = 0.388; )</td>
</tr>
<tr>
<td>Royalty (( \beta_1 ))</td>
<td></td>
<td>( q_1 = 0; q_2 = 0.703 )</td>
</tr>
<tr>
<td>Royalty (( \beta_1 ))</td>
<td></td>
<td>( \Pi^1(\beta_1, \beta_2) = 0.0753; )</td>
</tr>
<tr>
<td>Royalty (( \beta_1 ))</td>
<td></td>
<td>( \Pi^2(\beta_1, \beta_2) = 0.0726 )</td>
</tr>
</tbody>
</table>
Figure 1: Non-monotonicity in the value of information

\[ \text{slope} = mV - (m+n-1)I \]

\[ \text{slope} = -(1-m)V - (m+n-1)I \]

Figure 2: Fixed price contract

\[ \text{W(q)} \]

\[ \text{q1} \]

\[ \text{q1(\(\alpha\))} \]

\[ \text{q2(\(\alpha\))} \]
Figure 3: Royalty contract

\[ I \frac{(V-I)(m+n-1)}{V} \]

\[ W(q) \]

Slope = $\beta mqV$

\[ q_1(\beta) \]

\[ q_2(\beta) \]

Figure 4: Two-part tariff

\[ I \frac{(V-I)(m+n-1)}{V} \]

\[ W(q) \]

Slope = $\beta mqV$

\[ q_1(\alpha, \beta) \]

\[ q_2(\alpha, \beta) \]
Figure 5: Graphical illustration of Proposition 2

Figure 6: The optimal menu of contracts
Figure 7: Competition with homogenous information

Figure 8: Differentiated information
Figure 9: Graphical illustration of Lemma 2