

# **Effect of Information Revelation Policies Under Market Structure Uncertainty**

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## **Abstract**

Geographically dispersed sellers in electronic reverse-marketplaces such as those hosted by Freemarkets are uncertain about the number of other sellers in any given market session. Over the course of several market sessions, they learn about the competitive structure of their market. How sellers learn to reduce the level of market structure uncertainty, is dependent on the market transparency scheme (revelation policy) adopted. The revelation policies differ in terms of the level of competitive information revealed. Thus, they determine what sellers learn, how they bid in future, and, in general how the consumer surplus generated changes. In this paper, using game-theory, we compare a set of revelation policies commonly used in electronic reverse marketplaces on consumer surplus. Based on our analysis, we find that the policy that generates the least amount of market structure uncertainty for the sellers should be chosen to maximize consumer surplus. This contradicts the traditional view that under uncertainty sellers are worse off and the buyer better off. This paper provides insights into this apparently anomalous outcome.

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# 1 Introduction

FreeMarkets<sup>1</sup>, a successful B2B market-maker, creates electronic reverse marketplaces at the request of buyers. Among other attributes, buyers determine the “market transparency” scheme (or *information revelation policy*) to be used in the market. A revelation policy dictates the information about bids, winning bids, number of bidders etc. that are revealed to the geographically dispersed sellers in a market, at the beginning, in the middle, and at the end of a market session. At one end of the spectrum of available policies, the buyer can choose to accept sealed bids and notify its decision to each seller individually. Under this policy, no other competitive information is revealed to sellers. At the other end, the buyer can choose a revelation policy that allows the sellers to observe the bids submitted by their opponents in real-time, and react. Under this policy, all competitive information is revealed. Over the course of multiple sessions, the revelation policy adopted affects what sellers learn, how they bid in future, and the overall performance of the market. To our knowledge the impact of these revelation policies, assuming market structure uncertainty, has not been studied in any prior work leaving little guidance for the buyer in choosing the appropriate revelation policy. We focus on this issue and compare, using a consumer surplus metric, two of the many revelation policies available in Freemarkets:

1. Complete-Market Structure Information Setting (CIS): all quotes are revealed to all participants, implying that all sellers learn about the market structure.
2. Incomplete-Market Structure Information Setting (IIS): the only information revealed to the participants at the end of each market session is the winner’s quote. This means that the winning seller, which is already aware of its bid, does not learn anything about the market

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<sup>1</sup><http://www.freemarkets.com>

structure; however, losing bidders, if they exist, learn about the presence of at least one competitor, and can bid accordingly in future market sessions.

We chose these specific policies because they are commonly adopted in both traditional marketplaces and electronic marketplaces Thomas (1996).

While our work is motivated by a real-world electronic marketplace hosted by Freemarkets, results generated in this paper are applicable to any reverse-market setting. Note, however, comparing revelation policies is relevant only in the context of a web-based marketplace because of the unique nature of the web in controlling transparency to competition. For example, because of the computational power available, sellers can be informed about their rank relative to other competitors allowing them to respond real-time.

The rest of the paper is organized as follows. Section 2 describes the problem context. In Section 3, we review literature relevant to this topic. The problem context presented in Section 2 is modeled as a continuous price two-period game in Section 4 where we compare the performances of frameworks with and without market-structure uncertainty, and with and without cost-structure uncertainty. Finally, in Section 5, we conclude.

## **2 Problem Context**

Our game-theoretic model is a simplified abstract of the real-world scenario. Let us think about the problem context from the perspective of a coal reverse-marketplace (our assumptions are stated as bullet points). There are a certain number of sellers who can offer coal. Of these, only a subset of the sellers bid in auctions convened at the request of buyers. Typically, in each auction conducted in Freemarkets, maximum of three to four coal sellers participate. Exogenous factors such as the

distance between the coal mine and the supplier base, the type of the coal the buyer wants, limit seller participation. In our set-up:

- For sake of exposition, there can only be a maximum of two<sup>2</sup> sellers providing the product. In other words, we assume that the market can only be monopolistic or duopolistic in nature. The exogenous factors are assumed to influence the participation. It is assumed that with a probability of  $a$ , each seller bids <sup>3</sup>.

Anecdotal evidence from Freemarkets suggests that if a particular seller is chosen for a market session, then it is highly likely that it gets chosen in future market sessions also. In other words, there is a high correlation that the same seller is selected across market sessions.

- For the sake of tractability of the analytical model, we extend the anecdotal result and set the participation correlation to one in our set-up. Stated differently, market structure from the first auction, is assumed to continue in future auctions also.

If a seller participates, it is unaware of its nature of competition but can learn about the market structure depending on the information revelation policy adopted by the buyer. Each buyer is at the liberty of choosing its desired revelation policy unaware of the events in the previous market session.

- In our framework, there are exactly two buyers each initiating a period in our game-theoretic model. We are interested in studying the impact of the revelation policy chosen by the first

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<sup>2</sup>Refer to Greenwald *et al.* (2002) for n-seller-case. All results shown in this paper except one has been proved for the n-seller case. The result which could not be proved for the n-seller case will be marked so.

<sup>3</sup>Assuming asymmetrical values for the probability does not allow us to compare IIS and CIS schemes in the analytical realm.

buyer. The second buyer is assumed to be unaware of the events of the previous period when it arrives.

The reverse-auction executed is as follows:

- The auction conducted is a first price sealed bid auction. This is usually assumed in the reverse market context (e.g., Thomas (1996), Snir & Hitt (2002)).
- The price, representing the multiple-attributes of the bid, is assumed to be quoted simultaneously. Each seller's bid is based on the following:
  - Its belief about the market structure: In our set-up, sellers facing an uncertain number of competitors, hold symmetric first period belief about the market structure. Conditional on being present in the market, each seller's first period belief of being a monopolist is  $(1 - a)$  and let that be common knowledge.
  - Consumer Utility for the product: The consumer utility for the product is  $U \in \mathbb{R}_+$  and is assumed to be known to both sellers.
  - Cost of producing the product: We assume that the cost incurred in producing the product is a constant  $C$  and is also known to both sellers.
  - Revelation Policy in the marketplace: It is either IIS or CIS. Sellers are assumed to be aware of the revelation policy adopted.

Summarizing this: based on  $a$ ,  $U$ ,  $C$  and the policy adopted, sellers simultaneously bid prices.

After receiving all bids, the buyer chooses the seller that offers the best bid (in our set-up, it is the lowest price) as the winner (ties are broken randomly) and awards the contract. The winner

builds the product but incurs a production cost (in our set-up, it is  $C$ ). The product built is delivered to the buyer, who in turn, remunerates the winner. This point corresponds to the end of one period. At this point, bids submitted in that period are revealed allowing sellers to learn about the market-structure for the next period. This cycle repeats itself for the second period also.

Using this setting, we study the behavior of the sellers in CIS and in IIS. We begin by providing some intuition for our results. Analysis related to CIS is straightforward. Recall that in CIS, sellers are aware of their market structure information. In this case, the seller chooses an optimal strategy in the second period depending on whether it is a monopolist or in duopoly. Since this information is available independent of the outcome (win or lose) of the first period, the behavior of the sellers in the first period is equivalent to that of the single period game.

In contrast, uncertainty persists in the second period of IIS for the first-period-winner. To avoid this uncertainty, sellers are willing to pay in the first period to acquire information for the second period. They pay so, by bidding a higher price in the first period so that, by risking the loss in the first period game, they learn about their market structure. This leads to a lower overall expected consumer surplus in IIS than in CIS. Further, this behavior i.e., to explore with higher price bids under market structure uncertainty, is different from the case when sellers are uncertain about costs. Under cost structure uncertainty, sellers bid a price lower than their expected costs to learn about their cost-structure. Since the buyer is concerned about the lowest price observed, these two types of uncertainties differ in their impact on consumer surplus.

### 3 Literature Review

Given the context, let us review the literature relevant to it. Finance literature refers to Revelation Policies as “Trade-Transparencies”. There are a number of papers addressing the impact of pre-trade transparencies (Madhavan *et al.* (1999), Anand & Weaver (2001), Boehmer *et al.* (2002)) and post-trade transparencies (Bloomfield & O’Hara (1999), Flood *et al.* (1999)) on the efficiency of financial exchanges such New York Stock Exchange (NYSE) and Toronto Stock Exchange (TSE). However, the financial markets and electronic reverse-markets differ structurally. Typically, financial exchanges are double-sided auctions whereas the electronic reverse-markets are single sided auctions. The information revealed in the double-sided auction not only affects the sellers but also the buyers. Because of this characteristic, the problem is different from the single-sided auction we are interested in studying. The only financial market which operates as a single-sided auction is the primary bond market, say for US Treasury Bills. Even in these bond markets, the standard policy employed is to reveal the winner’s bid and the quantity.

To our knowledge, revelation policies in single-sided auctions have been studied only in Thomas (1996) and Koppius & van Heck (2002). Thomas (1996) compares the revelation policies in a setting where two sellers are certain about the presence of their opponent but are uncertain about each other’s cost type. Based on this, he demonstrates that the setting equivalent to our CIS generates higher consumer surplus than IIS. This setting is different from ours: in our framework, sellers are *not* certain about the presence of their opponent. Koppius & van Heck (2002) employ an experimental framework to compare different revelation policies on bidders’ profits. They show that the setting with the least uncertainty for the bidders generates the highest profit for the bidders. As mentioned earlier, our paper is different from these two papers in analytically comparing the effect

of revelation policies on a consumer surplus metric in a setting where bidders are uncertain about the nature of their competition.

Apart from these two, there are other papers which address “learning issues in single-sided auctions”. Prior research work in this domain can be categorized into two. The first category focuses on the bidders’ decision problem of whether or not to invest to learn the value of the auctioned item and use that information to participate in the auction (Schweizer & T.V. (1983), Milgrom (1981) and Guzman & Kolstad (1997)). The second dimension focuses on the bidding problem in a multi-period auction (game) as bidders learn about their opponents’ type across periods. Dekel *et al.* (2002), Snir *et al.* (1998) and Thomas (1996) study the equilibrium strategies in this context. Our work is on a third dimension addressing learning issues under market structure uncertainty (whether it is a monopoly or a duopoly or how many sellers are in the market). The only other work in this dimension would be that of Jansen & Rasmusen (2002) where they study the behavior of the firms in a single-period context. Our work extends Jansen & Rasmusen (2002) to a two-period game (reverse-auction) and studies the bidding behavior of the seller under different information revelation policies.

## 4 Game-Theoretic Model

Having differentiated our work from the existing literature, we focus on the problem context and compare the revelation policies analytically. Without loss of generality, we normalize the prices and the costs by the utility  $U$ . Then, the range of the price bid  $p$  is  $[0, 1]$ . Similarly the cost incurred is normalized by the utility and is represented as  $c = C/U$ . Using this set-up, we study market structure uncertainty in Subsection 4.1 and cost-uncertainty in Subsection 4.2.

## 4.1 Market Structure Uncertainty

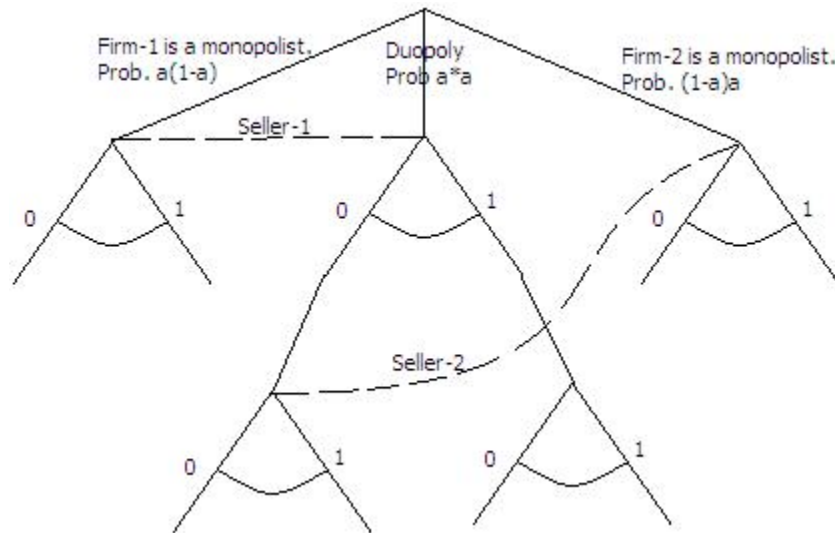


Figure 1: Market Structure Uncertainty - Extensive form of the Single Period Game. Nature makes the first move (root node), after which the sellers move simultaneously (non terminal nodes other than the root). The dotted lines denote each seller's information set, indicating that the seller is to move at each node cannot distinguish between that node and the one to which it is connected.

The extensive form for the single period game under market structure uncertainty is shown in figure 1. The branches in the figure correspond to: (1) both sellers are selected - happens with probability  $a^2$ . (2) seller-1 is selected but not seller-2, probability  $a(1 - a)$ . (3) seller-2 is selected but not seller-1, probability  $(1 - a)a$ . With uncertainty, sellers are uncertain about which branch they are on. When uncertainty is resolved, they know the branch they are on.

This single period game is extended to a two-period game in the context of CIS. The extensive form of CIS is shown in figure 2. We begin with the second period game where uncertainty is resolved. When uncertainty is resolved, sellers know which branch they are on. If a seller realizes its monopolistic position, it extracts the consumer surplus by charging  $p = 1$ . Instead, if the market is duopolistic, both sellers charge  $p = c$ .

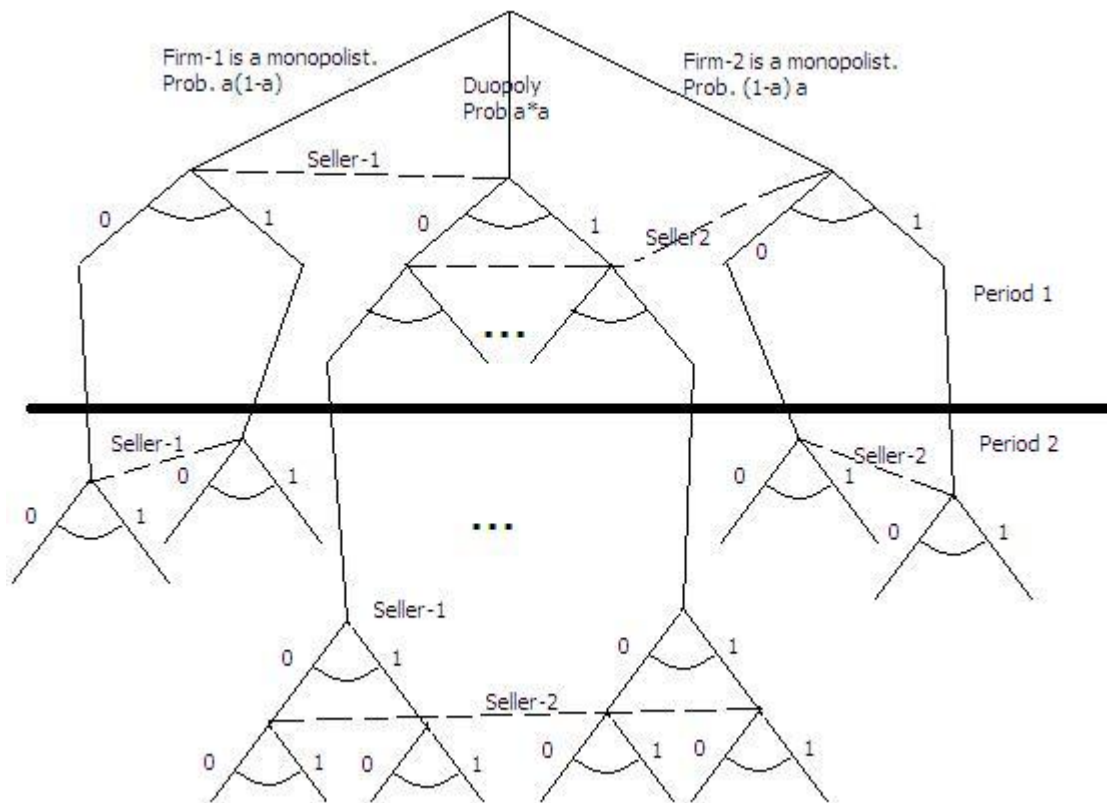


Figure 2: CIS - Extensive form of the Two Period Game. Sellers are uncertain about the market structure in the first period. In the second period, sellers are certain about the market structure.

Corresponding to this second period game, one can also determine the profit for each seller: If the seller is a monopolist, its profit is  $(1 - c)$ . If the market is duopolistic, the profit is 0. Thus, unlike in the discrete price model, duopoly profits are zero. Combining these two, the second period expected profit for any seller  $i$  is

$$\hat{\Pi}_{i,CM,2} = (1 - a)(1 - c) \quad (1)$$

This second period expected profit is independent of the first period outcome (win or lose). Also, this second period expected profit affects equilibrium of the first period game. We can characterize the equilibrium for the first period game base on the following lemma (see Appendix Appendix A for the proof):

**Lemma 4.1** *When a commitment is made to reveal market structure information at the end of the first period, the first period equilibrium is equivalent to that of the single period game.*

Intuitively, in CIS, sellers are provided information about market structure independent of whether they win or lose. That is, they do not have to pay to learn and therefore, the first period equilibrium is equivalent to that of the single period game. Since no pure strategy Bayesian-Nash equilibrium exists, the subgame-perfect Bayesian-Nash mixed strategy equilibrium for seller  $i$  is the cumulative bid distribution (see Appendix A for the proof)

$$F_{i,1}(p) = 1 - \frac{(1 - p)(1 - a)}{(p - c)a} \quad (2)$$

$F_{i,1}(p)$  is defined for  $p \in [(1 - a) + ca, 1]$ . The bid distribution is plotted for different values of  $a$  in figure 3. Notice that it matches with our intuition that when  $a$  is low, sellers bid only high prices. Based on this, one can compute the consumer surplus generated if the the probability

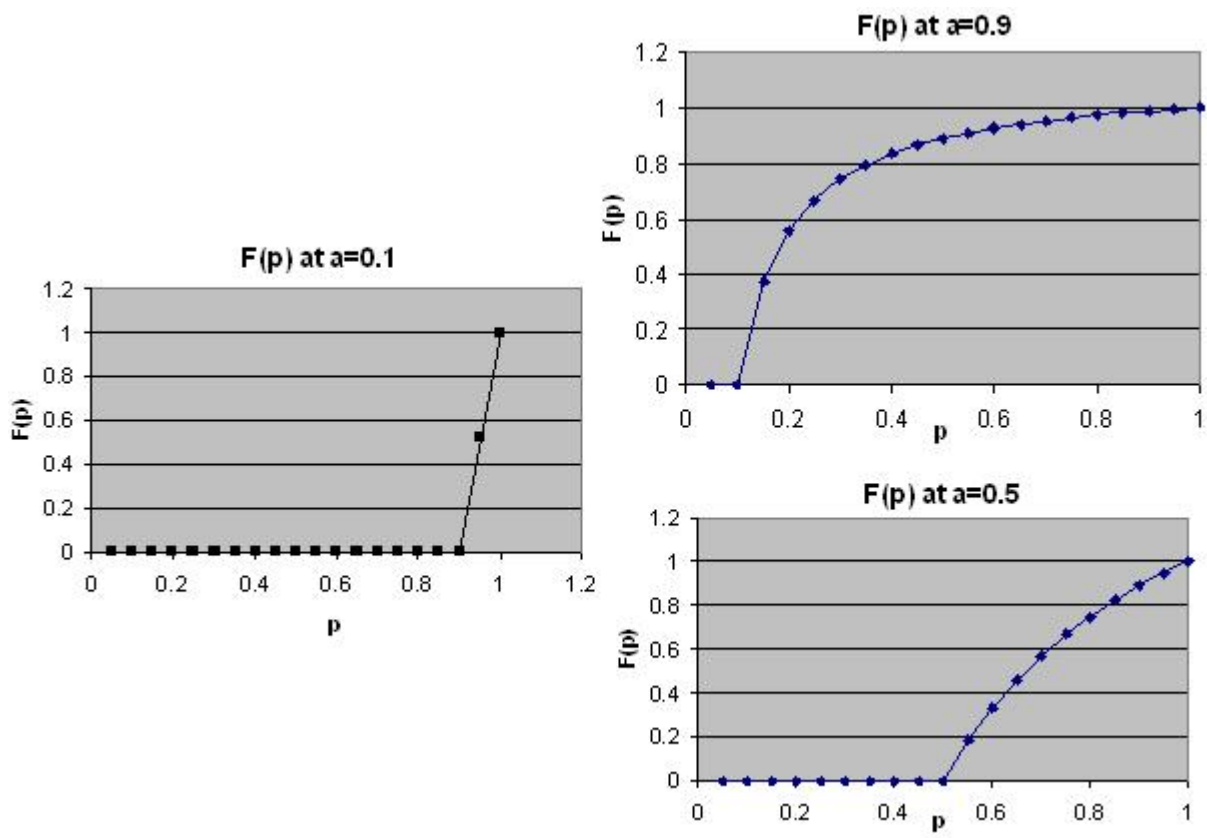


Figure 3:  $F_{i,1}(p)$  is shown for different values of  $a$ .

density function (pdf) of the bid distribution is known. We present the pdf function but defer the consumer surplus comparison to the latter part of this section. The pdf corresponding to equation 2 is

$$f_{i,1}(p) = \frac{(1-c)(1-a)}{(p-c)^2 a} \quad (3)$$

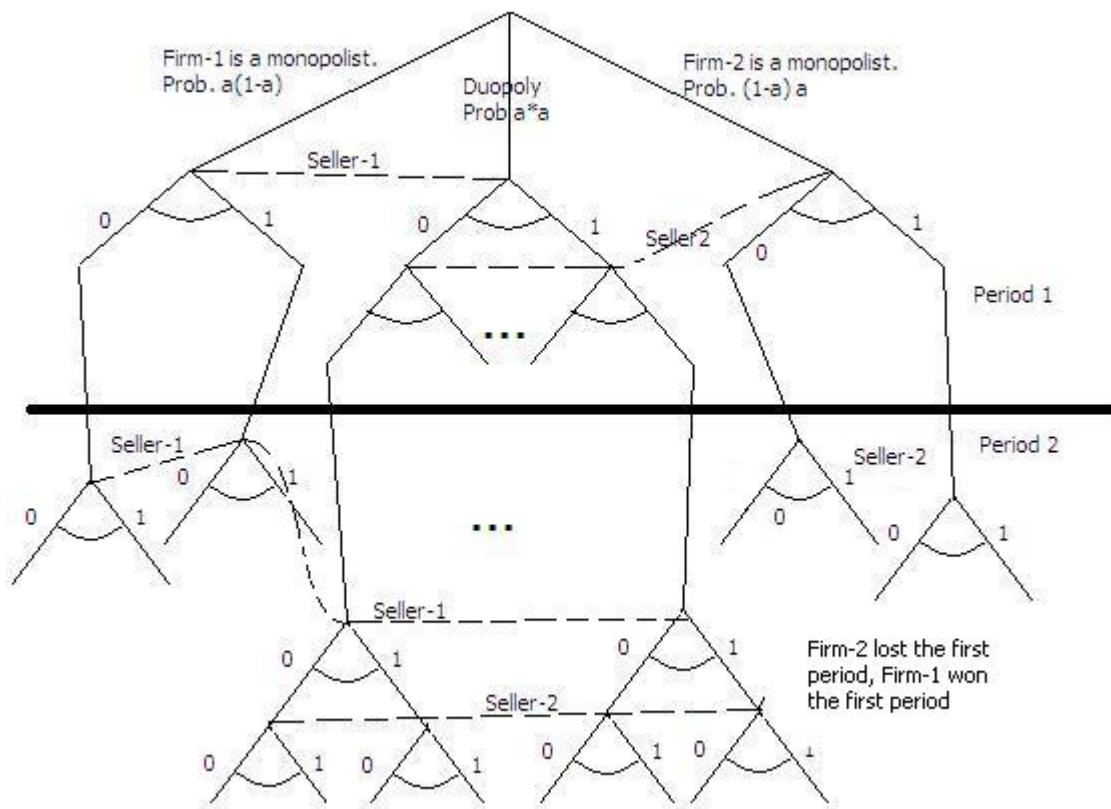


Figure 4: IIS - Extensive form of the Two Period Game. Sellers are uncertain about the market structure in the first period. In the second period, one seller - the winner is uncertain about the market structure while the other seller is unaware of the market structure.

After defining the equilibrium of CIS, we focus on IIS. The extensive form game for IIS is shown in figure 4. In the first period of IIS, both sellers are unaware of their market structures. But, the second period is an asymmetric game. Since the first period winner's bid is revealed at the end

of the first period, the loser from the first period, if there is one, learns that a competitor exists. In contrast, the winner is not aware of the market structure. However, the winner holds a belief  $x'$  that is also known to the loser. In such a case, only a Bayesian Nash mixed-strategy equilibrium exists for the winner,  $w$  and the loser  $l$  (See Appendix B for the proof). At equilibrium, the cumulative bid distribution for the loser is:

$$F_l(p) = 1 - \frac{(1-p)x'}{(p-c)(1-x')} \quad (4)$$

$F_l(p)$  is valid for  $p \in [x' + c(1-x'), 1]$ .

Corresponding to this distribution, the pdf is

$$f_l(p) = \frac{(1-c)x'}{(p-c)^2(1-x')} \quad (5)$$

Similarly, the cumulative bid distribution for the winner is:

$$F_w(p) = 1 - \frac{x'(1-c)}{(p-c)} \quad (6)$$

$F_w(p)$  is valid for  $p \in [x' + c(1-x'), 1]$ . At  $p = 1$ , a mass point with probability  $M_w = x'$  exists.

The pdf corresponding to this distribution is

$$f_w(p) = \frac{x'(1-c)}{(p-c)^2} \quad (7)$$

Figure 5 shows the distributions of the winner and the loser for  $x' = 0.5$ . Note that the winner's bid distribution is first degree stochastic dominant over the loser's bid distribution. Further, based on these equilibrium strategies, we can state that (see Appendix B for proof):

**Lemma 4.2** *The second period expected profits for the winner and the loser are equal to  $x'(1-c)$ .*

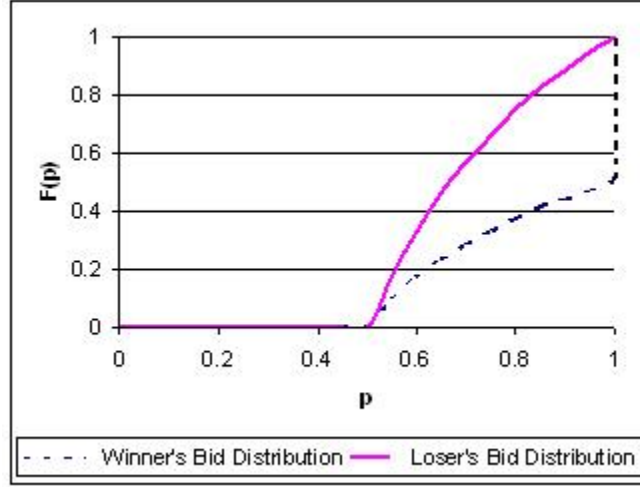


Figure 5:  $F_w(p)$  and  $F_l(p)$  is shown for  $x' = 0.5$ .

Although, the expected profits are the same for both sellers, the difference is in terms of whose  $x'$ , it is. Recall that  $x'$  is the belief held by the winner. Using a bayesian update, this belief can be represented as a function of the price bid,  $p$ , first period belief held, and the first period mixed strategy equilibrium  $F_{i,1}(p)$ :

$$x'(p) = \frac{(1 - a)}{(1 - a) + a(1 - F_{i,1}(p))} \quad (8)$$

Based on this definition, the first period equilibrium can be characterized as follows (proved in Appendix B):

**Lemma 4.3** *In IIS, the first period mixed strategy equilibrium,  $F_{i,1}(p)$  is a solution to a non-linear equation and is defined only for  $p \in [c + (1 - a)(1 - \log(1 - a))(1 - c), 1]$ .*

Even if one is not able to derive the expression for  $F_{i,1}(p)$ , one can compute it numerically for a given value of  $a$  (Figure 6 shows the bid distribution for the first period equilibrium computed numerically). But, this also implies that for comparing the performances of IIS and CIS, we cannot

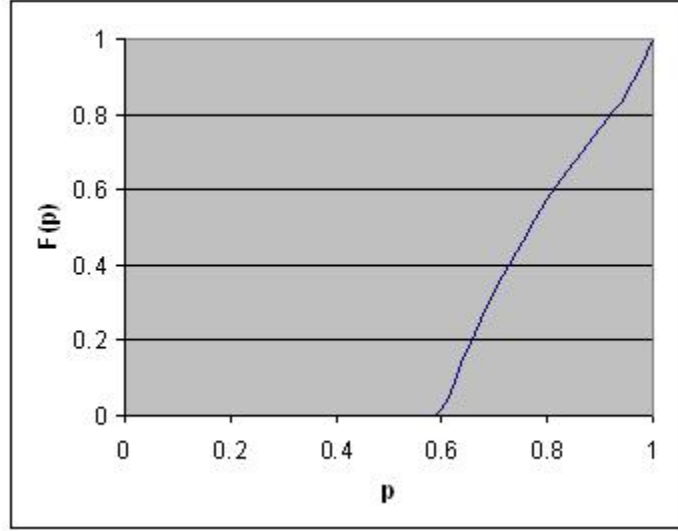


Figure 6: Bid Distribution for first period IIS - numerically computed for  $a = 0.75$ .

rely on the expression of the first period equilibrium. In fact, meaningful comparisons are possible using just the lower bounds of the first period IIS strategy sets.

Recall that our metric for comparison is consumer surplus and it is defined as follows. Since all prices are normalized by  $U$ , the consumer utility generated is also normalized to 1 for each period. Therefore, the total consumer surplus across both periods is given by  $CS = 2 - P_{paid}$  where  $P_{paid}$  is the total expected price paid across both periods.

For CIS, this  $P_{paid} = \hat{P}_{CM,1} + \hat{P}_{CM,2}$  i.e., the sum of the expected prices observed by the buyer in the first and second periods respectively:

$$\begin{aligned} \hat{P}_{CM,1} = & \frac{1}{a(2-a)} \left\{ a^2 \int_{((1-a)+c a)}^1 [2(1 - F_{i,1}(p)) f_{i,1}(p)] p dp \right. \\ & \left. + 2(1-a)a \int_{((1-a)+c a)}^1 f_{i,1}(p) p dp \right\} \end{aligned} \quad (9)$$

The numerator of this expression is explained as follows. The first term: with probability of  $a^2$ ,

both sellers are in the marketplace. In that case, the term  $2(1 - F_{i,1}(p))f_{i,1}(p)$  is the bid distribution of the minimum price observed by the buyer, when both sellers in the market use the cumulative bid distribution  $F_{i,1}(p)$ . The second term corresponds to the condition when each of the two sellers is a monopolist - which happens with probability,  $(1 - a)a$ . In such a case, the seller which is not aware of its market structure bids according to the same bid distribution  $F_{i,1}(p)$ . The factor 2 in the second term accounts for each seller being a monopolist. The denominator conditions the expected price on having at least one seller in the market.

Similarly, the expected minimum price observed in the second period of CIS is

$$\hat{P}_{CM,2} = \frac{1}{(2 - a)a} (2(1 - a)a + a^2 c) \quad (10)$$

A similar generic expression for the expected price cannot be computed for IIS because  $F_{i,1}(p)$ , the first period bid distribution for sellers in IIS, is a solution to a non-linear equation. However, we can compare the consumer surplus generated across both settings assuming the worst-case scenario for IIS:

1. Let the buyer observe a price of  $c + (1 - a)(2 - (1 - a))(1 - c)$ , a price lower than the support of the first period equilibrium bid distribution,  $c + (1 - a)(1 - \log(1 - a))(1 - c)$ .

In expected terms, conditioned on having at least one seller in the market

$$\hat{P}_{IM,1} = (c + (1 - a)(2 - (1 - a))(1 - c)) \quad (11)$$

2. After the end of the first period, we assume that the winner retains the same belief in the second period as in the first period i.e.,  $x' = (1 - a)$ . As before, this second period belief is assumed to be known to the loser.

Based on this, the expected minimum price observed by the buyer in the second period of IIS is

$$\begin{aligned} \hat{P}_{IM,2} = & \frac{1}{a(2-a)} \left\{ a^2 \int_{(1-a)+c}^1 [(1-F_l(p))f_w(p) + (1-F_w(p))f_l(p)] p dp \right. \\ & \left. + 2a(1-a) \left[ \int_{(1-a)+c}^1 f_w(p) p dp + M_w \right] \right\} \end{aligned} \quad (12)$$

The numerator of this expression is explained as follows. The first term: with probability  $a^2$ , both sellers are in the market.  $(1-F_l(p))f_w(p)$  refers to the probability that the winner,  $w$ , offers the lowest price in the second period. The term  $(1-F_w(p))f_l(p)$  is the probability that the loser,  $l$ , offers the lowest price in the second period. Using these, the expected lowest price observed by the buyer is calculated over all possible prices. The second term corresponds to the setting when the market is monopolistic and this happens with a probability of  $a(1-a)$ . In such a case, the winner who is not aware of the market structure continues to bid according to the distribution  $F_w(p)$ . We also account for the mass point at  $p=1$  by adding  $M_w$ .

Since the utility is assumed constant across both periods, the difference in the consumer surplus generated is the negative of the difference in the expected price:

$$D_M = \hat{P}_{IM,1} + \hat{P}_{IM,2} - [\hat{P}_{CM,1} + \hat{P}_{CM,2}] \quad (13)$$

This expression simplifies to

$$D_M = \frac{1}{a(2-a)} [a(1-a) (a(1-a(1-c)) - (1-a)(1-c) \log(1-a))] \quad (14)$$

Note that  $D_M > 0 \forall a \in [0, 1], \forall c \in [0, 1]$ . This implies that the expected consumer surplus generated in IIS is greater than that in CIS.

This is an interesting result which warrants further analysis. To investigate which period contributes to this result, we compare the consumer surplus generated period-by-period.

1. The First Period Comparison : Here again, we assume the worst-case scenario for IIS i.e., the first period expected price for IIS is  $\hat{P}_{IM,1} = \frac{1}{a(2-a)}\{c + (1-a)(2-(1-a))(1-c)\}$ .

This leads to a first period expected price difference

$$G_{IM,1} = \hat{P}_{IM,1} - \hat{P}_{CM,1} = \frac{1}{(2-a)a} [(1-a)^2 a^2(1-c)] \quad (15)$$

This is non-zero for any  $a \in [0, 1]$ ,  $c \in [0, 1]$  implying that the expected first period price of IIS is always higher than that of CIS.

2. The Second Period Comparison<sup>4</sup>: Similar to the earlier case, we assume the worst case scenario for IIS i.e., the second period updated belief  $x' = (1-a)$ . Based on that, we compute the expected second period price difference as

$$G_{IM,2} = \hat{P}_{IM,2} - \hat{P}_{CM,2} = \frac{1}{a(2-a)} [a(1-a)(ac - (1-a)(1-c)\log(1-a))] \quad (16)$$

Note that, even in the second period, the expected price (consumer surplus) is higher (lower) in IIS when compared to CIS.

Summarizing the results: in each period, the expected consumer surplus in IIS is lower than that in CIS. This implies that the first buyer's choice of the information revelation policy impacts not only itself but also the second buyer. Now let us analyze the seller-side.

The interesting behavior to notice is that sellers, when they face market structure uncertainty, have an incentive to bid higher prices in the first period in order to learn. Is this true across all

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<sup>4</sup>We could not compare the second period consumer surplus for the n-seller case.

types of uncertainty or is it unique to market structure uncertainty? To investigate this, we study the behavior of the sellers under cost structure uncertainty in the following section.

## 4.2 Cost Structure Uncertainty

We retain the same problem context from the previous section. However, sellers in this model are aware of their market structures but are uncertain about their costs. Sellers can be one of these two types: a low cost type or a high-cost type. We also assume that costs incurred  $c_L$  and  $c_H$  by the low-cost type and the high cost type respectively, are the costs normalized by the utility  $U$ . If  $b$  is the probability with which a seller is a low-cost type, then the expected cost is  $\hat{c} = bc_L + (1 - b)c_H = (1 - b)c_H$ . Similar to the earlier section, sellers can bid a price  $p$  in the range  $[0, 1]$ . Using this framework, we compare the consumer surplus generated in the following policies that are similar to the ones under market structure uncertainty:

1. Complete Cost Structure Information Setting (CCS): In this setting, at the end of the first period, both sellers become aware of their cost structure and that of their opponent. The intuition for this policy is as follows. If a seller wins the contract to build a certain product (say for example, a skyscraper) and if the resources available to both sellers is universally known, sellers can determine the cost incurred for both sellers based on the cost incurred by one.
2. Incomplete Cost Structure Information Setting (ICS): In this setting, at the end of the first period, the winner is aware of its cost structure while the loser is aware neither of its cost-structure nor that of the winner.

Results in this section can be summarized as follows. In the first period, both sellers are unaware of their cost structures but, in the second period, depending what they learn, their bidding behavior is different. Without uncertainty in the second period, both sellers bid the optimal price in the first period. With uncertainty, when a seller believes that with a high probability it is a high cost type, it is willing to incur a loss in the first period to learn about its cost for the second period. This leads to lower price and higher consumer surplus in ICS than in CCS.

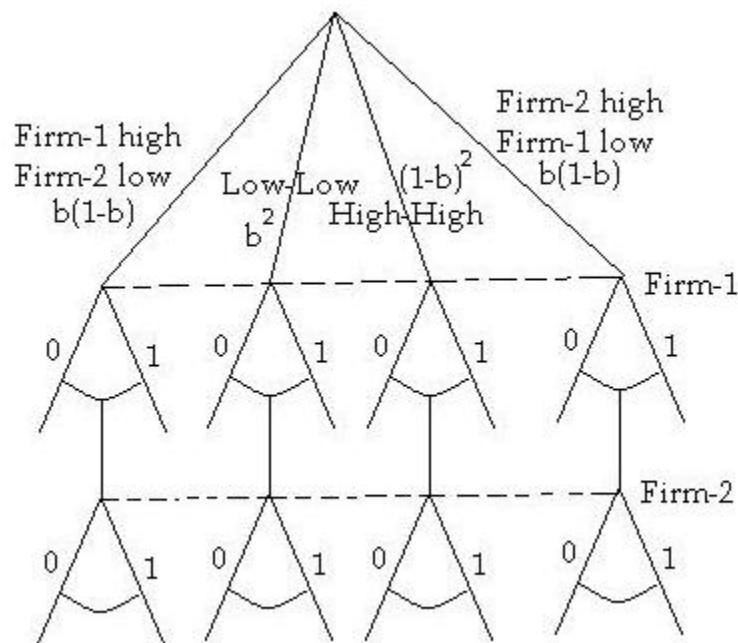


Figure 7: Cost Structure Uncertainty - Extensive Form of the Single Period Game

Let us begin our analysis by describing the extensive form of the single period game under cost uncertainty shown in figure 7. In this case, nature determines the cost-types for the sellers. Without any knowledge about their cost-types, sellers bid simultaneously. This single period game is extended to CCS and analyzed. In the second period of CCS, uncertainty is resolved and there are three possible scenarios:

1. Both sellers are low cost type. In this case, the equilibrium strategy for both sellers is to bid  $c_L$ . This case occurs with a probability of  $b^2$ .
2. Both sellers are high cost type which happens with a probability of  $(1 - b)^2$ . In this case, the equilibrium strategy for both sellers is to bid  $c_H$ .
3. One seller is a high-cost type and the other is a low cost type. This happens with a probability of  $2b(1 - b)$ . In this case, the equilibrium strategy for the low-cost seller is to bid  $c_H - \delta$  whereas for the uncertain seller, the equilibrium is to bid  $c_H$ .

Since cost information is available to both sellers in the second period, the bidding behavior of the sellers in the first period of CCS is similar to that of the single period game i.e., the equilibrium is for both sellers to bid  $\hat{c}$ .

Having defined the equilibrium strategy in CCS, we proceed to ICS. In the second period, one seller - the loser-seller  $l$  which lost the first period game - is unaware of both its cost structure and that of its opponent while the winner,  $w$ , is aware of its cost structure (whenever there is a need to differentiate between types of the winner, we use  $w_l$  to represent the low-cost type and  $w_h$  to represent the high cost type).

The equilibrium results are summarized below (see Appendix C for the proof). When the winner realizes his type as high cost, the equilibrium for  $w_h$  is to bid  $c_H$ . But, if the winner realizes his type as low cost,  $w_l$ , it plays a mixed strategy according the following cdf

$$F_w(p) = 1 - \frac{(1 - b)(c_H - p)}{(p - c)} \quad (17)$$

$F_w(p)$  is defined only for  $p$  in the range  $[\hat{c}(1 + b), c_H]$ . Corresponding to this, the pdf is

$$f_w(p) = \frac{(1-b)c_H}{(p-c)^2} \quad (18)$$

Similarly, the cdf for the mixed strategy equilibrium for the loser from the first period,  $l$ , is given by

$$F_l(p) = 1 - \frac{\hat{c}(1+b)}{p} \quad (19)$$

Note that  $F_l(p)$  is defined only for  $p$  in  $[\hat{c}(1+b), c_H]$  and it has a mass point at  $p = c_H$  with probability  $M_l = 1 - a^2$ . Corresponding to  $F_l(p)$ , the pdf is

$$f_l(p) = \frac{\hat{c}(1+b)}{p^2} \quad (20)$$

The mixed strategy equilibrium can be explained intuitively. The only type of uncertainty that exists in this framework is the cost-type and it persists only for  $l$ . In contrast, the winner from the first period,  $w$ , is aware of its cost-type. Suppose,  $l$  knew that  $w$  is a low-cost type, then the equilibrium is for  $l$  to bid  $\hat{c}$  and, for  $w_l$ , to bid  $\hat{c} - \delta$ . Similarly, if  $l$  knew that  $w$  is a high-cost type, then the equilibrium is for  $l$  to bid  $c_H - \delta$  and for  $w_h$  to bid  $c_H$ . But, since  $l$  is not aware of its opponent type, it tries to mix between the strategies  $c_H$  and  $\hat{c}$ . Although this fixes the strategy for  $w_h$ , this provides room for  $w_l$  to secure higher profits by mixing strategies according to the cdf  $F_w(p)$ .

Based on these expressions for the equilibrium, we compute the difference in the expected second period profits of the winner and the loser as

$$G_{IC,2} = (1-b)c_H b^2 \quad (21)$$

Since for  $G_{IC,2} > 0$  any  $b \in [0, 1]$ , winning the first period game is more profitable than losing. Therefore, sellers in the first period of ICS bid an equilibrium price of  $\hat{c} - G_{IC2}$ . Note that this means that sellers are willing to bid a price lower than their expected cost whereas in CCS sellers never bid less than their expected cost.

Now, let us step back and contrast the first period bidding behavior of sellers in a framework with and without market structure uncertainty against a framework with and without cost structure uncertainty. Under market structure uncertainty sellers bid higher with uncertainty than without it. In contrast, under cost structure uncertainty, sellers bid lower with uncertainty than without it. The key difference is how sellers are willing to pay to overcome uncertainty. Under market structure uncertainty, sellers pay by bidding a higher price and thereby, risking their loss in the period. But, under cost structure uncertainty, sellers pay by bidding a price lower than their expected costs. This explains the differing impacts of uncertainty on consumer surplus.

## 5 Conclusion

In conclusion, we have addressed an important real-world problem i.e., the impact of market structure uncertainty on consumer surplus, which has not been studied in the literature. Specifically, we compare two of the many policies that are possible in e-marketplaces. Using the properties of the equilibrium, we observe results that appear counter-intuitive to our traditional view: the setting with market structure uncertainty generates lower consumer surplus than that without it. Insights gained, explain why, in order to maximize consumer surplus, it may be best to choose a policy that generates the least level of market structure uncertainty for sellers.

Further, our analysis also emphasizes the focus on the “nature of uncertainty” when comparing

performances of settings with and without uncertainty. From our baseline analysis on market structure uncertainty, we contrast the behavior of the sellers under cost-structure uncertainty. With cost structure uncertainty, sellers bid a lower price in order to “learn” about their costs. This leads to higher consumer surplus in the first period when compared to the setting without it. But, with market structure uncertainty, it is the opposite. Sellers “learn” only by bidding a high price and thereby decreasing the consumer surplus generated in the first period than in the setting without it.

This stylized model can be extended further to make it more realistic. Some of the suggestions include: a) In this paper, we set the correlation of selection probability for the sellers across the two periods to be one for tractability reasons. In reality, it is not so. We intend to investigate this by comparing the policies for any exogenously set correlation value. b) Also, for tractability reasons, we assume that sellers are symmetrical. We intend to compare the two settings relaxing this assumption. c) This paper compares only two of the many policies facilitated in marketplaces like Freemarkets. Another extension would be to model other revelation policies used in Freemarkets and study their impact relative to those studied in this paper.

## A Second period of CIS Continuous Price Model

**Proposition A.1** *There is no pure strategy Nash equilibrium*

**Proof:** Let  $\Pi_i(p_i, p_j)$  represent the profit for seller  $i$  when seller  $i$  bids  $p_i$  and its opponent seller  $j$  bids  $p_j$ . Let the pure strategy equilibrium be  $(p_i^*, p_j^*)$  for both sellers  $i$  and  $j$ . Because of the symmetric nature,  $p_i^* = p_j^*$ . If such a pure strategy equilibrium exists, then there cannot exist any price  $p_i$  such that  $\Pi_i(p_i, p_j^*) > \Pi_i(p_i^*, p_j^*)$ .

$$\Pi_i(p_i^*, p_j^*) = (1 - a)(p_i^* - c) + (1 - (1 - a)) \frac{(p_i^* - c)}{2} \quad (\text{A-1})$$

Let there be a  $p_i = p_i^* - \epsilon$ ,  $\epsilon > 0$ , then,

$$\Pi_i(p_i, p_j^*) = (1 - a)(p_i^* - c - \epsilon) + (1 - (1 - a))(p_i^* - c - \epsilon) \quad (\text{A-2})$$

From equation A-1 and equation A-2, we have  $\Pi_i(p_i, p_j^*) > \Pi_i(p_i^*, p_j^*)$  if  $\epsilon < \frac{(1-(1-a))(p_i^*-c)}{2}$ .

Since such an  $\epsilon$  exists, therefore no pure strategy equilibrium exists. **QED**

Having proved that a pure strategy Nash equilibrium does not exist, we focus on the mixed strategy equilibrium. The equilibrium for any seller,  $i$ , is defined as the pair  $(\mu_i^*, S_i^*)$  where  $\mu_i^*$  is the probability measure defined over the strategy set  $S_i^* \subseteq p$  such that, given  $(\mu_j^*, S_j^*)$  for the opponent,  $j$ ,  $(\mu_i^*, S_i^*)$  is the best response. To determine the equilibrium, consider the expected profit for seller  $i$ . With probability  $(1 - a)$ , when the seller is a monopolist, any bid,  $p$ , secures the seller a profit of  $(p - c)$ . With probability  $a$ , the seller has an opponent,  $j$  whose equilibrium bid distribution is  $F_j(p)$ . In such a case, seller  $i$  secures  $(p - c)$  only if it outbids  $j$  - the probability that  $i$  outbids  $j$  is  $(1 - F_j(p))$ . Therefore, the profit is given by

$$\Pi_i(p) = (1 - a)(p - c) + a(1 - F_j(p))(p - c) \quad (\text{A-3})$$

Since the expected profits are similar between sellers  $i$  and  $j$ , the equilibrium strategies are also symmetric. The nature of the equilibrium can be further characterized based on the propositions below

**Proposition A.2** *There are no gaps in the strategy sets  $S_i^*$  and  $S_j^*$ .*

**Proof:** On the contrary, let there be a gap between  $\tilde{p}$  and  $\tilde{\tilde{p}}$  such that  $\inf(S_i^*) < \tilde{p} < p' < \tilde{\tilde{p}} < \sup(S_i^*)$ . The  $p'$  continues to be the lowest price for any price above  $\tilde{\tilde{p}}$ . Similarly  $p'$  fails to be the lowest price for any price lesser than  $\tilde{p}$ . However, in the circumstances when the price bid by opponent is greater than  $\tilde{\tilde{p}}$ ,  $p'$  can generate more profits than  $\tilde{p}$ . Therefore, such a gap cannot exist

**QED**

**Proposition A.3**  $\sup(S_i^*) = \sup(S_j^*) = 1$

**Proof:** Because of the symmetric nature,  $\sup(S_i^*) = \sup(S_j^*)$  and let it be  $= \hat{p}$ .

Consider the case when  $\hat{p} > 1$ . It is not possible to generate any profit when the price bid  $p = \hat{p}$ , since the price is above buyer's reservation price. Therefore  $\hat{p} \geq 1$ .

Let  $\hat{p} < 1$ , the profit for seller  $i$  if it bids  $\hat{p}$  is  $(\hat{p} - c)(1 - a)$ . This is because when seller  $j$  is present,  $F_j(\hat{p}) = 1$  (From the earlier proposition A.2 and the expected profit is the profit that it secures if it is a monopolist only. This expected profit  $(\hat{p} - c)(1 - a) < (1 - c)(1 - a)$ , the profit from bidding the reservation price of the consumer. Therefore  $\hat{p} = 1$ . **QED**

Based on these propositions, one can compute the symmetric mixed strategy equilibrium for seller  $j$  as a distribution that generates the same profit for its opponent independent of the action

taken by the opponent. In our set-up, equation A-3 generates the same profit for all  $p$  including  $p = 1$ . But, at  $p = 1$ ,  $F_j(p = 1) = 1$  and therefore,  $\Pi_i(1) = (1 - a)(1 - c)$ . Since the expected profits are the same independent of the price, equation A-3 becomes

$$((1 - a) + a(1 - F_j(p)))(p - c) = \Pi_i(1) = (1 - a)(1 - c) \quad (\text{A-4})$$

Rearranging the terms, we have

$$F_j(p) = 1 - \frac{(1 - p)(1 - a)}{(p - c)a} \quad (\text{A-5})$$

**Proposition A.4**  $\inf(S_i^*) = \inf(S_j^*) = (1 - a) + ca$

**Proof:** We know  $F_j(p)$  is continuous (proposition A.2) and increasing ( $f_j(p) = \partial(F_j(p))/\partial p > 0$  for all  $p$  in the range  $[0, 1]$ ). Based on that, one can compute the  $\inf(S_j)$  by setting  $F_j(p) = 0$ . That yields  $p = (1 - a) + ca$ . **QED**

## B IIS Continuous Price Model

**Proposition B.1** *There is no pure strategy Nash equilibrium.*

**Proof:** Let  $\Pi_w(p_w, p_l)$  represent the profit for the winner  $w$  when the winner bids  $p_w$  and the loser bids  $p_l$ . If there exists a pure-strategy equilibrium, it can be in one of two ways i.e.,  $p_l^* = p_w^*$  or  $p_l^* > p_w^*$ . It can be easily shown that  $p_l^* \neq p_w^*$  in the same manner as the proof for proposition A.1. So, we focus on the case when  $p_l^* > p_w^*$ ,

In that case, the profit is

$$\Pi_w(p_w^*, p_l^*) = (1 - a)(p_w^* - c) + a(1 - F_l(p))(p_w^* - c) \quad (\text{B-1})$$

If we choose a  $p_w = p_w^* + \epsilon < p_l^*$ , then

$$\Pi_w(p_w, p_l^*) = (1 - a)(p_w^* - c + \epsilon) + a(1 - F_l(p))(p_w^* - c + \epsilon) \quad (\text{B-2})$$

From this, it is trivial to show that  $\Pi_w(p_w, p_l^*) > \Pi_w(p_w^*, p_l^*)$ . **QED**

Since there is no pure strategy equilibrium, we compute the mixed strategy as the pair  $(\mu_l^*, S_l^*)$ ,  $S_l \subseteq p$  for the loser from the first period  $l$ , and  $(\mu_w^*, S_w^*)$ ,  $S_w \subseteq p$  for the winner from the first period  $w$ .

To compute the equilibrium, we determine the second period expected profits for both  $l$  and  $w$ .

For  $w$ , the second period expected profit is

$$\Pi_w(p) = x'(p - c) + (1 - x')(1 - F_l(p))(p - c) \quad (\text{B-3})$$

where  $F_l(p)$  represents the bid distribution for  $l$ .

For the  $l$ , the expected profit is

$$\Pi_l(p) = (1 - F_w(p))(p - c) \quad (\text{B-4})$$

where  $F_w(p)$  represents the bid distribution of  $w$ . Note that neither  $x'$  nor  $(1 - a)$  appears in this profit function because the losing seller knows that it is in a duopolistic market.

In this set-up, the equilibrium solution can be further characterized based on the following propositions:

**Proposition B.2** *There are no holes in the strategy sets  $S_l^*$  and  $S_w^*$ .*

The proof for proposition B.2 is similar to that for proposition A.2.

**Proposition B.3** *Neither seller has mass points a) in the interior or b) at the lower boundary of the other's support or c) at the upper boundary of other's support if that boundary has a mass point for the other seller.*

**Proof:** Let us represent  $p' = \inf(S_w^*)$  and  $p'' = \sup(S_w^*)$ . Assume to the contrary that there exists a mass point equilibrium for the losing seller  $l$  at the price  $p' < p^* < p''$  with probability of  $\gamma$ .

Consider the profits for the winner when bidding  $p^* - \epsilon$  and  $p^* + \epsilon$ . These are

$$x'(p^* - \epsilon - c) + (1 - F_l(p^* - \epsilon))(1 - x')(p^* - \epsilon - c) \quad (\text{B-5})$$

and

$$x'(p^* + \epsilon - c) + (1 - F_l(p^* - \epsilon))(1 - x')(p^* + \epsilon - c) \quad (\text{B-6})$$

Subtracting equation B-5 from B-6 yields

$$\approx -2\epsilon + (1 - x')\epsilon\gamma + x'p^*\gamma \quad (\text{B-7})$$

Therefore, the winner will find it advantageous to move its equilibrium to some point  $p'' < p^*$ . This is contrary to our earlier assumption that such an equilibrium exists. Therefore, there cannot be a mass point in the interior of the other seller's support. If such a mass point exists at  $\inf(S_l^*)$ ,

the only equilibrium possible is a pure strategy equilibrium. We had proved that no pure strategy equilibrium exists and therefore, there can be no mass point at the lower boundary.

When  $p^* = p''$ ,  $w$  can do better by bidding a price  $p^* - \epsilon$ . This proof holds when the other seller has mass point at  $p''$ . The only condition when the proof is not applicable if the winner has a mass point at  $p''$ .

Using a similar proof, we can prove that all the above conditions are applicable for the losing seller. **QED**

**Proposition B.4** *If one seller has a mass point at  $p''$ , the other seller will charge  $p''$  with zero density in the equilibrium.*

**Proof:** Let seller  $w$  have a mass point at  $p''$  with a probability a weight of  $\gamma$ . The difference between the expected profits for the losing seller between setting  $p'' + \epsilon$  and  $p''$  is given by  $(1 - x')(p''/2 - \epsilon)$  which is strictly positive for a small  $\epsilon$ . Therefore, if  $w$  has a mass point at  $p''$ , the losing seller  $l$  is better off coming arbitrarily close to  $p''$ . Converse of this, can be proved similarly showing that if a mass point exists for the losing seller, the winner's bid distribution will come arbitrarily close to  $p''$ . **QED**

**Proposition B.5**  $\sup(S_w^*) = \sup(S_l^*) = 1$ .

**Proof:** First, we will prove that  $\sup(S_w^*) = \sup(S_l^*)$  and then we will prove that  $\sup(S_w^*) = 1$ .

Suppose  $\sup(S_w^*) < \sup(S_l^*)$ , then the low-cost winner can increase its profit by bidding a price closer to  $\sup(S_l^*)$ . Similarly, if,  $\sup(S_w^*) > \sup(S_l^*)$ , the losing seller can increase its price close to the  $\sup(S_w^*)$ . Thus the only possibility is  $\sup(S_w^*) = \sup(S_l^*)$ . Let us represent  $\sup(S_w^*) = \sup(S_l^*) = \hat{p}$ .

Let  $\hat{p} > 1$ . In such a case, it is not possible to generate any profit when the price bid  $p = \hat{p}$ , since the price is above buyer's reservation price. Therefore  $\hat{p} \leq 1$ .

Let  $\hat{p} < 1$ , the profit for the seller  $w$  if it bids  $\hat{p}$  is  $(\hat{p} - c)(1 - a)$ . This is because when the opponent is present,  $F_l(\hat{p}) = 1$  (because  $\sup(S_w^*) = \sup(S_l^*)$ ) and the expected profit is just the profit the winner secures if it is a monopolist. This expected profit  $(\hat{p} - c)(1 - a) < (1 - c)(1 - a)$ , the profit from bidding the reservation price of the consumer. Therefore  $\hat{p} = 1$ . **QED**

To determine the strategy for  $l$ , we equate equation B-3 to the expected profit for the winner at  $p = 1$  which is  $x'(1 - c)$ . Based on this,

$$x'(p - c) + (1 - x')(1 - F_l(p))(p - c) = x'(1 - c) \quad (\text{B-8})$$

Rearranging the terms, we have

$$F_l(p) = 1 - \frac{(1 - p)Xm'}{(p - c)(1 - Xm')} \quad (\text{B-9})$$

Corresponding to this distribution, the pdf is

$$f_l(p) = \frac{(1 - c)Xm'}{(p - c)^2(1 - Xm')} \quad (\text{B-10})$$

From this, we can state that

**Proposition B.6**  $\inf(S_w^*) = \inf(S_l^*) = x' + c(1 - x')$

**Proof:**  $\inf(S_l^*)$  can be calculated by equating  $F_l(p) = 0$ . This yields  $\inf(S_l^*) = x' + (1 - x')c$ .

Having proved that  $\inf(S_l^*) = x' + (1 - x')c$ , we focus on proving that  $\inf(S_l^*) = \inf(S_w^*)$ . Let us assume the contrary i.e.,  $\inf(S_l^*) \neq \inf(S_w^*)$ . Let  $\inf(S_w^*) < x' + (1 - x')c$ . In such a case, the winner from the first period can be better off with a bid  $p = x' + (1 - x')c$  since the loser will not bid any value below  $x' + (1 - x')c$ . This rules out  $\inf(S_w^*) < x' + (1 - x')c$ . Similarly,

if  $\inf(S_w^*) > x' + (1 - x')c$ , the loser will move its boundary point to  $\inf(S_w^*)$  and have a mass point at the boundary. Since we know that there can be no mass point at the lower boundary,  $\inf(S_w^*) = x' + (1 - x')c$ . **QED**

To determine the strategy for  $w$ , we use the result from proposition B.6. We compute the profit for the  $l$  at the boundary of the winner's distribution  $F_w(p)$  which is at  $p = x' + c(1 - x')$ . From the definition of the mixed strategy equilibrium, the expected profit at this point for the  $w$  is the same as that for all other prices also. Therefore,

$$(1 - F_w(p))(p - c) = x'(1 - c) \quad (\text{B-11})$$

$$F_w(p) = 1 - \frac{x'(1 - c)}{(p - c)} \quad (\text{B-12})$$

The pdf corresponding to this distribution is

$$f_w(p) = \frac{x'(1 - c)}{(p - c)^2} \quad (\text{B-13})$$

Based on these distributions, we can compute the expected profits.

## Proof for lemma 4.2

Note that since the winner  $w$  is not aware of the market structure, the second period bid distribution is the same  $F_w(p)$  and it is independent of whether the market is monopolistic or duopolistic. Further,  $F_w(p) \neq 1$  at  $p = 1$ . This implies that there is mass point at  $p = 1$  with probability  $M_w = x'$  to make the distribution  $F_w(p) = 1$  at  $p = 1$ . Based on this we can compute the expected profits for the winner and the loser in equilibrium.

$$\Pi_w = \int_{x'+c(1-x')}^1 \{a^2(1 - F_l(p))f_w(p) + 2a(1 - a)f_w(p)\} (p - c) dp + M_w(1 - c) \quad (\text{B-14})$$

Simplifying this, we have,

$$\Pi_w = x'(1 - c) \quad (\text{B-15})$$

For  $l$ , the expected profit is

$$\Pi_l = \int_{x'+c(1-x')}^1 (1 - F_w(p)) f_l(p)(p - c) dp \quad (\text{B-16})$$

Simplifying this, we have

$$\Pi_l = x'(1 - c) \quad (\text{B-17})$$

### Proof for Lemma 4.3

The expected profits equal  $x'(1 - c)$ , where  $x'$  is the second period beliefs held by the winner. Second period beliefs are dependent on first period beliefs and bids,  $p_i$  and  $p_j$ , and the equilibrium behavior of the sellers. If we assume sellers play a pure strategies, then the winning seller in the first period, say seller  $i$ , wins because either it is a monopolist, or it outbid its opponent  $j$ , which we assume occurs with probability  $\delta_{ij}$ ; note that  $\delta_{ij}$  depends on  $p_i$  and  $p_j$ . Thus, seller  $i$ 's second period belief  $x'_i$ , conditioned on winning in the first period, can be represented in a Bayesian manner as follows:

$$x'_i(a, p_i, p_j) = \frac{(1 - a)}{(1 - a) + a\delta_{ij}} \quad (\text{B-18})$$

where

$$\delta_{ij} = \begin{cases} 1 & \text{if } p_i < p_j \\ \frac{1}{2} & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

accounts for the possibility of ties.

In IIS, seller  $i$ 's expected profits  $\Pi_i$  as a function of seller  $i$ 's price  $p_i$  and seller  $j$ 's price  $p_j$  are calculated as follows:

$$\Pi_i(p_i, p_j) = (1-a)((p_i - c) + x'_i(1-c)) + a[(p_i - c + x'_i(1-c))\delta_{ij} + (0 + x'_j(1-c))(1 - \delta_{ij})] \quad (\text{B-19})$$

where  $\delta_{ij}$  is defined as before.

Equation B-19 can be understood as follows. Seller  $i$  believes itself to be a monopolist with probability  $(1 - a)$ . Thus, with probability  $(1 - a)$ , it earns profits of  $p_i - c$  in the first period and  $x'_i(1 - c)$  in the second period. Seller  $i$  believes the marketplace is a duopoly with probability  $a$ . Thus, with probability  $a$ , seller  $i$  is either the lower priced seller in the first period, in which case it earns profits of  $p_i - c$  in the first period and  $x'_i(1 - c)$  in the second period, or seller  $i$  is the higher priced seller in the first period, in which case it earns zero profits in the first period and  $x'_j(1 - c)$  in the second period. The  $\delta_{ij}$  terms in this Equation B-19 account for the possibility of ties.

**Proposition B.7** *There is no pure strategy equilibrium in the first period in IIS.*

**Proof:** Suppose not: i.e., suppose there exists pure strategy equilibrium  $(p_i^*, p_j^*)$ . The proof proceeds by establishing the existence of  $p_i$  s.t.  $\Pi_i(p_i, p_j^*) > \Pi_i(p_i^*, p_j^*)$ .

Note the following: if seller  $i$  wins in the first period by outbidding its opponent, then

$$x'_i(a, p_i, p_j) = \frac{(1-a)}{(1-a) + a} = (1-a) \quad (\text{B-20})$$

If seller  $i$  wins in the first period, but both sellers bid the same price, then

$$x'_i((1-a), p_i, p_j) = \frac{(1-a)}{(1-a) + \frac{1}{2}negProb} = \frac{2(1-a)}{1 + (1-a)} \quad (\text{B-21})$$

The symmetric case: If  $p_i^* = p_j^* = p^*$ , then

$$\begin{aligned} \Pi_i(p_i^*, p_j^*) &= (1-a) \left( p^* - c + \left( \frac{2(1-a)}{1 + (1-a)}(1-c) \right) \right) + (1 - (1-a)) \\ &\quad \left( \frac{1}{2} \left( p^* - c + \frac{2(1-a)}{1 + (1-a)}(1-c) \right) + \frac{1}{2} \left( 0 + \frac{2(1-a)}{1 + (1-a)}(1-c) \right) \right) \end{aligned} \quad (\text{B-22})$$

Now if  $p_i = p^* - \epsilon$ , for some  $\epsilon > 0$ , then

$$\Pi_i(p_i, p_j^*) = p^* - \epsilon - c + (1 - a)(1 - c) \quad (\text{B-23})$$

From equations B-22 and B-23,  $\Pi_i(p_i, p_j^*) > \Pi_i(p_i^*, p_j^*)$  whenever  $\epsilon < a[(p^* - c)/2] - (1 - a)(1 - c)(1 - (1 - a))/(1 + (1 - a))$ . Such an  $\epsilon$  exists, whenever  $p^* > c + 2(1 - a)(1 - c)/(1 + (1 - a))$ . But note that  $p_i = p_j = p$  is not a pure strategy equilibrium for  $p \in [0, c + 2(1 - a)(1 - c)/(1 + (1 - a))]$ , since

$$\Pi_i(p_i, p_j) \leq \left( (1 - a) + \frac{2(1 - a)}{1 + (1 - a)} \right) (1 - c) \quad (\text{B-24})$$

but  $\Pi_i(1, p_j) = [2(1 - a) + (1 - a)(1 - (1 - a))](1 - c) > \Pi_i(p_i, p_j)$  whenever  $0 < (1 - a) < 1$ .

The asymmetric case: Without loss of generality, assume  $p_i^* < p_j^*$ . Choose  $p_i = p_i^* + \epsilon < p_j^*$ , for some  $\epsilon > 0$ . Now

$$\Pi_i(p_i^*, p_j^*) = p_i^* - c + (1 - a)(1 - c) \quad (\text{B-25})$$

and

$$\Pi_i(p_i, p_j^*) = p_i - c + (1 - a)(1 - c) \quad (\text{B-26})$$

Since  $p_i > p_i^*$ , it follows that  $\Pi_i(p_i, p_j^*) > \Pi_i(p_i^*, p_j^*)$ . **QED**

Having argued that no pure strategy equilibrium exists, we now study the mixed strategy equilibrium of the first period of IIS: i.e.,  $F_{i,1}$  and  $F_{j,1}(p)$ . Rewriting equations B-18 and B-19 in terms of mixed strategies yields:

$$x'_i(a, p_i) = \frac{(1 - a)}{(1 - a) + a(1 - F_{j,1}(p_i))} \quad (\text{B-27})$$

and

$$\begin{aligned}\Pi_i(p_i) &= (1-a)(p_i - c + x'_i(p_i)(1-c)) + \\ &a[(1 - F_{j,1}(p_i))(p_i - c + x'_i(p_i)(1-c)) + F_{j,1}(p_i)(0 + x'_j(p_j)(1-c))] \quad (\text{B-28})\end{aligned}$$

Note that although at the end of the first period, seller  $i$  can observe  $p_j$  and compute  $x'_j(p_j)$ , ex ante, this information is not available. Therefore we rewrite equation B-28 as

$$\begin{aligned}\Pi_i(p_i) &= (1-a)(p_i - c + x'_i(p_i)(1-c)) + \\ &a\{(1 - F_{j,1}(p_i))(p_i - c + x'_i(p_i)(1-c)) + \\ &F_{j,1}(p_i)(0 + \hat{x}'_j(p_i)(1-c))\} \quad (\text{B-29})\end{aligned}$$

where  $\hat{x}'_j$  denotes the expected value of  $x'_j(a, p_j)$ . Now if we restrict our attention to the symmetric mixed strategy equilibrium (i.e., let  $F_{i,1}(p) \equiv F_{j,1}(p)$ ), and if  $\inf(S_j^*) = \underline{p}_j$ , then  $\hat{x}'_j$  when seller  $i$  bids  $p_i$  can be computed as follows:

$$\begin{aligned}\hat{x}'_j(p_i) &= \int_{\underline{p}_j}^{p_i} \frac{(1-a)}{(1-a) + a(1 - F_{i,1}(p))} f_{i,1}(p) dp \\ &= \frac{(1-a)}{a} [-\log((1-a) + a(1 - F_{i,1}(p)))] \quad (\text{B-30})\end{aligned}$$

Having defined the profits, we can characterize the equilibrium using the following propositions:

**Proposition B.8**  $\sup(S_i^*) = \sup(S_j^*) = 1$ .

**Proof:** Suppose not: i.e., suppose  $\sup(S_i^*) = \bar{p}_i < 1$  then,  $F_{j,1}(\bar{p}_i) = 1$ . Thus,  $\Pi_i(\bar{p}_i) = (1-a)(\bar{p}_i - c + (1-a)_i(\bar{p}_i)(1-c)) + a\hat{x}'_j(1-c) < 2(1-a)(1-c) + a(1-c)\hat{x}'_j = \Pi_i(1)$ , since  $\bar{p}_i + (1-a)_i(\bar{p}_i) < 2$ . Therefore,  $\bar{p}_i = 1$ . The argument is analogous for seller  $j$ . **QED**

**Proposition B.9**  $\inf(S_i^*) = \inf(S_j^*) \geq c + (1 - a^2)(1 - c)$ .

**Proof:** At the upper boundary,  $\Pi_i(1) = 2(1 - a)(1 - c) + (1 - c)a\hat{x}'_j(1)$ , since winning by bidding at the upper boundary reveals that the marketplace is monopolistic. At the lower boundary,  $\Pi_i(\underline{p}_i) = \underline{p}_i - c + (1 - a)(1 - c)$ , since winning at the lower boundary reveals no information. Setting these two expressions equal to one another and plugging in Equation B-30 yields  $\underline{p} = c + (1 - a)(1 - c) + a(1 - c)\hat{x}'_j(1) = c + ((1 - a) + (1 - a)[- \log(1 - a)])(1 - c)$ . Since  $- \log(1 - a) \geq a$ , it follows that  $\underline{p}_i \geq c + ((1 - a) + (1 - a)a)(1 - c) = c + (1 - a^2)(1 - c)$ . **QED**

## C ICS Continuous Price Model

**Proposition C.1** *When the winner from the first period realizes its type to be high-cost, its pure strategy equilibrium is to bid  $c_H$ . However, no pure strategy Nash equilibrium exists for the loser or for the winner if it realizes its type to be low-cost.*

**Proof:** Let us assume that the losing seller plays a pure strategy equilibrium  $P_l^* > c_H$ . Then, the equilibrium for the winner is to bid  $c_H$  independent of its cost type and this generates zero profits for the losing seller. For this reason, any  $P_l^* > c_H$  is ruled out for the loser.

If  $P_l^* < c_H$ , then the response for the high-cost type first period winner  $w_h$  is to bid  $c_H$ .

We extend this further to prove that the pure strategy equilibrium does not exist for both the losing seller  $l$  and the low-cost type winner  $w_l$ . Let  $P_l^* < c_H$ , be such that  $P_l^* = P_w^*$ , i.e., the low-cost type winner and the losing seller play the same strategy. Then, the expected profit for  $l$  is

$$\Pi_l(P_l^*) = a \frac{(P_l^* - c)}{2} + (1 - a)(P_l^* - c)$$

We can demonstrate that a  $P_l$  exists such that  $\Pi_l(P_l) > \Pi_l(P_l^*)$ . Let  $P_l = P_l^* - \epsilon$ ,  $\epsilon > 0$ .

$$\Pi_l(P_l) = a(P_l^* - c - \epsilon) + (1 - a)(P_l^* - c - \epsilon)$$

This shows that for  $0 < \epsilon < a(P_l^* - c)/2$ ,  $\Pi_l(P_l) > \Pi_l(P_l^*)$ , thus proving that an equilibrium  $P_l^* = P_w^*$  does not exist.

Consider the case  $P_l^* < P_w^*$ . Now, the expected profit for  $l$  is

$$\Pi_l(P_l^*) = a(P_l^* - c) + (1 - a)(P_l^* - c)$$

Then for  $P_l = P_l^* + \epsilon < P_w^*$

$$\Pi_l(P_l) = a(P_l^* - c + \epsilon) + (1 - a)(P_l^* - c + \epsilon) > \Pi_l(P_l^*)$$

This means that there always exists an  $\epsilon$  that can generate higher profits.

This proves that no pure strategy equilibrium exists for the losing seller. A similar proof exists for stating that no pure strategy equilibrium exists for the high-cost seller. **QED**

Given that there is no pure strategy equilibrium for the low-cost type  $w_l$  and  $l$ , we focus on the mixed strategy equilibrium. Let the strategy pair for  $l$  be  $(\mu_l^*, S_l^*)$  and the strategy pair for the low cost type winner  $w_l$  be  $(\mu_w^*, S_w^*)$ . In this setting,

**Proposition C.2**  $\sup(S_w^*) = \sup(S_l^*) = c_H$

**Proof:** First, assume that a set  $S'_l$ , a subset of the strategy set  $S_l^*$  that represents bids greater than  $c_H$  exists. Our proof will demonstrate that  $S'_l$  has to be null for the loser to maximize profits.

For the sake of convenience, let  $S''_l = S_l^* - S'_l$ , represent the set of bids  $p < c_H$  and let  $\mu_l(p)$  represent the probability that a price  $p$  in the strategy set  $S_l^*$  is chosen. Then, the profit for the losing seller can be written in the following manner.

$$\Pi_{S_l} = \sum_{\forall p \in S''_l} \mu_l(p - \hat{c}) (p - \hat{c}) + 0$$

This is because, from proposition C.1, only a pure strategy equilibrium exists for the high cost winner. If the losing seller uses the same  $\mu_l(p)$  distribution for all prices below  $c_H$  and chose to bid  $c_H$  with probability,  $1 - \sum_{\forall p \in S''_l} \mu_l$ , then the expected profit for the losing seller is

$$\Pi_{noS_l} = \sum_{\forall p \in S''_l} \mu_l(p - \hat{c}) + (1 - \sum_{\forall p \in S''_l} \mu_l)(c_H - \hat{c})$$

It's easy to see that  $\Pi_{noS_l} > \Pi_{S_l}$

**QED**

To determine the mixed strategy equilibrium, we compute the second expected profits for  $l$  and  $w_l$ . For the  $l$ , any bid  $p < c_H$  will win if the first period winner is a high cost type - this happens with probability  $(1 - b)$ . With probability  $b$ , the winner can be a low cost type, in which case, it can win only if it outbids its opponent. If  $F_w(p)$ , represents the cumulative bid distribution of the low-cost type winner, then the losing seller can outbid its opponent with a probability of  $1 - F_w(p)$ . Combining the two terms, the expected profit for  $l$  is

$$\Pi_l(p) = (1 - F_w(p)) b (p - \hat{c}) + (1 - b) (p - \hat{c}) \quad (\text{C-1})$$

Similarly if  $F_l(p)$  represents the bid distribution of the loser, one can also compute the expected profit for  $w_l$  as

$$\Pi_w(p) = (1 - F_l(p))(p - c_L) \quad (\text{C-2})$$

Note that expected profit for a high cost type winner,  $w_h$  is 0.

The equilibrium strategies for this game can be further described using the propositions below:

**Proposition C.3** *There are no holes in the strategy set  $S_w$  and  $S_l$ .*

Proof for this proposition is similar to that of A.2

**Proposition C.4** *There are no mass points a) in the interior or b) at the lower boundary of other's support or c) at the upper boundary of other's support if that boundary is a mass point for the other seller.*

Proof for this proposition is similar to that of B.3

**Proposition C.5** *If one seller (the low cost winner or the loser) has a mass point at  $p''$ , then the other seller has zero density at that point in equilibrium.*

Proof for this proposition is similar to that of B.4

To calculate the distribution for the winner,  $w_l$  we equate equation C-1, to the expected profit for  $l$  at  $p = c_H$ . Therefore, we have

$$(1 - F_w(p)) b (p - c) + (1 - b) (p - c) = (1 - b) (c_H - \hat{c}) \quad (\text{C-3})$$

$$F_w(p) = 1 - \frac{(1 - b)(c_H - p)}{b(p - \hat{c})}$$

Corresponding to this distribution, the pdf is

$$f_w(p) = \frac{(1 - b)c_H}{(p - c)^2} \quad (\text{C-4})$$

Based on this, we can state that

**Proposition C.6**  $\inf(S_w) = \inf(S_l) = \hat{c} (1 + b)$ .

This proof is similar to the proof for B.6.

We exploit this proposition to compute the bid distribution for  $l$ . First, we compute the expected profit for  $w_l$  at  $p = \hat{c} (1 + b)$ . Based on the definition for the mixed strategy equilibrium, this profit  $\hat{c} (1 + b) - c_L$  is the profit that seller  $w_l$  can secure independent of the price it bids. Therefore, equation C-2 becomes

$$(1 - F_l(p))(p - c_L) = \hat{c} (1 + b) - c_L \quad (\text{C-5})$$

Substituting  $c_L = 0$  and rearranging the terms, we have

$$F_l(p) = 1 - \frac{\hat{c} (1 + b)}{p}$$

The pdf corresponding to that distribution is

$$f_l(p) = \frac{\hat{c} (1 + b)}{p^2} \quad (\text{C-6})$$

Note that this distribution has mass point at  $p = c_H$  with probability  $M_l = 1 - b^2$ .

Based on these distributions, we can compute the difference in the expected profits between the winner and the loser. For the loser:

$$\begin{aligned}
 \Pi_l &= b \left[ \int_{\hat{c}(1+b)}^{c_H} \{ (1 - F_w(p)) f_l(p) \} (p - \hat{c}) dp \right] + (1 - b) \left[ \int_{\hat{c}(1+b)}^{c_H} f_l(p) (p - \hat{c}) dp \right] \\
 &\quad + (1 - b)(c_H - \hat{c}) M_l \\
 &= b\hat{c}
 \end{aligned} \tag{C-7}$$

and for the winner:

$$\begin{aligned}
 \Pi_w &= b \left[ \int_{\hat{c}(1+b)}^{c_H} \{ ((1 - F_l(p)) + M_l) f_w(p) \} (p - c_L) dp \right] + (1 - b)(c_H - c_H) \\
 &= b(1 + b)\hat{c}
 \end{aligned} \tag{C-8}$$

and therefore the difference  $G_{IC2} = b^2\hat{c}$ .

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