Effect of Information Revelation Policies on Cost Structure Uncertainty

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Abstract

Geographically dispersed sellers in electronic reverse-marketplaces such as those hosted by Freemarkets are uncertain both about the number of opponents participating and their cost-structure in any given market session. Over the course of several market sessions, they learn about the competitive structure of their market. How sellers learn to reduce the level of uncertainty is dependent on the revelation policy adopted. The revelation policies differ in terms of the level of competitive information revealed. Thus, they determine what sellers learn, how they bid in future, and, in general how the consumer surplus generated changes. In this paper, using game-theory, we compare a set of revelation policies commonly used in electronic reverse marketplaces on consumer surplus. Specifically, we focus only on the effect of revelation policies when firms are uncertain about their opponent's cost. Based on our analysis, we provide intuition as to why under certain conditions one setting is better than the other.

1 Introduction

FreeMarkets[1], a successful Business-to-Business (B2B) market-maker, creates web-based reverse marketplaces involving geographically dispersed sellers. Sellers in this market face uncertainty both about the number of competitors (referred to as market structure uncertainty) and their opponents' cost structure. Their ability to overcome these uncertainties is dependent on the "market transparency" scheme, or the *information revelation policy*, choice made by the buyer. A revelation policy determines the nature of information about bids – winning bids, number of bidders etc. – that are revealed to geographically dispersed sellers in a web-based market, in the beginning, in the middle and at the end of a session. At one end of the spectrum of available policies, the buyer can choose to accept sealed bids and inform each seller only about whether it won or lost in a given market-session. Under this policy, competitive information is not revealed to sellers. At the other end, the buyer can choose a revelation policy that allows sellers to observe the bids submitted by their opponents in real-time and react. Under this policy, sellers are aware of the number of other sellers, their bidding patterns and winning bid prices. Over the course of multiple market-sessions, the revelation policy adopted affects what sellers learn, how they bid in the future and the overall performance of the market including buyer surplus.

To our knowledge, this problem – the impact of these revelation policies – has not been studied in any prior work. Thus little guidance is available for the buyer in choosing the appropriate policy. One of the reasons that this problem has not been studied thus far, could be that these diverse revelation policies were not feasible to implement without the Internet or information technologies. Comparing revelation policies is relevant only in the context of a web-based marketplace because of the unique nature of the web in controlling transparency to competition. For example, revelation

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[1] http://www.freemarkets.com

policies such as the one where sellers are informed about their rank relative to other competitors are possible only because of the computational power available. Given the current importance and the need for such an analysis, this paper compares the expected price paid by the buyer under different revelation policies.

In order to understand the effect of information revelation policies, one needs to employ a framework closer to reality i.e., one where sellers face uncertainties about both their market structure as well as their opponents' cost structure. As a first step, we break this rich problem into two relevant sub-problems. In Arora *et al.* (2002), we study the effect of revelation policies on the expected price paid by the buyer using a framework where sellers are uncertain just about their market structure. In our current paper, we study the effect of revelation policies on the expected price paid by the buyer using a framework where sellers are uncertain about their opponents' cost-structure. For this, we consider the following two revelation policies:

1. Complete Information Setting (CIS): At the end of a reverse-auction, all quotes are revealed to all participants.
2. Incomplete Information Setting (IIS): At the end of a reverse-auction, only the winner's bid is revealed to all participants.

These policies are among the many policies available to a buyer in reverse-markets hosted by Freemarkets. We chose to study these specific policies because they are commonly adopted in both traditional marketplaces and electronic marketplaces (Thomas (1996)). The information available to each seller under each policy is explained in detail while describing the problem context in Section 2.

While our work is motivated by a real-world electronic marketplace hosted by Freemarkets, analysis and results presented in our current paper are applicable to any reverse-market setting that can create these different information regimes. Our game-theoretic analysis provides the buyer with insights, which the buyer can use while choosing the optimal information revelation policy. It demonstrates how sellers respond to different information revelation policies, when and how they attempt to fake their cost structures, and how all these factors impact the expected price paid by the buyer. We find that under certain conditions, CIS is better than IIS and vice-versa. Further, we extend the results from our current paper to argue that studying the effect of revelation policies in a richer context with both cost structure uncertainty and market structure uncertainty is analytically intractable.

The paper is organized as follows. Section 2 describes the problem context. In Section 3, we review literature most relevant to this topic and position our current paper with respect to prior work. After that, the problem context described in Section 2 is modeled as a game-theoretic problem and solved in Section 4. Based on the equilibrium calculations, we compare the price paid by the buyer in CIS and IIS in Section 5. Finally in Section 6, we conclude.

2 Problem Context

We begin this section by describing a typical reverse-marketplace for coal. Buyers sequentially arrive at the electronic reverse-marketplace and initiate market-sessions. For each market-session convened at its request, the buyer is at the liberty of choosing its desired revelation policy. On the seller-side, there are a certain number of geographically distributed sellers who can offer coal. Only a subset of them bid in each market-session. For example, in each auction conducted by Freemarkets, a maximum of three or four coal sellers participate. Exogenous factors such as the type of coal the buyer wants or the distance between the coal mine and the buyer site limit seller participation. Participating sellers are unaware of their opponents' cost structure and learn about it across market-sessions. At the beginning of a market-session, each participating seller submits a multi-dimensional bid which includes coal content, ash content and water content of its coal, and the price quote. After reviewing all bids, the buyer chooses the best bid and awards the contract to the winner. At the end of the market-session, submitted bids are revealed according to the information revelation policy chosen by the buyer.

We portray this mechanism in our problem context in the following manner:

- There are exactly two buyers, each initiating one *period*. The impact of the first buyer's choice of revelation policy is studied in this paper. Note that we assume that the second buyer is unaware of the events of the previous period.

Let there be a certain number of sellers in the "world" and let the sellers be heterogeneous.

- Among the sellers in the "world", let there be two low-cost sellers and the rest, high-cost sellers. This is common knowledge. Further private information that each seller possesses is information about its own cost type.

- Let the cost of manufacturing/delivering the product for the low-cost type be 0 and that for the high-cost type be $1 + \delta$ (δ being an infinitely small number). This is assumed to be common knowledge.

- Among all the sellers in the "world", let it also be given that $M > 1$ sellers participate in the reverse-auction.

In reverse-auctions hosted by Freemarkets, M tends to vary with product type. For coal reverse-markets, $M = 3$ or 4, whereas for metal-castings-reverse-markets, $M = 20$[2]. Exogenous factors such as the distance between the supplier and the customer, the type of the product the buyer wants may limit seller participation. In our set-up:

- Each low-cost seller bids in a market-session with an exogenous[3,4] *participation probability* of r s.t. $0 < r < 1$. All sellers are assumed to be aware of the value of r, but they are unaware of the realized value of the participation probability for their opponent for that market session[5]. In this manner, we capture the uncertainty each seller faces.

- Within each period, participating sellers simultaneously submit a sealed single dimensional bid – price, $p \in \Re+$.

Notice that we make assumptions only about the participation probability of low-cost sellers. The participation probability of high-cost sellers affects neither the bidding behavior of the high-cost type nor that of the low-cost type. High-cost sellers in the reverse-auction will always bid the Bertrand price of $p = 1 + \delta$. But prices bid by low-cost sellers are only dependent on each low cost seller's belief about the presence of its low-cost opponent and not on the participation probability of high-cost sellers. At one end, if both low-cost sellers are present and they are aware of the presence of each other, the equilibrium price bid is $p = 0$. At the other end, if only one low-cost

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[2] Based on personal communication with Freemarkets Inc.

[3] Assuming asymmetrical values for the probability value leads to intractability. In the interest of staying in the analytical realm, we assume symmetrical participation probabilities.

[4] An exogenous participation probability value is assumed for simplicity. Typically, the market-maker sends invitations to sellers. The set of sellers invited to participate may vary depending on what the incentives are for the market-maker. Incentives for some market-makers may be aligned with consumer surplus (e.g. Freemarkets). Other market-makers may have an incentive to maximize social welfare (e.g. marketplaces such as Transora, ForestExpress where the marketplace is owned by consortia of both buyers and sellers). After receiving the invitation, a seller may choose to accept or reject the invitation. Its decision could be based on: production capacity constraints, expected profits from participation etc. The outcome of this series of complex processes is assumed to be captured by the variable r in our model.

[5] For example when $a = \frac{1}{2}$, sellers know that nature tosses a coin for each low-cost seller and allows that seller to participate only if the outcome is a head. But each low-cost seller does not know if the outcome was a head or a tail when nature tossed a coin to decide whether or not to permit its low-cost opponent to participate in that market session.

seller is present and it is aware of its cost-dominance over other sellers, then it bids $p = 1$. In a geographically dispersed setting, since each low-cost seller is uncertain about the presence of its low-cost opponent, prices bid are between 0 and 1. These prices are dependent on

- Its belief about the presence or absence of its low-cost competitor. This belief is r in the first period. It evolves across periods, depending on the knowledge gained at the end of each period. Note that the information revelation policy adopted determines the knowledge gained.

- Revelation Policy in the marketplace: It can either be IIS or CIS. Sellers are assumed to be aware of the revelation policy adopted.

After receiving all bids, the buyer chooses the seller that offers the lowest price as the winner (ties are broken randomly) and awards the contract. The winner builds the product but incurs a production cost. The built product is delivered to the buyer who, in turn, remunerates the winner. This point corresponds to the end of one period. At this point, bids submitted in that period are revealed according to the revelation policy adopted. Depending on the information available, sellers learn about their opponents. In IIS, only the winner's bid is revealed to all sellers. If a seller, which lost the reverse-auction, observes a price $p \leq 1$ bid by its opponent, then it learns about the presence of the low-cost winner. This is particularly significant when the loser is the other low-cost seller since it can use this information for its future bids. The low-cost winner, however, continues to be uncertain about the presence of its low-cost opponent. In CIS, all bids are revealed to everybody. This allows all sellers to be aware of the number of low-cost sellers which bid $p \leq 1$ in the first period before the second period begins. After the first period, the second period is executed similarly.

Participation for the second period is assumed in the following manner. Anecdotal evidence from Freemarkets suggests that if a particular seller is chosen for a market session, then it is highly likely that the same seller gets chosen in future market sessions. In other words, there is a high participation correlation across periods.

- For the sake of tractability of the analytical model, we extend the anecdotal result and set the participation correlation to one. Stated differently, the same sellers from the first period are assumed to participate in future periods.

We model this description game-theoretically, based on which we compare the price paid by the buyer in IIS and CIS. Before we present our analysis, we review the literature relevant to our analysis in the following section.

3 Literature Review

Revelation policies in financial markets are referred to as "Trade-Transparencies" by "Market Micro-Structure" literature. Depending on whether information about outstanding orders or completed orders are revealed, these policies are respectively called "pre-trade transparency" and "post-trade transparency". Information revealed under these transparency schemes includes the number of securities purchased/sought, price and other details of the limit order. A number of papers have studied the impact of these trade transparencies. In this section, we review a few of them and differentiate our work from prior work done in this area.

We begin with the literature on post-trade transparency schemes. Flood *et al.* (1999) use an experimental study resembling a foreign exchange market – multiple dealer market with inter-dealer trading – to study the impact of post-trade transparency policies. They show that opaque markets are more efficient but have higher spreads than transparent ones. This result contradicts Bloomfield & O'Hara (1999) who also use experimental economics. Bloomfield & O'Hara (1999) find that in a regular market without inter-dealer trading, post-trade transparency improves informational efficiency but with higher bid-ask spreads. Next, we review the literature on pre-trade transparencies. Madhavan *et al.* (1999), Anand & Weaver (2001) use data from the Toronto Stock Exchange and demonstrate that opaque markets lead to less efficiency. This again contradicts Boehmer *et al.* (2002) who use data from the New York Stock Exchange.

Note that these results are not applicable to electronic reverse-markets because of structural differences between the two types of markets. Typically, financial exchanges are double-sided auctions[6] whereas electronic reverse-markets are single-sided auctions. Information revealed in a double-sided auction not only affects the behavior of sellers but also that of buyers. In a single-sided reverse-auction, information revealed affects the behavior of sellers only. These differences make revelation-policy-comparisons in our procurement e-marketplace context different from that in financial markets.

To our knowledge, revelation policies in single-sided auctions have been studied only in Thomas (1996) and Koppius & van Heck (2002). Koppius & van Heck (2002) use experimental data to compare revelation policies on bidders' profits. They show that the setting that creates the least level of uncertainty for the bidders generates the highest profit for the bidders. Our paper is different from Koppius & van Heck (2002) in the following manner: it analytically studies the effect of revelation policies on the expected price paid by the buyer, the parameter of the interest to the decision-maker – the buyer. Further, our paper incorporates uncertainty about cost structure explicitly.

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[6] The only financial market which operates as a single-sided auction is the primary bond market (e.g. US Treasury Bills). Even in these bond markets, the standard policy is to reveal the winner's bid and the quantity purchased.

Thomas (1996), is the most revelant paper. It compares a similar set of revelation policies with a two-seller framework. Each seller is aware of the presence of its opponent and its own cost-structure, but is unaware of its opponent's cost structure. It is known to both sellers that, with a probability of $\frac{1}{2}$, their opponent is a low-cost type. Using this model, Thomas (1996) shows that the expected price paid by the buyer in CIS is lower than in IIS, a result similar to what Arora *et al.* (2002) have shown but under market structure uncertainty. We demonstrate that when Thomas (1996) setting is generalized further, the results may not hold good. In fact, Thomas (1996) framework is a special case of our current paper, when $r = \frac{1}{2}$ and $M = 2$.

4 Game-Theoretic Model

In this section, we model the problem context described in Section 2 game-theoretically and compute the equilibrium strategies. These strategies will be used in the next section for the consumer surplus comparison.

Recall that participating high-cost sellers always bid a Bertrand Price of $1 + \delta$ and the only interesting behavior to study is that of low-cost sellers. For computing the equilibrium strategies of low-cost sellers in IIS and CIS, we use the following result:

Lemma 4.1 *If sellers S_α and S_β hold asymmetric respective beliefs α and β ($\alpha \leq \beta$ without loss of generality) about the presence of their opponents, then the equilibrium strategies for the single period is a mixed strategy one defined between $[1 - \alpha, 1]$. The cdfs are:*
For seller S_β

$$F_\beta(p) = 1 - \frac{(1-\alpha)}{\alpha}\frac{(1-p)}{p} \tag{1}$$

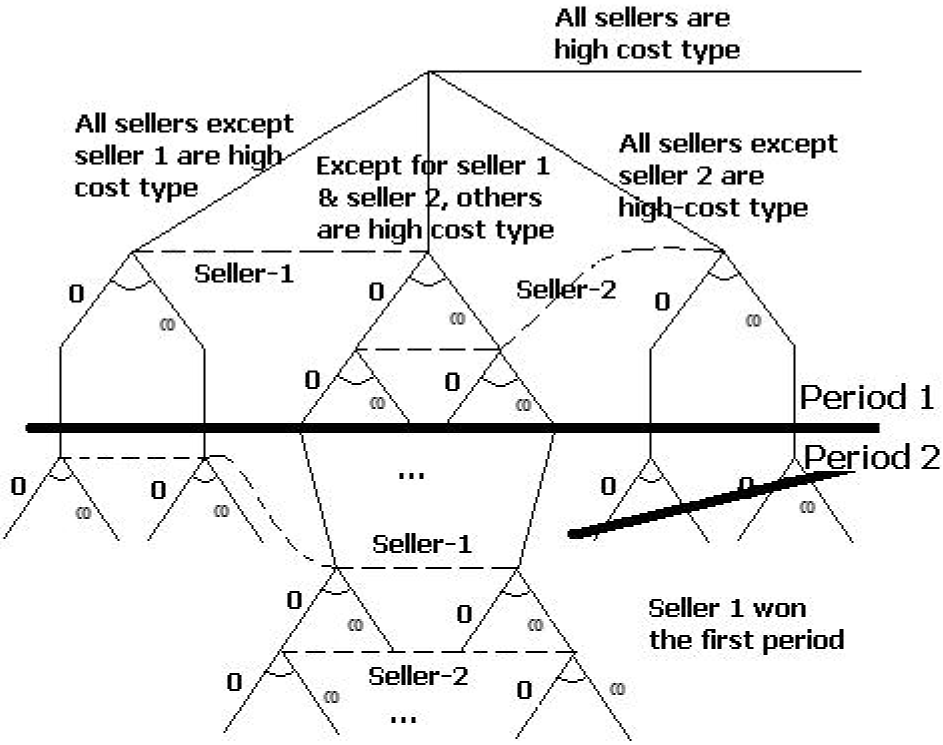

Figure 1: IIS - Extensive form of the Two Period Game captures the actions of the low-cost sellers only. At the end of the first period, the winner's bid is revealed to all sellers.

For seller S_α

$$F_\alpha(p) = 1 - \frac{(1-\alpha) - (1-\beta)p}{\beta p} \tag{2}$$

with a mass point of $\frac{\beta-\alpha}{\beta}$*.*

Expected profits for each seller is equal to $(1-\alpha)$*.*

Now, let us begin with IIS.

4.1 IIS Game

Figure 1 shows the extensive form of IIS. The root node in the figure corresponds to Nature's move. Nature chooses one low-cost seller, two low-cost sellers or no low-cost seller at all. If a low-cost seller is selected, it is uncertain about whether it is the only low-cost seller in the reverse-market or not. This uncertainty is represented by the dotted line connecting the branch where it is the only low-cost seller to the branch where both low-cost sellers are present. In this game, sellers choose their bid prices and a winner is chosen based on these bids. At the end of the first period, the winner's bid is revealed to all sellers, allowing other sellers to learn about the winner's cost structure if the winner bid a price $p \leq 1$ in the first period. If the other low-cost seller – the loser from the first period – also exists in the first period, it becomes aware of its opponent's cost structure whereas the winner continues to be uncertain. The equilibrium for this asymmetric second period game is determined first before computing the same for the first period game. However, we summarize the results chronologically.

In the first period, a participating low-cost seller is uncertain about the presence of its low-cost competitor. With probability r, it believes that its opponent exists. Based on this belief, we characterize the first period equilibrium as:

Lemma 4.2 *Only a mixed strategy equilibrium exists where each low-cost seller mixes prices in the range* $[(1-r)(1-\log(1-r)), 1]$. *The cdf (cumulative density function) for the first period bid distribution,* $F_{IIS,1}(p)$, *is a solution to the following non-linear equation*

$$(1-r)+\log(1-r) = [(1-r)+r[1-F_{IIS,1}(p)]]\,p_i-(1-r)\log([(1-r)+r[1-F_{IIS,1}(p)]]) \quad (3)$$

Although, the expression for $F_{IIS,1}(p)$ cannot be computed analytically, one can compute this distribution numerically. Figure 2 shows $F_{IIS,1}(p)$ computed for $r = 0.75$.

After comparing the bids submitted, the buyer chooses the seller that bids the lowest price as the winner. At the end of the first period, the winner's bid is revealed to all sellers. If the winner's bid price happens to be a price $p \leq 1$, then all sellers become aware of the winner's cost-type. This

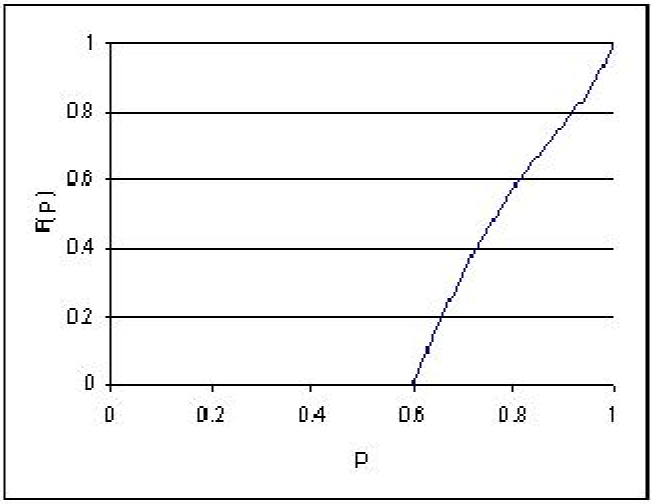

Figure 2: Bid Distribution for first period IIS - numerically computed for $r = 0.75$.

information puts the low-cost loser, if one exists, at an advantage over the low-cost winner for the second period game. The low-cost winner from the first period continues to be uncertain about its low-cost opponent and it believes that with a probability of $r'(p_1)$, it is the only low-cost seller in the market. This $r'(p_1)$ is defined as follows:

$$r'(p_1) = \frac{(1-r)}{(1-r) + r(1 - F_{\text{IIS},1}(p_1))} \tag{4}$$

The probability of being the only low-cost seller, conditional on winning the first period, is the ratio of the probability of being the only low-cost seller to the probability of winning the first period.

Since low-cost sellers are identical, the value of $r'(p_1)$ becomes known to the losing-low-cost seller as soon as the winner's first period bid is revealed. Conditional on this second period belief, $r'(p_1)$ (for the rest of this section, we will refer to this simply as r'), we characterize the second period equilibrium of low-cost sellers using lemma 4.1. The first period winner corresponds to seller S_α with a belief of $\alpha = 1 - r'$ while the first period loser corresponds to seller S_β with a belief of $\beta = 1$.

Lemma 4.3 *Both low-cost sellers – the loser and the winner – mix their bid prices between* $[r', 1]$ *for the second period game. The cdf of the bid distributions are as follows:*
For the first period loser, the cdf of the second period bid distribution is

$$F_l(p) = 1 - \frac{(1-p)r'}{p(1-r')} \tag{5}$$

Similarly, the cdf for the first period winner's second period bid distribution is:

$$F_w(p) = 1 - \frac{r'}{p} \tag{6}$$

with a mass point of $M_w = r'$ *at* $p = 1$. *Both these distributions are defined only for* $[r', 1]$.

Corresponding to these distribution, the pdfs (probability density functions) are

$$f_l(p) = \frac{r'}{p^2(1-r')} \tag{7}$$

$$f_w(p) = \frac{r'}{p^2} \tag{8}$$

We will use these distributions for the consumer surplus comparison in the next section. But in the following subsection, we compute the equilibrium for sellers in CIS.

4.2 CIS Game

Let us begin this subsection by providing some intuition. Recall that in CIS, all bids are revealed to all sellers. Because of this policy, under certain conditions, low-cost sellers have an incentive fake their cost-type in the first period. This happens only if the probability of having a low-cost opponent is high. To provide intuition, consider two low-cost sellers – seller A and seller B – along with other high cost sellers in the "world". Let seller B be present in the reverse-market and let it fake its cost-type. This serves to deceive seller A, if it exists, into believing that with a higher probability it is the only low-cost seller in the reverse-auction since it does not observe any price $p \leq 1$ bid by any seller in the first period. This belief gives way for seller B, to low-ball and outbid its rival in the second period. Note that this happens only if the probability of having an opponent is high. Instead if the probability of having an opponent is low, there is no incentive for the low-cost sellers to fake. This threshold probability value, beyond which low-cost sellers have an incentive to fake, is determined in section 4.2.1. After that, the equilibrium for CIS when the probability values are below and above the threshold are computed in section 4.2.2 and section 4.2.3 respectively.

4.2.1 Condition when Low-Cost Sellers Fake

We derive the condition by computing the value of r beyond which a seller has an incentive to deviate from a separating equilibrium in the first period.

Lemma 4.4 *There exists a threshold value of* r

$$r_{th} = \frac{M-1}{2M-1} \tag{9}$$

below which a separating equilibrium exists and above which no separating equilibrium exists.

4.2.2 Equilibrium when Low-Cost Sellers do not Fake

Consider the condition $r < r_{th}$ i.e., when sellers have no incentive to fake. We begin with the second period game where information about all bids are revealed at the end of the first period. Under this condition, if a low-cost seller does not observe a first period price $p \leq 1$ bid by any opponent, it can be sure that it is the only low-cost seller. Similarly, if it observes price $p \leq 1$ bid by its opponent, it can be sure of the low-cost opponent's presence. In this case, the equilibrium for the second period game is:

Lemma 4.5 *If a low-cost seller realizes that it is the only low-cost seller in the reverse-market, it bids* $p = 1$ *in the second period. Instead, if both low-cost sellers realize the presence of the other in the reverse-market, the equilibrium for both sellers is to bid* $p = 0$ *in the second period.*

Based on this, the expected second period profit for each seller is: $\Pi_2 = (1 - r)$. These second period profits are taken into account while computing the first period equilibrium.

Lemma 4.6 *The first period Bayesian-Nash mixed strategy equilibrium is a cdf given by the expression*

$$F_{\text{CIS}}(p) = 1 - \frac{(1-p)(1-r)}{p\,r} \tag{10}$$

defined between $[(1 - r), 1]$.

Corresponding to this, the pdf is:

$$f_{\text{CIS}}(p) = \frac{(1-r)}{p^2 r} \tag{11}$$

Note that results under this condition – when sellers have no incentive to fake – are similar to that under CIS in Arora *et al.* (2002). In the next subsection, we derive the equilibrium for CIS under the condition when sellers have an incentive to fake.

4.2.3 Equilibrium when low-cost sellers fake

This situation occurs only when $r > r_{th}$ i.e., the condition corresponding to no separating equilibrium. The equilibrium in this setting can be characterized as follows:

Lemma 4.7 *Only a semi-pooling equilibrium exists in the first period. In other words, we show that the only equilibrium that exists is one where sellers mix between faking and bidding prices* $p \leq 1$.

Given that low-cost sellers fake in the first period, let γ represent the probability with which a low-cost seller bids $1 + \delta$. Our first period equilibrium computation involves determining the expression for γ and the expression for how a low-cost seller bids $p \leq 1$. Note that this first period equilibrium is dependent on the second period profits. So let us compute the second period profits.

Given γ, if a seller does not observe any opponent bidding $p \leq 1$ in the first period, it updates its second period belief of being the only low-cost seller in the reverse-auction to

$$\gamma' = \frac{(1-r)}{(1-r) + r\,\gamma} \tag{12}$$

Instead if it observes a price $p \leq 1$ at the end of the first period, $\gamma' = 1$.

From the perspective of the second period game, depending on whether one seller faked, both sellers faked or both sellers showed their true cost-structures in the first period, three different conditions are possible. Table 1 shows the different conditions and the beliefs held by the sellers before the second period begins (Note that α and β are the beliefs held by the two sellers that their opponent exists, it is inverse of the belief about their monopolistic position). The expected profits

Possibilities for the First Period Game	Second Period Game	Expected Second Period Profits using lemma 4.1
One seller bids $p \leq 1$	Asymmetric Game Equilibrium: set $\alpha = 1 - \gamma'$ and $\beta = 1$. Seller which bid $p \leq 1$ believes with a probability of γ', it the only low-cost type .	γ'
Both sellers bids $p \leq 1$	Equilibrium: bid $p = 0$.	0
Both sellers bids $p = 1 + \delta$	Symmetric Game Equilibrium: set $\alpha = 1 - \gamma'$ and $\beta = 1 - \gamma'$.	γ'

Table 1: Second Period Game

for all these three conditions are computed using the lemma 4.1. Note that the second period game corresponding to the condition where one seller fakes while its opponent plays a price $p \leq 1$ in the first period is an asymmetric game.

Based on these second period profits, we compute the first period equilibrium.

Lemma 4.8 *The semi-pooling equilibrium for this first period game includes a bid distribution $F(p)$ according to which sellers choose a price $p \leq 1$ and a probability γ with which they choose to fake:*

$$F(p) = 1 - \frac{(1-r)(1-p) + r\gamma}{r\,p} \tag{13}$$

$$\gamma = \frac{-(3M-2)(1-r) + \sqrt{M(1-r)(4(M-1) + M(1-r))}}{2\,(M-1)r} \tag{14}$$

$F(p)$ is defined only for the price range $[p_{\min}, 1]$ where

$$p_{\min} = \frac{1}{2}\left(-1 + r + \sqrt{\frac{(1-r)(M(5-r)-4)}{M}}\right) \tag{15}$$

The pdf distribution corresponding to $FCIS(p)$ is

$$f(p) = \frac{(1-r)}{p^2 r} + \frac{r\,\gamma}{p^2} \tag{16}$$

Similarly γ is positive only for $r \geq \frac{M-1}{2M-1}$. To track the variation of γ with respect to r, we compute:

$$\frac{\partial \gamma}{\partial r} = \frac{(3M-2)\sqrt{(M(5-r)-4)(1-r)} - \sqrt{M}(M(5-r)-4)+2r)}{2(M-1)r^2\sqrt{(M(5-r)-4)(1-r)}} \tag{17}$$

Based on this, we can say that γ increases for $r \in [\frac{M-1}{2M-1}, \frac{5M-4}{6M-4}]$. For $r \in [\frac{5M-4}{6M-4}, 1]$, γ decreases. Intuitively, as the probability of having an opponent increases, sellers have an incentive to fake their cost-structures for the first period. But when the probability is very high, then the advantage decreases. In the extreme case when $r = 1$, sellers are aware of their low-cost opponent's presence with certainty and therefore, they perceive no advantage in faking their cost-structures in the first period.

Next, let us study the sensitivity of γ with respect to the number of competitors in the reverse-market, M.

$$\frac{\partial \gamma}{\partial M} = \frac{1}{-\frac{1}{2}(M(5-r)-4)) + \sqrt{\frac{M}{(1-r)}}(M(5-r)-4))(2-3M+Mr)r} < 0 \tag{18}$$

This result can be explained intuitively. The higher the value of M, the lower is the expected profit from faking in the first period. As M increases i.e., the expected profit from faking decreases, it is traded off against the advantage that sellers perceive from faking. This, in turn, leads to lower γ.

5 Consumer Surplus Comparison

Based on these equilibrium computations, we compare the consumer surplus generated between CIS and IIS. Since the equilibrium for CIS varies depending on whether r is below or above r_{th}, we compare consumer surplus for each condition separately. Under each condition, we compute the difference between the expected prices which is equivalent to the negative of the difference between the consumer surpluses.

5.1 Comparison when Sellers did not Fake

Let us consider the case when low-cost sellers do not fake i.e., for $r < r_{th}$. In this case, the consumer surplus comparison is straightforward. It is very easy to show that the first buyer benefits from choosing CIS over IIS. This choice not only affects the buyer favorably, but it also affects the second buyer favorably. This can be demonstrated in each period by assuming the worst-case scenario for IIS (best-case scenario from the consumer surplus perspective):

1. The First Period Comparison: Note that in the first period of IIS, no price below $\{(1-r)(2-(1-r))\}$ is bid. As a best case scenario, we assume that this is the first period minimum price observed by the buyer when at least one low-cost seller is present and $1+\delta$ when neither low-cost seller is present i.e.,

$$\hat{P}_{IM,1} = \{(1-r)(2-(1-r))\}\{r(2-r)\} + (1-r)^2(1+\delta) \tag{19}$$

For CIS, the expected price paid is calculated as follows:

$$\begin{aligned}\hat{P}_{CM,1} \;&=\; \{r^2 \int_{(1-r)}^{1} [2(1-F_{\text{CIS}}(p))\, f_{\text{CIS}}(p)]\, p\, dp + 2(1-r)r \int_{(1-r)}^{1} f_{\text{CIS}}(p)\, p\, dp\} \\ &\quad + (1-r)^2(1+\delta) \qquad (20)\end{aligned}$$

This expression can be explained as follows. The first term: with a probability of r^2, both low-cost sellers are in the marketplace. In such a case, $2(1 - F_{\text{CIS}}(p))f_{\text{CIS}}(p)$ is the bid distribution of the minimum price observed by the buyer. The second term corresponds to the minimum price observed by the buyer when one low-cost seller is present in the reverse-market. This happens with of probability of $2r(1-r)$. The last term accounts for the condition when neither low-cost seller is present in the reverse-market.

Based on these, the difference between the expected prices of CIS and IIS is computed as

$$G_{IM,1} = \hat{P}_{IM,1} - \hat{P}_{CM,1} = \left[(1-r)^2\, r^2\right] \qquad (21)$$

$G_{IM,1} > 0$ for any $r \in [0,1]$, implying that the expected first period price of IIS is always higher than that of CIS.

2. The Second Period Comparison: Similar to the earlier case, we assume the best case scenario for IIS i.e., the second period updated belief $r' = r$. Based on this, the expected minimum price observed by the buyer in the second period of IIS is

$$\begin{aligned}\hat{P}_{IM,2} \;&=\; \{r^2 \int_{(1-r)}^{1} [(1-F_l(p))\, f_w(p) + (1-F_w(p))\, f_l(p)]\, p\, dp \\ &\quad + 2r(1-r)[\int_{(1-r)}^{1} f_w(p)\, p\, dp + M_w]\} + (1-r)^2(1+\delta) \qquad (22)\end{aligned}$$

This expression is explained in the following manner. The first line corresponds to the case when both low-cost sellers are present in the market and this happens with probability r^2. In this case, the lowest second period price may either be offered by the first period winner or the first period loser. With a probability of $(1 - F_l(p))f_w(p)$ the winner, w, offers the lowest price in the second period. Similarly, with a probability of $(1-F_w(p))f_l(p)$ the loser, l, offers the lowest price in the second period. Using these, the expected lowest price observed by the buyer is calculated over all possible prices. The first term in the second line corresponds to the setting when the reverse-market has only one low-cost seller and this happens with a probability of $2r(1 - r)$. The winner, which is uncertain if it is the only low-cost seller, continues to bid according to the distribution $F_w(p)$. We also account for the mass point at $p = 1$ by adding M_w. The last term in the second line corresponds to the condition when no low-cost seller is present in the reverse-market and the winner is a high-cost seller bidding $1 + \delta$.

The second period expected minimum price for CIS involves: with a probability of r^2, both sellers are aware of the presence of the other and bid $p = 0$. With a probability of $2r(1 - r)$, sellers realize their status as the only low-cost sellers and bid $p = 1$. With probability of $(1 - r)^2$, neither low-cost seller is present in the market and a high-cost seller bidding $1 + \delta$ wins. Therefore, the expected minimum price for the second period CIS is

$$\hat{P}_{CM,2} = 2r(1 - r) + (1 - r)^2(1 + \delta) \tag{23}$$

Based on these, we compute the expected second period price difference as

$$G_{IM,2} = \hat{P}_{IM,2} - \hat{P}_{CM,2} = [r\ (1-r)(\log(1-r) - (1-r))] \tag{24}$$

Even in the second period, the expected price (consumer surplus) is higher (lower) in IIS when compared to CIS.

Note that these comparisons are independent of the value of M.

5.2 Comparison when Sellers Fake

Unlike in the earlier section, the comparison is not straightforward. Recall that the equilibrium for CIS, under this condition, includes a probability of faking γ that is dependent on M and r. Taking this into account, we compute the expected minimum price of CIS as

$$\hat{P}_{CM,1} = \{r^2[\int_{p_{\min}}^{1} [2(1-F(p))\ f(p)]\ p\ dp + \gamma^2] + 2(1-r)r[\int_{p_{\min}}^{1} f(p)\ p\ dp + \gamma] + (1-r)^2(1+\delta)\} \tag{25}$$

where $f(p)$ is the pdf of the bid distribution $F(p)$. This expression is similar to equation 20 except for γ which accounts for faking. This expression can be simplified by substituting values for $F(p)$ and γ. Recall that the expected first period IIS price is given by equation 19. Using these results from CIS and IIS, one cannot categorically conclude if IIS is better than CIS or not, unlike in the earlier section. We find that the comparison, assuming even the best case scenario for IIS, leads to IIS performing better than CIS under certain circumstances and vice versa. Given this, we numerically compute the expected prices for CIS and IIS for $M = \{2, 10, \infty\}$ and for $r = \{0.75, 0.825, 0.9, 0.975\}$ in Table 2.

a	Incomplete Information Setting (IIS)	Complete Information Setting (CIS)		
		$M=\infty$	$M=10$	$M=2$
0.75	0.660253	0.565873	0.566297	0.5625
0.825	0.552301	0.533621	0.529324	0.508982
0.9	0.445542	0.457328	0.44884	0.412432
0.975	0.224889	0.270242	0.260834	0.219375

Table 2: Expected prices for different values of M and a.

Based on this, we observe that the consumer surplus comparison between CIS and IIS is dependent on M and r. Note that this is unlike the condition $r < r_{th}$ where CIS is always better than IIS from a consumer surplus perspective. The reason for the result in this subsection is: for very high values of r, sellers benefit from faking and this leads to higher expected prices in CIS. The same reason accounts for CIS to be worse off than IIS from a social welfare perspective. This social welfare comparison is executed in the following subsection.

5.3 Social Weflare Comparison

The expected price paid is simply a transfer of rent from the buyer to the winning seller. Assuming the buyer utility for the product is constant, the higher the probability of a low-cost seller winning, the better is the social welfare. Given this, we compare the expected probabilities of low-cost sellers winning market sessions in CIS and IIS.

Low-cost sellers always win both periods in IIS so long as at least one of the low-cost sellers is present in the reverse-market. It is the same case in CIS for $r < r_{th}$. This means that the social welfare is identical for CIS and IIS for $r < r_{th}$. When $r > r_{th}$, the comparison needs further explanation.

We noted that low-cost sellers always win both periods of IIS if either of them is present. For

CIS, independent of whether both low-cost sellers faked, one low-cost seller faked or both low-cost sellers bid prices $p \leq 1$ in the first period, the second period winner is always a low-cost seller. Because of this, the second period social welfare in CIS, under the condition $r > r_{th}$, is identical to IIS. However, the CIS first period social welfare compares to that in IIS in the following manner.

The probability of a high-cost seller winning the first period of IIS is the probability that no low-cost seller is present in the market which corresponds to:

$$Prob_{\text{IIS}} = (1-r)^2 \tag{26}$$

But the probability of a high-cost seller winning the first period of CIS is given by:

$$Prob_{\text{CIS}} = (1-r) + 2r(1-r)\left[\gamma\frac{M-1}{M}\right] + r^2\left[\gamma^2\frac{M-2}{M}\right] \tag{27}$$

Based on the last two equations, one can conclude that $Prob_{\text{CIS}} > Prob_{\text{IIS}}$. Stated differently, the probability that a low-cost seller wins the market-sessio is higher in IIS than in CIS. This means that the first period in CIS has a lower expected social welfare than in IIS. Further discussions on this result is deferred to the next section.

6 Discussion and Conclusion

In this paper, we address an important IT-enabled real-world problem. Prior to the arrival of the Internet, information revelation policies were not feasbile and the need for such a study did not exist. However with the current scenario, a buyer arriving at an electronic reverse-marketplace needs guidance to choose the appropriate information revelation policy. In this paper, we address this issue by considering a simple framework to study the effect of information revelation policies in an e-marketplace framework where sellers are uncertain about the cost structures of their opponents. Specifically, we compare the expected price paid by the buyer in CIS and IIS, two of the many policies available to a buyer in a reverse-market hosted by Freemarkets.

Our game-theoretic analysis provides the buyer with insights which the buyer can use while choosing the optimal information revelation policy. It demonstrates how sellers respond to different information revelation policies, when and how they attempt to fake their cost structures, and how all these factors impact the expected price paid by the buyer. In this two-buyer, two low-cost seller framework, we derive the threshold probability value below which sellers have no incentive to fake. In such a case, we show that the expected price paid by the buyer is better in CIS than in IIS. Instead, if the probability of observing an opponent is higher than the threshold value, one cannot categorically conclude if one setting is better than the other even for the first period game. We numerically demonstrate that under certain conditions, the expected price paid by the buyer is higher in CIS than in IIS and vice versa. Although CIS may sometimes be better from a consumer surplus perspective, it is not so from a social welfare perspective. A social welfare maximizing market-maker tends to prefer IIS over CIS under the framework where sellers are uncertain about their opponents' cost structure.

Our inability to analytically compare CIS and IIS even in this simplistic setting, poses a problem to our larger interest which is to understand the impact of revelation policies in a framework involving both types of uncertainties – uncertainty about market structure (uncertainty about the number of competitors) and uncertainty about opponent's cost structure. Introducing market structure uncertainty into the framework will involve computing the equilibrium bidding behavior for different values of M and for different numbers of low-cost sellers participating. Then based on the equilibrium strategies, the expected prices paid by the buyer in CIS and IIS will be compared. The comparison in this richer context becomes analytically intractable forcing us to seek alternative methodologies. In fact, we intend to use a computational marketplace to study the impact of revelation policies. Such an approach will also permit us to relax assumptions made for the game-theoretic models and study the impact of revelation policies using a framework closer to reality: a) In this paper, we set the participation correlation across the two periods to be one for tractability reasons. In reality, it is not so. We intend to investigate the effect of this correlation computationally. b) Also, for tractability reasons, we assumed that sellers are symmetrical. We intend to compare the two settings by relaxing this assumption within a computational framework that lends itself to a simulation-based analysis. c) Another extension would be to model other revelation policies in reverse-marketplaces and study their impact relative to those studied in this paper.

References

Anand, A. & Weaver, D. (2001). Should order exposure be mandated? the toronto stock exchange solution, working Paper, Syracuse University.

Arora, A., Greenwald, A., Kannan, K.N. & Krishnan, R. (2002). Effect of information revelation policies under market structure uncertainty, working Paper, Carnegie Mellon University.

Bloomfield, R. & O'Hara, M. (1999). Market transparency: Who wins and who loses? *Review of Financial Studies*, **12**, 5–35.

Boehmer, E., Saar, G. & Yu, L. (2002). Lifting the veil: Analysis of pre-trade transparency, working Paper, NYSE.

Flood, M., Huisman, R., Koedijk, K. & Mahieu, R. (1999). Quote disclosure and price discovery in multiple-dealer financial markets. *Review of Financial Studies*, **12**, 37–59.

Koppius, O.R. & van Heck, E. (2002). Information architecture and electronic market performance in multidimensional auctions, working Paper, Erasmus University.

Madhavan, A., Porter, D. & Weaver, D. (1999). Should securities markets be transparent, working Paper, Baruch College.

Thomas, C. (1996). Market structure and the flow of information in repeated auctions, working Paper, Federal Trade Commission.

A Proof of Lemma 4.1

In this section, we compute the equilibrium strategies for R_α and R_β when the firms hold their respective beliefs of α and β they are the only low-cost seller in the reverse-market. For this, we use Π_α and Π_β to represent the expected profits for R_α and R_β respectively. Note that these proofs focus only the behavior of the low-costsellers present in the reverse-market.

A.1 Equilibrium Strategies

Proposition A.1 *All firms earn positive profits at equilibrium.*

Proof: Suppose not: i.e., suppose $p^*_\alpha = 0$. The proof proceeds by establishing the existence of p_α s.t. $\Pi_\alpha(p_\alpha, p^*_\beta) > \Pi_\alpha(0, p^*_\beta)$. If $p^*_\beta = 1$ and $p_\alpha = 1$, then

$$\Pi_\alpha(p_\alpha, p^*_\beta) = (1 - \alpha) + \alpha\frac{1}{2} > 0 = \Pi_\alpha(0, p^*_\beta) \tag{App-1}$$

Otherwise, if $0 < p^*_\beta < 1$ and $p_\alpha = p^*_\beta - \epsilon$, for some ϵ, then

$$\Pi_\alpha(p_\alpha, p^*_\beta) = (1 - \alpha)p_\alpha + \alpha p_\alpha = p_\alpha > 0 = \Pi_\alpha(0, p^*_\beta) \tag{App-2}$$

Otherwise, if $p^*_\beta = 0$ and $p_\alpha = 1$, then

$$\Pi_\alpha(p_\alpha, p^*_\beta) = (1 - \alpha) > 0 = \Pi_\alpha(0, p^*_\beta) \tag{App-3}$$

From this proof, we know that a $\underline{p}$ exists below which the R_α never bids. In such a case R_β can always secure a profit of at least $\Pi_\beta(\underline{p} - \epsilon) = \underline{p} - \epsilon > \Pi_\beta(0) = 0$. **QED**

Proposition A.2 *There is no pure strategy equilibrium for this game.*

Proof: Let $\Pi_\alpha(p_\alpha, p_\beta)$ represent the profit for seller R_α when it bids p_α and its opponent R_β bids p_β. Let the pure strategy equilibrium for sellers R_α and R_β be (p^*_α, p^*_β). If such a pure strategy equilibrium exists, then there cannot exist any other price p_α such that $\Pi_\alpha(p_\alpha, p^*_\beta) > \Pi_\alpha(p^*_\alpha, p^*_\beta)$. The Symmetric Case: If $p^*_\alpha = p^*_\beta$

$$\Pi_\alpha(p^*_\alpha, p^*_\beta) = (1-\alpha)(p^*_\alpha) + \alpha\frac{(p^*_\alpha)}{2} \tag{App-4}$$

Let there be a $p_\alpha = p^*_\alpha - \epsilon$, $\epsilon > 0$, then,

$$\Pi_\alpha(p_\alpha, p^*_\beta) = (1-\alpha)(p^*_\alpha - \epsilon) + \alpha(p^*_\alpha - \epsilon) \tag{App-5}$$

From equation App-4 and equation App-5, we have $\Pi_\alpha(p_\alpha, p^*_\beta) > \Pi_\alpha(p^*_\alpha, p^*_\beta)$ if $\epsilon < \frac{\alpha(p^*_\alpha)}{2}$. Since such an ϵ exists, therefore no pure strategy equilibrium exists.

The asymmetric case: Without loss of generality, assume $p^*_\alpha < p^*_\beta$. Choose $p_\alpha = p^*_\alpha + \epsilon < p^*_\beta$, for some $\epsilon > 0$. Now

$$\Pi_\alpha(p^*_\alpha, p^*_\beta) = (1-\alpha)p^*_\alpha + \alpha p^*_\alpha = p^*_\alpha \tag{App-6}$$

and

$$\Pi_\alpha(p_\alpha, p^*_\beta) = (1-\alpha)p_\alpha + \alpha p_\alpha = p_\alpha \tag{App-7}$$

Since $p_\alpha > p^*_\alpha$, it follows that $\Pi_\alpha(p_\alpha, p^*_\beta) > \Pi_\alpha(p^*_\alpha, p^*_\beta)$. This proves that no pure strategy equilibrium exists for R_α. The proof for R_β is based on the proof that R_α does not have a pure strategy equilibrium price.

Suppose not: Let R_β fix the price at p_β while R_α plays a mixed strategy. In such a case, R_α can improve its profits by bidding 1 if $\Pi_\alpha(p_\beta, 1) = (1-\alpha) > \Pi_i(\underline{p}-\epsilon) = \underline{p}-\epsilon$ or bid $\underline{p}-\epsilon$ otherwise. In either case, R_α will start to play a pure strategy to maximize his profits. But this cannot be true because we earlier proved that R_α mixes strategies. **QED**

Since there is no pure strategy equilibrium, let us represent the equilibrium as a pair (f_α, S_α) for R_α and (f_β, S_β) for R_β. The expected profits are given by:

$$\Pi_\alpha(p) = (1-\alpha)p + \alpha(1 - F_\beta(p))p \tag{App-8}$$

where $F_\beta(p)$ represents R_β's equilibrium strategy and $(1-\alpha)$ represents R_α's beliefs about being the only low-cost seller in the reverse-market. Similarly, $F_\alpha(p)$ – the equilibrium strategy of R_α – affects the expected profits for R_β:

$$\Pi_\beta(p) = (1-\beta)p + \beta(1 - F_\alpha(p))p \tag{App-9}$$

where β represents R_β's belief about being the only low-costseller.

Proposition A.3 *The strategy sets S^*_α and S^*_β are convex, with identical upper boundaries.*

Proof: The proof proceeds by first showing that there are no holes in $T = S^*_\beta \cap S^*_\alpha$, and by then showing that there are no holes in $T' = S^*_\beta \setminus T$ (or $T'' = S^*_\alpha \setminus T$).

No holes in T: Let $p_l = \inf(T)$ and $p_h = \sup(T)$. Assume to the contrary that there exists price $p \notin T$ and interval $I = (h_{\min}, h_{\max})$, with $p_l < h_{\min} < p < h_{\max} < p_h$.

If $p \notin T$, then either $p \notin S^*_\alpha$ but $p \in S^*_\beta$, or $p \notin S^*_\beta$ but $p \in S^*_\alpha$, or $p \notin S^*_\beta \cup S^*_\alpha$.

It cannot be that $p \notin S^*_\beta$ and $p \in S^*_\alpha$, because if R_β places no mass on interval I, then R_α also places no mass on interval I. Let $q_l = \sup\{p \in S^*_\beta | p_l < p < h_{\min}\}$ and let $q_h = \inf\{p \in S^*_\beta | p_h >$

$p > h_{\max}\}$. R_α's profits $\Pi_\alpha(p) = (1-\alpha)p + \alpha(1 - F_\beta(p))p$ are increasing in p for $p \in I$, since $F_\beta(h_{\max}) - F_\beta(h_{\min}) = 0$. Thus, R_α can achieve greater profits by charging q_h with probability $F_\beta(q_h) - F_\beta(q_l)$, leaving no mass in the interval I. Along similar lines, it can be shown that $p \notin S^*_\alpha$ and $p \in S^*_\beta$ cannot exist. The only remaining possibility is that $p \notin S^*_\beta \cup S^*_\alpha$.

But it also cannot be that neither player is randomizing over the interval I. If R_α charges $h_{\min}$, it earns profits $\Pi_\alpha(h_{\min}) = (1-\beta)h_{\min} + \beta(1 - F_\beta(h_{\min}))h_{\min}$. But if R_α charges $h_{\max}$, then it earns profits $\Pi_\alpha(h_{\max}) = (1-\beta)h_{\max} + \beta(1 - F_\beta(h_{\max}))h_{\max}$. Since $F_\beta(h_{\min}) = F_\beta(h_{\max})$, it follows that $\Pi_\alpha(h_{\max}) > \Pi_\alpha(h_{\min})$, violating the assumption that (f_α, S^*_α) and (f_β, S^*_β) comprise a mixed strategy Nash equilibrium.

No holes in T': The set T' includes the prices charged by R_β, but not by R_α. There are no holes in T', since otherwise R_β firm could make itself better off by moving the mass from the lower end of the hole to the upper end, without changing its expected rewards conditioned on the market being in duopoly, or improving its expected profit conditional on the market being a monopoly. Similarly, there are no holes in T''. **QED**

Proposition A.4 *Neither firm's equilibrium strategy has a mass point a) in the interior, or b) at the lower boundary of the other's support. or c) at the upper boundary of other's support, if that boundary has a mass point for the other firm.*

Proof: WLOG, assume to the contrary that there exists a mass point in equilibrium for R_β at price $\underline{p}_\alpha \leq p^*_\beta < \overline{p}_\alpha$, where $\underline{p}_\alpha = \inf(S^*_\alpha)$ and $\overline{p}_\alpha = \sup(S^*_\alpha)$, with probability of $\kappa > 0$. By proposition A.1, $\underline{p}_\alpha = \inf(S^*_\alpha) > 0$ and by Proposition A.3, there are no "holes" in the strategy set S^*_α. Based on this, we can consider the profits for firm R_α when bidding $p^*_\beta - \epsilon$ and $p^*_\beta + \epsilon$, for

$\epsilon > 0$, namely:

$$\Pi_\alpha(p^*_\beta - \epsilon) = (1-\alpha)(p^*_\beta - \epsilon) + \alpha(1 - F_\beta(p^*_\beta - \epsilon))(p^*_\beta - \epsilon) \tag{App-10}$$

and

$$\Pi_\alpha(p^*_\beta + \epsilon) = (1-\alpha)(p^*_\beta + \epsilon) + \alpha(1 - F_\beta(p^*_\beta + \epsilon))(p^*_\beta + \epsilon) \tag{App-11}$$

Subtracting equation App-11 from equation App-10 yields

$$\begin{aligned}\Pi_\alpha(p^*_\beta - \epsilon) - \Pi_\alpha(p^*_\beta + \epsilon) &= -2\epsilon + p^*_\beta\alpha\left[F_\beta(p^*_\beta + \epsilon) - F_\beta(p^*_\beta - \epsilon)\right] \\ &\quad +\epsilon\alpha\left[F_\beta(p^*_\beta + \epsilon) + F_\beta(p^*_\beta - \epsilon)\right] \\ &\approx -2\epsilon + p^*_\beta\alpha\kappa + \epsilon\alpha\left[F_\beta(p^*_\beta + \epsilon) + F_\beta(p^*_\beta - \epsilon)\right]\end{aligned}$$

For $0 < \epsilon < (p^*_\beta\alpha\kappa)/(2 - \alpha[F_\beta(p^*_\beta + \epsilon) + F_\beta(p^*_\beta - \epsilon)])$, this quantity is strictly positive, which suggests that firm R_α could earn strictly greater profits by shifting some mass from above p^*_β to below p^*_β. Thus, R_β cannot have a mass point in the interior of firm R_α's support at equilibrium.

Note that this result is applicable even when R_β has a mass point at $p^*_\beta = \underline{p}_\alpha$. In such a case, firm R_α can obtain higher expected profits from bidding a price $p = \underline{p}_\alpha - \epsilon$ which condradicts the the support assumed earlier.

If R_β has a mass point at $p^*_\beta = \overline{p}_\alpha$ with probability κ, where firm R_α also has a mass point, then firm j can increase its profits by bidding $p^*_\beta - \epsilon$ with probability κ and p^*_β with zero density. The proof demonstrates that no mass point exists for R_β.

To prove that no mass point exists for R_α, let us assume that mass point exists for R_α at $\inf(S^*_\beta) \leq p_\alpha < \sup(S^*_\beta)$ with probability κ.

The expected profits for R_β from bidding $p_\alpha + \epsilon$ and $p_\alpha - \epsilon$ are as below:

$$\begin{aligned} \Pi_\beta(p_\alpha - \epsilon) &= (1 - F_\alpha(p_\alpha - \epsilon))(p_\alpha - \epsilon) && \text{(App-12)} \\ \Pi_\beta(p_\alpha + \epsilon) &= (1 - F_\alpha(p_\alpha + \epsilon))(p_\alpha + \epsilon) && \text{(App-13)} \end{aligned}$$

For $0 < \epsilon < p_\alpha\kappa/(2 - (F_\alpha(p_\alpha + \epsilon) + F_\alpha(p_\alpha - \epsilon))$, $\Pi_\beta(p_\alpha - \epsilon) > \Pi_\beta(p_\alpha + \epsilon)$ implying that R_β will have an incentive to move a certain mass point from below p_α to above p_α. But, we had proved that no mass point exists for R_β which implies no exists for R_α also. **QED**

Corollary A.5 *If firm j – R_α or R_β has a mass point at $\overline{p}_i = \sup(S_i^*)$, then its opponent – firm i – charges $\overline{p}_i$ with zero probability at equilibrium: i.e., firm i randomizes in the interval $[\underline{p}_i, \overline{p}_i)$.*

Proof: Let a mass point exist for R_α at the upper boundary $\overline{p}$ of R_β's strategy set with probability be κ. The expected profits for R_β for the loser from bidding $\overline{p}$ are

$$\Pi_\beta(\overline{p}) = ((1 - \beta) + \beta\frac{\kappa}{2})\overline{p} \tag{App-14}$$

But the profit from bidding $\overline{p} - \epsilon$ is

$$\Pi_\beta(\overline{p} - \epsilon) = ((1 - \beta) + \beta\kappa)(\overline{p} - \epsilon) \tag{App-15}$$

For $0 < \epsilon < \frac{\overline{p}\kappa\beta}{((1-\beta)+\kappa\beta)2}$, $\Pi_\beta(\overline{p} - \epsilon) > \Pi_\beta(\overline{p} + \epsilon)$ i.e., R_β will try to bid as close to $\overline{p}$ as possible but bid $\overline{p}$ with probability of 0.

A similar proof can be shown for R_α. **QED**

Proposition A.6 $\sup(S_\alpha^*) = \sup(S_\beta^*) = 1.$

Proof: Suppose $\sup(S_\alpha^*) = \overline{p}_\alpha < 1$. Since, $\Pi_\alpha(\overline{p}_\alpha) = (1-\alpha)\overline{p}_\alpha < (1-\alpha) = \Pi_\alpha(1)$, $F_\beta(\overline{p}_\alpha) = 1$. Therefore, $\overline{p}_\alpha = 1$. By Proposition A.5, $\sup(S_\beta^*) = 1$. **QED**

To determine R_β's equilibrium strategy, we use Equation App-8, which describes the expected profits of R_α. In particular, $\Pi_\alpha(p) = (1-\alpha)$ at $p = 1$, since $F_\beta(1) = 1$. Thus,

$$(1-\alpha) = (1-\alpha)p + \alpha(1 - F_\beta(p))p \tag{App-16}$$

Rearranging terms,

$$F_\beta(p) = 1 - \frac{(1-p)(1-\alpha)}{p\alpha} \tag{App-17}$$

Corresponding to this cdf is the pdf

$$f_\beta(p) = \frac{(1-\alpha)}{p^2\alpha} \tag{App-18}$$

The lower boundary of $F_\beta(p)$ occurs at $(1-\alpha)$. Similarly, the lower boundary of $F_\alpha(p) = (1-\alpha)$, as shown in the following proposition.

Proposition A.7 $\inf(S_\alpha^*) = \inf(S_\beta^*) = (1-\alpha)$.

Proof: To determine $\inf(S_\beta^*)$, let $F_\beta(p) = 0$, and compute $\inf(S_\beta^*) = (1-\alpha)$. Now let $\underline{p} \equiv \inf(S_\alpha^*)$. If $\inf(S_\alpha^*) \neq \inf(S_\beta^*)$ (that is, if $\underline{p} \neq (1-\alpha)$), then either $\underline{p} < (1-\alpha)$ or $\underline{p} > (1-\alpha)$.

If $\underline{p} < (1-\alpha)$, then R_α can increase its expected profits by increasing $\underline{p}$ to $(1-\alpha)$: i.e., R_α's strategy is not in equilibrium. Contradiction.

If $\underline{p} > (1-\alpha)$, then R_β can increase its expected profits by increasing $\inf(S_\beta^*)$ to $\underline{p}$. But then there would be a mass point at $\underline{p}$. By proposition A.4, there can be no mass point at the lower boundary of the other firm's support. Contradiction. **QED**

Having derived the equilibrium strategy for R_β in the first period, we now compute the equilibrium strategy for R_α. By proposition A.6, the lower boundary of R_α's distribution $F_\alpha(p)$ occurs at $(1-\alpha)$, which implies that $F_\alpha((1-\alpha)) = 0$. Thus, $\Pi_l^2(p) = (1-\alpha)$ at $p = (1-\alpha)$. Equating Equation App-9 to the profits earned by bidding $(1-\alpha)$ yields

$$(1-\alpha) = (1-\beta)p + \beta(1-F_\alpha(p))p \tag{App-19}$$

It follows that

$$F_\alpha(p) = 1 - \frac{(1-\alpha)-(1-\beta)p}{\beta\, p} \tag{App-20}$$

Corresponding to this cdf is the pdf

$$f_\alpha(p) = \frac{(1-\alpha)}{\beta p^2} \tag{App-21}$$

Note that R_α's equilibrium distribution has a mass point equal to $\frac{((1-\beta)-(1-\alpha))}{\beta}$ at $p = 1$, since $F_\alpha(1) = 1 - \frac{((1-\beta)-(1-\alpha))}{\beta}$. This mass point ensures that the distribution $F_\alpha(p) = 1$ at $p = 1$. Based on these distributions, we can compute the expected profits.

A.2 Expected Profits are the same

Lemma A.8 *The second period expected profits for R_α and R_β are equal to $(1-\alpha)$.*

Proof: $\Pi_\alpha(p) = \Pi_\beta(p) = (1-\alpha)$, for all prices p. **QED**

B Proof of Lemma 4.2 and Lemma 4.3

We begin with the second period and compute the second period equilibrium strategies. Based on that, we compute the first period equilibrium.

B.1 Second Period Game

This game is solved by substituting $\alpha = 1 - r'$ and $\beta = 1$ in the asymmetric game. Correspondingly l corresponds to R_β and w corresponds to R_α. Therefore, the cdf for the loser's distribution is given by:

$$F_l(p) = 1 - \frac{(1-p)r'}{p(1-r')} \tag{App-22}$$

Corresponding to this cdf is the pdf

$$f_l(p) = \frac{r'}{p^2(1-r')} \tag{App-23}$$

For the winner w, the cdf of the distribution is

$$F_w(p) = 1 - \frac{r'}{p} \tag{App-24}$$

Corresponding to this cdf is the pdf

$$f_w(p) = \frac{r'}{p^2} \tag{App-25}$$

Note that the winner's equilibrium distribution has a mass point equal to r' at $p = 1$, since $F_w(1) = 1 - r'$. This mass point ensures that the distribution $F_w(p) = 1$ at $p = 1$. Based on these distributions, we can compute the expected profits.

B.2 Expected Profits are the same

Lemma B.1 *The second period expected profits for the winner and the loser are equal to* r'.

Proof: $\Pi_w^2(p) = \Pi_l^2(p) = x'$, for all prices p. **QED**

B.3 First Period Game

These second period profits determine the first period behavior. Before we begin the equilibrium calculations for the first period game, let us characterize the second period belief if sellers had a pure strategy equilibrium (p_i for seller i and p_j for seller j) in the first period.

If we assume sellers play pure strategies, then the winning seller in the first period, say seller i, wins because either it is the only low-cost seller in the reverse-market, or it outbid its low-cost opponent j. It can outbid its opponent with probability δ_{ij} which depends on p_i and p_j. Thus, seller i's second period belief about being the only low-cost seller – r'_i – conditioned on winning in the first period, can be represented in a Bayesian manner as follows:

$$r'_i(a, p_i, p_j) = \frac{(1-r)}{(1-r) + r\delta_{ij}} \tag{App-26}$$

where

$$\delta_{ij} = \begin{cases} 1 & \text{if } p_i < p_j \\ \frac{1}{2} & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

accounts for the possibility of ties.

In IIS, seller i's expected profits Π_i as a function of seller i's price p_i and seller j's price p_j are calculated as follows:

$$\Pi_i(p_i, p_j) = (1-r)((p_i) + r'_i) + r[(p_i + r'_i)\delta_{ij} + (0 + r'_j)(1-\delta_{ij})] \tag{App-27}$$

where δ_{ij} is defined as before.

Equation App-27 can be understood as follows. Seller i believes itself to be the only low-cost seller with probability $(1-r)$. Thus, with probability $(1-r)$, it earns profits of p_i in the first period and r'_i in the second period. Seller i believes that its low-cost opponent is present with a probability of r. Thus, with probability r, seller i is either the lower priced seller in the first period, in which case it earns profits of p_i in the first period and r'_i in the second period, or seller i is the higher priced seller in the first period, in which case it earns zero profits in the first period and r'_j in the second period. The δ_{ij} terms in this Equation App-27 account for the possibility of ties.

Proposition B.2 *There is no pure strategy equilibrium in the first period in* IIS.

Proof: Suppose not: i.e., suppose there exists pure strategy equilibrium (p_i^*, p_j^*). The proof proceeds by establishing the existence of p_i s.t. $\Pi_i(p_i, p_j^*) > \Pi_i(p_i^*, p_j^*)$.

Note the following: if seller i wins in the first period by outbidding its opponent, then

$$r'_i(r, p_i, p_j) = \frac{(1-r)}{(1-r)+r} = (1-r) \tag{App-28}$$

If seller i wins in the first period, but both sellers bid the same price, then

$$r'_i(a, p_i, p_j) = \frac{(1-r)}{(1-r)+\frac{1}{2}r} = \frac{2(1-r)}{1+(1-r)} \tag{App-29}$$

The symmetric case: If $p_i^* = p_j^* = p^*$, then

$$\begin{aligned}\Pi_i(p_i^*, p_j^*) &= (1-r)\left(p^* + \left(\frac{2(1-r)}{1+(1-r)}\right)\right) + (1-(1-r)) \\ &\left(\frac{1}{2}\left(p^* + \frac{2(1-r)}{1+(1-r)}\right) + \frac{1}{2}\left(0 + \frac{2(1-r)}{1+(1-r)}\right)\right) \qquad \text{(App-30)}\end{aligned}$$

Now if $p_i = p^* - \epsilon$, for some $\epsilon > 0$, then

$$\Pi_i(p_i, p_j^*) = p^* - \epsilon + (1-r) \qquad \text{(App-31)}$$

From equations App-30 and App-31, $\Pi_i(p_i, p_j^*) > \Pi_i(p_i^*, p_j^*)$ whenever $\epsilon < r[(p^*)/2] - (1-r)r/(1+(1-r))]$. Such an ϵ exists, whenever $p^* > 2(1-r)/(1+(1-r))$. But note that $p_i = p_j = p$ is not a pure strategy equilibrium for $p \in [0, 2(1-r)/(1+(1-r))]$, since

$$\Pi_i(p_i, p_j) \leq ((1-r) + \frac{2(1-r)}{1+(1-r)}) \qquad \text{(App-32)}$$

but $\Pi_i(1, p_j) = [2(1-r) + (1-r)r] > \Pi_i(p_i, p_j)$ whenever $0 < (1-r) < 1$.

The asymmetric case: Without loss of generality, assume $p_i^* < p_j^*$. Choose $p_i = p_i^* + \epsilon < p_j^*$, for some $\epsilon > 0$. Now

$$\Pi_i(p_i^*, p_j^*) = p_i^* + (1-r) \qquad \text{(App-33)}$$

and

$$\Pi_i(p_i, p_j^*) = p_i + (1-r) \qquad \text{(App-34)}$$

Since $p_i > p_i^*$, it follows that $\Pi_i(p_i, p_j^*) > \Pi_i(p_i^*, p_j^*)$. **QED**

Having argued that no pure strategy equilibrium exists, we now study the mixed strategy equilibrium of the first period of IIS: i.e., $F_{\text{IIS}}(p)$ and $F_{\text{IIS}}(p)$. Rewriting equations App-26 and App-27 in terms of mixed strategies yields:

$$r_i'(a, p_i) = \frac{(1-r)}{(1-r) + r(1 - F_{\text{IIS}}(p_i))} \tag{App-35}$$

and

$$\begin{aligned}
\Pi_i(p_i) &= (1-r)(p_i + r_i'(p_i)) + \\
&\quad r[(1 - F_{\text{IIS}}(p_i))(p_i + r_i'(p_i)) + F_{\text{IIS}}(p_i)(0 + r_j'(p_j))]
\end{aligned} \tag{App-36}$$

In abuse of notation, we use $r_i'(p_i)$ to represent $r_i'(a, p_i)$ hereafter. Note that although seller i can observe p_j at the end of the first period and compute $r_j'(p_j)$, this information is not available ex ante. Therefore, we compute the expected value of $r_j'(p_j)$ conditional on $p_i > p_j$.

Now, if we restrict our attention to the symmetric mixed strategy equilibrium (i.e., let $F_{\text{IIS}}(p) \equiv F_{\text{IIS}}(p)$), and if $\inf(S_j^*) = \underline{p}_j$, then $\hat{r'}_j$ conditional on $p_i > p_j$ can be computed as follows:

$$\begin{aligned}
\hat{r'}_j(p_i) &= \frac{1}{F_{\text{IIS}}(p)} \int_{\underline{p}_j}^{p_i} \frac{(1-r)}{(1-r) + r(1 - F_{\text{IIS}}(p))} \, f_{\text{IIS}}(p) \, dp \\
&= \frac{1}{F_{\text{IIS}}(p)} \frac{(1-r)}{r} [-\log((1-r) + r(1 - F_{\text{IIS}}(p))]
\end{aligned} \tag{App-37}$$

Substituting for $\hat{r'}_j(p_i)$ and $r_i'(p_i)$, equation App-36 is rewritten as

$$\Pi_i(p_i) = [(1-r) + r(1 - F_{\text{IIS}}(p_i))] \, p_i + (1-r) \, [1 - \log((1-r) + r(1 - F_{\text{IIS}}(p_i))] \tag{App-38}$$

Having defined the profits, we can characterize the equilibrium using the following propositions:

Proposition B.3 *No holes exist in the strategy sets S_i and S_j*

Proof: The proof for this is very similar to that of the asymmetric single period game but with a different expected profit function. **QED**

Proposition B.4 $\sup(S_i^*) = \sup(S_j^*) = 1$.

Proof: Suppose not: i.e., suppose $\sup(S_i^*) = \overline{p}_i < 1$ then, $F_{\text{IIS}}(\overline{p}_i) = 1$. Thus, $\Pi_i(\overline{p}_i) = (1 - r)(\overline{p}_i + (1 - r)_i(\overline{p}_i)) + r\hat{x}'_j < 2(1 - r) + r\hat{r'}_j = \Pi_i(1)$, since $\overline{p}_i + (1 - r)_i(\overline{p_i}) < 2$. Therefore, $\overline{p}_i = 1$. The argument is analogous for seller j. **QED**

Substituting $F_{\text{IIS}}(1) = 1$ does not allow us to compute a closed form expression for $F_{\text{IIS}}(p)$. However, we can obtain a lower bound using the following proposition:

Proposition B.5 $\inf(S_i^*) = \inf(S_j^*) \geq (1 - r^2)$.

Proof: At the upper boundary, $\Pi_i(1) = 2(1 - r) + r\hat{r'}_j(1)$, since winning by bidding at the upper boundary reveals that the marketplace is monopolistic. At the lower boundary, $\Pi_i(\underline{p}_i) = \underline{p}_i + (1 - r)$, since winning at the lower boundary reveals no information. Setting these two expressions equal to one another and plugging in Equation App-37 yields $\underline{p} = (1 - r) + r\hat{r'}_j(1) = ((1 - r) + (1 - r)[-\log(1 - r)])$. Since $-\log(1 - r) \geq r$, it follows that $\underline{p}_i \geq ((1 - r) + (1 - r)r) = (1 - r^2)$. **QED**

C Proof of Lemma 4.4

Proof: Let the separating equilibrium for the first period game be a mixed strategy one and the cdf of the bid distribution be $F(p)$. By definition, $F(1) = 1$. In such a case, the low-cost seller A, if it exists, follows $F(p)$. Based on this, seller B's total expected profits from bidding $p < 1 + \delta$ in the first period are:

$$\Pi_{B,separate}(p) = (1 - r)(p + 1) + r((1 - F(p))p + 0) \tag{App-39}$$

This is explained in the following manner. With probability of $(1 - r)$, seller A does not exist. In such a case, it secures the price p it bids in the first period. It also secures a profit of 1 for the second period which is explained as follows. Since seller B observes no price $p \leq 1$ bid by any opponent in the first period, it knows that it is the only low-cost seller in the market and therefore, it bids a price of 1 in the second period. With probability of r, seller A exists. In this case, seller B outbids seller A with probability $(1 - F(p))$ and secures p. For the second period, both realize the existence of the other and therefore, they bid 0 in the second period.

Using this expression and the property that $F(1) = 1$, we compute the expected profits of the separating equilibrium as $2(1 - r)$.

Now consider the case when seller B deviates from the separating equilibrium and masquerades itself as a high cost seller in the first period game. The total expected profits for seller B in this case are

$$\Pi_{B,fake} = r0 + (1 - r)\frac{1}{M} + 1 \tag{App-40}$$

This can be explained period-by-period. With a probability of r, seller A exists. In this case, seller B loses the first period since seller A wins with a price of $p \leq 1$ while bidding according to the separating equilibrium. This corresponds to the first term. When seller A is not present, seller B wins the first period game with a probability of $\frac{1}{M}$. This accounts for the second term. The third term, 1, corresponds to seller B's expected profit from winning the second period, independent of whether or not seller A is present in the reverse-market. If seller A exists, seller B loses the first period. But it deceives seller A into believing that seller A is the only seller in the market. Because of this, seller A bids 1 in the second period and seller B outbids seller A with a bid of $1-\epsilon$ (Ignore ϵ because of its smallness). If seller A does not exist, seller B observes no price $p \leq 1$ bid by any opponent and therefore, it knows that it is the only seller in the market. In this case, it bids and secures a profit of 1 in the second period.

Combining equation App-39 and equation App-40, a seller finds it advantageous to fake only if:

$$\Pi_{B,fake} > \Pi_{B,separate}(p) \quad \text{(App-41)}$$

$$r > \frac{M-1}{2M-1} \quad \text{(App-42)}$$

QED

D Proof for lemma 4.6

In this section, we prove that the equilibrium for CIS setting is similar to that of the single period game with symmetric beliefs. Then, we derive the equilibrium for the single period game with symmetric beliefs.

D.1 CIS Equilibrium is equivalent to the Single Period Game.

Lemma D.1 *When a commitment is made to reveal market structure information at the end of the first period, the first period equilibrium is equivalent to that of the single period game.*

Proof: The total expected profits for any seller i is

$$\begin{aligned} \Pi^{i1}_{\text{CIS}}(p) &= (1-r)(p+1) + r(1-F_{\text{CIS}}(p))(p+0) \\ &= (1-r)p + r(1-F_{\text{CIS}}(p))p + (1-r) \end{aligned} \qquad \text{(App-43)}$$

If the expected profits for all prices are increased by a constant, they can be neglected for the equilibrium computation. Without it, the equation for the first period expected profits is equivalent to that of the single period game with symmetric beliefs. **QED**

D.2 Equilibrium for a Single Period Game

We use the asymmetric game result and set $\alpha = \beta = r$. Therefore, the cdf of the mixed strategy equilibrium is

$$F_j(p) \equiv F_i(p) \equiv F_{\text{CIS}}(p) = 1 - \frac{(1-p)(1-r)}{p\, r} \qquad \text{(App-44)}$$

This is defined over $[r, 1]$. Note that no mass point exists for either player.

E Proof of Lemma 4.7

Proof: Suppose not: Let a pure pooling equilibrium exists where both types of sellers – high-cost and low-cost – bid $1+\delta$ in the first period. In such a case, the expected profits are

$$\Pi_{pool} = \frac{1}{M} + (1-r) \qquad \text{(App-45)}$$

This expression is explained in the following manner. All M sellers bid $1+\delta$ in the first period.

Therefore, the first period expected profit is $\frac{1}{M}$ (ignore δ because of its smallness). In the second period, since both low-cost sellers continue to be uncertain about the existence of the other, their belief about being the only low-cost seller continues to be the same as in the first period i.e., $(1-r)$. From lemma 4.1, we know that the expected profits are equal to $(1-r)$ for both sellers.

Instead, consider the case when one seller deviates from the pooling equilibrium by bidding 1 in the first period. In the process, the seller secures an expected profit of 1 for the first period but reveals its cost-structure to its opponents. In such a case, the second period is an asymmetric game. The seller that bid 1 and won the first period is uncertain if it is the only low-cost seller in the reverse-market. It believes that it is so with a probability of $(1-r)$. But, the losing low-cost seller, if it exists, is certain about the presence of the low-cost winner. In this game, the expected profits for each seller is the same and equal to $(1-r)$ (using lemma 4.1). Therefore, the total expected profits for the seller that deviates from the pooling equilibrium are

$$\Pi_{deviate} = 1 + (1-r) \tag{App-46}$$

Since $\Pi_{deviate} > \Pi_{pool}$, sellers have an incentive to deviate from a pooling equilibrium. This implies that only a semi-pooling equilibrium exists where sellers alternate between faking and playing a price $p \leq 1$. **QED**

F Proof for Lemma 4.8

Let us begin by defining the first period expected profits corresponding to the condition when sellers choose not to fake (this happens with a probability of $1-\gamma$). If $F(p)$ represents the cdf of the bid distribution according to which prices $p \leq 1$ are bid, the expected profits for seller B are

$$\begin{aligned} \Pi_B(p) &= (1-r)(p+\gamma') + r\,\gamma(p+\gamma') \\ &\quad +r\,(1-\gamma-F(p))(p+0) + rF(p)(0+0) \end{aligned} \tag{App-47}$$

This expression can be explained as follows. The first term corresponds to the case when seller A is not present and the second term corresponds to the case when seller A is present but bids $1+\delta$. In each case, seller B bids a price $p \leq 1$ and secures profit equal to the price it bid in the first period. In addition, it secures a second period profit of γ'. Therefore, both these cases have $(p+\gamma')$ in their expressions. The third term corresponds to the condition when seller A also bids a price $p \leq 1$ in the first period but seller B is the winner. In this case, seller B secures the price it bid for the first period but bids a Bertrand price in the second period to secure 0 profits. Finally, the last term corresponds to the case when seller A bids a price $p \leq 1$ in the first period and outbids seller B.

Simplifying equation App-47, we get

$$\Pi_B(p) = \{(1-r) + r(1-F(p))\}\,p + (1-r) \tag{App-48}$$

By definition, the expected profits in a pooling-equilibrium at any p is the same as under $p=1$. Subsituting $F(1) = 1-\gamma$, equation App-48 becomes

$$\Pi_B(1) = 2(1-r) + r\gamma \tag{App-49}$$

In addition, we can write the expression for $\Pi_B(1+\delta)$, the expected profit for seller B from faking as

$$\Pi_B(1+\delta) = (1-r)(\frac{1+\delta}{M} + \gamma') + r\,\gamma(\frac{1+\delta}{M} + \gamma') + r(1-\gamma)(0+\gamma') \qquad \text{(App-50)}$$

The first term corresponds to the setting when seller A is not present. This happens with a probability of $(1-r)$, where seller B secures an expected profit of $\frac{1+\delta}{M}$ in the first period and an expected profit of γ' in the second period. The second term corresponds to the setting where seller B has its low-cost opponent also bidding $1+\delta$ in the first period but seller B wins the first period. In the second period, the expected profit for both sellers is γ'. Finally, the third term corresponds to the setting where its low-cost opponent bids a price $p \leq 1$ in the first period.

Substituting the expression for γ' from equation 12, we have (ignoring δ because of its smallness)

$$\Pi_B(1+\delta) = \frac{1-a}{1-a+a\,\gamma} + 1 - a + a\,\gamma\frac{1}{M} \qquad \text{(App-51)}$$

By equating equation App-48 to equation App-49 and equation App-49 to equation App-51, we have two equations which can be used to solve for γ and $F(p)$. The solution is:

$$F(p) = 1 - \frac{(1-r)(1-p) + r\gamma}{r\,p} \qquad \text{(App-52)}$$

$$\gamma = \frac{-(3M-2)(1-r) + \sqrt{M(1-r)(4(M-1)+M(1-r))}}{2\,(M-1)r} \qquad \text{(App-53)}$$

QED