Mathematical Models for Reconstruction Planning in Urban Areas

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Abstract
In the aftermath of a natural disaster, urban planners need to decide how neighborhoods should be restored under limited resources. An ill-planned reconstruction strategy and implementation process may result in socially inefficient land uses and subject neighborhoods to excessive future risk of disaster damage. In this study, we develop a prescriptive mathematical model for the design of redevelopment strategies that minimize future risk of flood and maximize net social benefit under spatial and equity constraints. This model, based on political districting models, is implemented using mathematical programming and a local search heuristic. Computational results are encouraging both for model outputs and solution efficiency. We generate an approximation to a Pareto frontier that provides decisionmakers with considerable flexibility regarding redevelopment strategies.
I. Introduction

Planning models for post-disaster recovery and reconstruction have received significant attention from operations research and management science. Opricovic et al. (2002) has developed a multi criteria decision model for reducing post-disaster reconstruction cost, which they applied to a post-earthquake reconstruction problem in Central Taiwan. Kozin and Zhou (1990) have examined the lifeline (electricity, water, gas, etc) restoration process immediately after a severe earthquake using Markov decision processes and discrete event simulation. However, these studies are not sufficient for us to apply to our planning problem: traditional administrative region boundaries are inflexible, and generating decision alternatives in MCDM is expensive. What we mean by traditional boundary’s insufficiency is that redevelopment planning in post-disaster areas can not be answered by whether or not to develop existing districts. We need to identify the common characteristics of neighborhoods and aggregate them into new districts in order to assign appropriate land use for those neighborhoods. Thus, we develop planning districts which may or may not be the same as administrative boundary. In planning context it is natural that we have several competing objectives such as politics, environmental planning, and human factors such as residents’ preference. Thus, a multi-criteria approach is essential. However, problem instances can result in millions of alternatives of land aggregation even with a small instance (25 land parcels clustered into 5 districts can create maximum $2.43647 \times 10^{15}$ alternatives). Thus, a mathematical programming approach would appear to be essential.

The goal of this study is to develop a mathematical modeling framework for post-disaster reconstruction planning that addresses issues related to social benefit, environmental preservation and social equity. We solve a stylized mathematical program for post-disaster reconstruction planning, evaluate the results, and discuss computational efficiency, policy relevance, and alternative solution algorithms. Our model can be extended to diverse regions facing post-disaster reconstruction with limited resources.

In real life, reconstruction planning is stochastic multi-phase procedure; decision makers develop the planning strategy in the first phase, and based on the planning outcomes,
make second phase decisions. However, our single phase mathematical modeling is likely to provide sufficient insight to the overall planning strategy by providing the meanings of competing objectives and the interpretation of test results. Hence our research that applies math modeling to post-disaster area restoration planning contributes to the development of new planning approach.

Our study of reconstruction planning for post-disaster urban areas has its motivation in Hurricane Katrina, which hit the U.S. Gulf Coast on August 29, 2005 and resulted in property damage and human suffering unprecedented in American history: Hurricane Katrina was the third-strongest landfalling U.S. hurricane on record and the costliest natural disaster in U.S. history. According to the Federal Emergence Management Agency (hereafter FEMA) flood and damage assessments data as of February 12, 2006, total of 1,197,499 homes were damaged, 515,249 of which were located in Louisiana, 134,564 in Orleans Parish alone. This latter number accounts for 71.5% of the total occupied units in Orleans Parish, which is coextensive with the city of New Orleans (see Appendix A). Orleans Parish is most severely damaged by hurricane Katrina; 105,323 units are major/severely damaged, which counts 51.4% of Louisiana’s major/severely damaged units. In addition, the damage in New Orleans is mainly because most part (80%) of the city was under water for a long term unlike other Gulf coast regions. Kates et al (2006) estimate that reconstruction from Hurricane Katrina may take more than a decade to complete1. Local government and FEMA agree that more than $100 billion will be required to rehabilitate the affected areas.

The cost of natural disaster reconstruction is especially high in urban areas where there is a large concentration of people with a heavy dependency on infrastructure and services (Boulle et al 1997), due to high population densities, concentrations of employment and level of developed infrastructure. Hence post-disaster urban reconstruction requires strategic planning and close attention to the needs in preferences of displaced residents.

Conventionally, urban planning aims to develop a spatial structure of land uses which is preferred to the pattern that would exist without planning (Hall, 1975). There are

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1 Historically, reconstruction phase takes ten times longer than emergency period and restoration period (Kates et al, 2006).
multiple applications of operations research that are salient to urban planning: land allocation/acquisition, reserve design, and political districting. We discuss these below to motivate our solution approach.

*Land allocation model* seeks to select land parcels to minimize total cost and adverse environmental impacts subject to spatial constraints (Wright *et al* 1983, Gilbert *et al* 1985). Diamond and Wright (1991) applied the implicit enumeration method to the land acquisition model. Also, it is introduced as an optimization problem by Wright, ReVelle, and Cohon (1983), which is multiobjective (cost minimization, total area maximization, and compactness maximization) zero-one programming.

*Reserve design* is the process of choosing land parcels to be preserved for endangered species protection (Camm *et al*, 2002) or nature reserves. This is done by selecting one or more clusters of land parcels whose structure protects certain ecological features. Model objectives focus on benefit maximization or cost minimization, with a secondary focus on equity. The reserve design and land allocation/acquisition problem are closely related since land allocation models incorporate many of the same spatial attributes important to reserve design, such as proximity, compactness, and contiguity (Williams *et al*, 2005).

The *districting problem* is to provide a strategy to generate land clusters (districts) that address certain spatial and equity concerns. The school districting problem and facility districting problems (Keeney, 1972) show that the districting problem can be applied to the problems in which public benefit and fairness are central. We discuss details of each of these categories in section IV.

All of these approaches are relevant, but none, alone, are sufficient to solve our problem. The reserve design and the land allocation problem does not take exhaustive land selection into account. To solve our problem, we need to select all parcels in the target area either for human use or for passive land use, which will be clustered with adjacent parcels with the same land use. We believe that the modified political districting model is best suited for adaptation for our problem.

Our mathematical model is not intended to tell administrators and planners how to redevelop specific portions of New Orleans. Instead, our research is intended to help
decisionmakers make better decisions generally; that is, to make targeted neighborhoods better off than they would have been absent this planning by evaluating tradeoffs between various objectives, and addressing impacts of various constraints.

Our model provides redevelopment strategies for neighborhoods damaged by natural disasters as decisions regarding whether to rebuild a set of neighborhood for human habitation or not. The alternative to human habitation is for the land to be uninhabited, and perhaps improved somewhat for recreation or environmental safety purposes. In principle, a social decision maker would assign land uses to neighborhoods to maximize the aggregate viability of the entire region, i.e. the ability of the neighborhoods under consideration to provide adequate shelter and economic opportunities to allow population growth and social vitality.

We construct and evaluate a neighborhood development model to maximize the total viability (net social benefit) of neighborhoods through assignment of land uses under a number of constraints that include budget, contiguity and spatial orientation. *Net Social Benefit* indicates social benefit minus social cost (Boardman et al, 2001). This quantity is hard to estimate with the incomplete data typical of post-disaster reconstruction. In our problem, net social benefit specifically addresses desire of displaced residents to return to their former neighborhoods, as well as *neighborhood viability* which we model as a function of population, average income level, and physical/social services.

We assume that the study area is a square grid composed of identically-sized cells called *land parcels*. Each parcel is assumed to be assigned to a single land use that is either human use or passive use. *Human use* denotes residential and commercial uses; *passive use* denotes parkland, wetland or undeveloped brownfields. Section II will validate this assumption.

*Contiguity* refers to the requirement that some neighborhoods with common land uses be adjacent to each other. Contiguity is important in two respects: land preservation and economies of scale. Enforcing contiguity in neighborhoods assigned to passive use may increase the protective capacity of these neighborhoods in case of a flood. Also, assigning the same land use to adjacent neighborhoods may result in a total redevelopment cost that is less than that associated with separate development efforts in
non-adjacent neighborhoods. Solving the neighborhood development problem without land contiguity constraints, e.g. as an assignment problem would likely result in a “checkerboard pattern” pattern of land uses that is suboptimal in terms of aggregate viability.

We assume that each land parcel is associated with a district in which all constituent parcels share same land use. In other words, all land parcels in the area are subject to selection, assignment to a land use, and spatial constraints. This can be interpreted as dividing the target area to a number of clusters that are mutually exclusive and exhaustive such that each cluster satisfies its desired spatial characteristics. Support for these modeling assertions is provided in section IV.

We apply our model to data from New Orleans, simplified appropriately. Also, we test a greedy heuristic as an alternative solution, and present the result in later section. Preliminary results indicate that human habitat land uses should be assigned to the land parcels of higher elevation and higher net social benefit. Also, consistent with modeling assumption, each of the resulted districts has same land use, and connected land parcels. The southern part of the city of New Orleans, which is close to Mississippi river, is recommended for redevelopment by the model. This area has higher elevation relative to other parts of the city, high pre-Katrina population, and more schools and hospitals opened post-Katrina. That means those areas have more demand from people, less damaged by the flood, and perhaps more viable.

The structure of the remainder of the paper is as follows: In section II, we address the impact of disaster, and reconstruction efforts that have been made since Katrina from the social and urban study perspective, and include controversial issues in New Orleans reconstruction planning. Section III illustrates our model’s development with respect to environmental science’s perspective, as well as Louisiana’s coastal preservation project. Section IV introduces the details of our model formulation. Section V explains about test data used followed by computational results in section VI. We discuss and compare the test results in Section VII. Section VIII concludes and identifies future study directions.
II. Social impacts and Planning Post-disaster Reconstruction

The goals of land use planning are to monitor and guide continuing change to best benefit the community. To do so, land planners must balance three competing sets of land values: social, market, and environmental (Kaiser et al, 1995). In this study, we focus on social and environmental benefits to neighborhoods. Complex issues of environmental preservation are modeled as reducing the risk of future flooding; since we focus on urban redevelopment planning after catastrophic flooding, our modeling goals have focused more on strategies to minimize future risks rather than on environmental restoration.

When planning for reconstruction in urban areas, we need to account for existing ownership of properties, and previous land usage. “Planning” does not necessarily mean the complete reallocation of land use and it has to produce fair benefits across the region. Historically, few communities are abandoned completely after disaster or entirely relocated to new sites for several reasons. First, land ownership is a key motivation for redevelopment. Second, emotional or ideological attachment to place may be quite high. Third, the local population is usually keen to restore the pre-existing pattern of economic activities and social relations in order to regenerate a sense of community. Finally, some places are historically meaningful and are worth rebuilding for that reason alone. New Orleans is a large, modern city, yet very much a localized place where its residents have lived in the same neighborhoods for generations (Hartman and Squires, 2006). Most of previous residents want to come back to the same neighborhood and enjoy its unique cultural characteristics.

Tangible losses associated with floods are represented by physical damage to property including loss of income or services and clean-up costs (Smith 2001, Nott 2006). Intangible losses include physical, emotional and psychological health problems suffered by flood-affected people (Smith 2001, Nott 2006). In the aftermath of Hurricane Katrina, over one million people have been displaced and forced to begin life anew in unfamiliar place a long way – geographically, socially and culturally – from home (Hartman and Squires, 2006). As of October 13, 2005, Katrina victims were scattered widely across the south (see Appendix C). Besides Louisiana (38.6%) and Mississippi (28.3%), other
destinations for Katrina evacuees include Texas (11.6%), Alabama (8.1%), Georgia (2.6%), and Florida (2.6%).

In addition to emotional motivations, there is economic and historical importance in rebuilding New Orleans; the city is the center of a metropolitan area with over $500 billion in real estate assets, and it has 19 districts in National Register of Historic Places\(^2\) with 38,000 properties; as many as 25,000 properties were damaged by hurricane Katrina (Bring New Orleans Back Commission, 2006; hereafter BNOBC).

Unfortunately the development process of post Katrina rebuilding strategy has not been smooth. On September 1, 2006 New Orleans City Council adopted the 3-foot rule (the bottom floor of a house must be at least 3 feet off the ground), which was criticized as ‘poor substitute for a comprehensive rebuilding plan\(^3\)’; FEMA deputy director Doug Bellomo has stated that a new catastrophic event could result in new floodings even with these new rules. BNOBC’s Action Plan on Jan. 19\(^{th}\), 2006 raised contentious issues; foremost of these is the perception that if not enough people come back to the neighborhood, the neighborhood is not worthy to redevelop, and therefore government’s buyout is inevitable. According to the action plan, neighborhoods where less than 50% of the previous residents returned would be razed without question. This plan made people hesitate to return because their effort to rebuild their home might be bulldozed in case they had not enough people in the neighborhood. The final plan was announced on March 29, 2007\(^4\) saying that the city has decided to concentrate resources on 17 zones. This plan is at least the fourth iteration.

An example of the complications of post-hurricane reconstruction is The Broadmoor neighborhood of Orleans Parish. Broadmoor was heavily damaged by Hurricane Katrina, and the neighborhood was a candidate for conversion into green space due to its low elevation and the large damage magnitude according to the BNOBC’s urban planning report. The Broadmoor residents’ association would not accept the plan; and some of the

\(^2\) National Register of Historic Places is the nation’s official list of cultural resources worthy of preservation. [http://www.nationalregisterofhistoricplaces.com](http://www.nationalregisterofhistoricplaces.com/)

\(^3\) USA Today, “Our view on Katrina recovery: New Orleans plan: Rebuild in flood zones, hand you the bill” (Oct 25, 2006)

residents are rebuilding their own home at their own expenses to prove the neighborhood is worthy of redevelopment. Broadmoor is representative of similar incidents happening throughout New Orleans. This implies that we need to consider pre-Katrina population, and other measures to provide an equitable planning framework. We do this by associating net social benefit value with population data, and by enforcing the total net social benefit value of each district must be within an allowed range.

From the social impacts from the disaster and overall reconstruction planning process, in summary we assume that the city of New Orleans should be rebuilt, and all neighborhoods should be evaluated fairly.

III. Environmental Science and Gulf Coast Development

In the land use planning literature, there are several types of land use: residential use, commercial use, open spaces, etc., and these are often competing for the same locations (Carsjens and Van der Knaap 2002). The United Nation’s International Decade for Natural Reduction (hereafter IDNDR: International Strategy for Disaster Reduction) provides policy guidance for development that accounts for risks imposed by natural disasters. One of IDNDR strategies for disaster reduction in human settlements is land use planning (Boulle et al 1997); suitable land uses must be defined and enforced for hazard-prone land. To simplify our planning problem, we define two land use types – human habitation and passive land use – which is a similar categorization to that of Hanink and Cromley (1998).

For ecological and environmental reasons – protect species from negative outside impact, reserve design models and land allocation models emphasize the importance of contiguity of reserved (selected) parcels (Williams et al 2005, Wright et al 1983). It is also important in the agriculture literature; isolated farm parcels that are not contiguous with other farmed parcels often experience negative impacts such a complains from neighbors and lack of support that negatively affect farming operations (Bryant and Johnston 1992, Scarfo 1990) in addition to the management issues associated with farming isolated parcels of land (Brabec and Smith 2002).
We incorporate other spatial configurations in our model; distance to water source and elevation of the land parcels. New Orleans has developed in a delta plain and is bordered by Lake Pontchartrain to the north and the Gulf of Mexico to the east. The city’s nickname, “Crescent City”, denotes the course of the Mississippi river around and through the city. The most vulnerable landscapes for floods are low-lying parts of floodplains, low-lying coasts and deltas, low-lying inland shorelines and alluvial fences (Smith 2001). Therefore New Orleans is highly vulnerable for flood risk. Indeed, New Orleans has flooded nine times between 1735 and 1927; except for the recent hurricanes, flood losses have occurred mainly as a result of rainfall events (NRC 2006).

The 100-year flood plain depends on proximity to water source and the elevation of the land parcels. In addition, flood insurance and floodplain management are associated with the elevation of the region (National Flood Insurance Program, NFIP, by FEMA).

Wetland preservation is one way to reduce flood risk in delta and coastal areas (US Geological Survey: hereafter USGS). Wetlands are vital for absorbing and storing floodwaters, and it provides a natural defense against storm surge. Coastal marshes are particularly valuable for preventing loss of life and property by moderating extreme floods and buffering the land from storms as well as help maintain desirable water quality (USGS National Wetlands Research Center). However, urbanization results in taking up all the unused lands which is risk-prone, hazardous, or environmentally protective and important for ecological perspective, including wetlands. According to the USGS Coastal Louisiana land loss report, Louisiana has lost 1,900 square miles of wetland in the past seven decades (1932 to 2000), an area the size of the state of Delaware, and is predicted to lose approximately 700 square miles of land in the next 50 years. Creating wetland “buffers” between populated areas and water sources may reduce the risk of flood (National Research Council (NRC) 2006).

In summary we assume that land parcels close to the water sources are less preferable for human habitation, lower elevation is less preferable for human habitation, and that parcels of the same use, of the same district are contiguous.
IV. Details of the model

We revisit alternative modeling approaches relevant to post-disaster reconstruction in order to motivate specific features of a novel planning model.

*Reserve design* is the study of designing nature reserve using mathematical optimization models. A fundamental decision to make is which land parcels to select for reserves under spatial constraints. Often time the objectives are to maximize biodiversity within the reserve areas, to minimize reserve cost, or to maximize selected reserve areas, etc. Recently, reserve design models adapt spatial attributes such as contiguity, compactness, reserve size, number of reserves, reserve proximity, reserve shape, core areas with buffer zones, etc (Williams, 2005). Most reserve design models are discrete optimization models because of the nature of the decision, i.e.; whether or not a land parcel is selected for inclusion in a reserve. Depending on spatial attributes, models can be formulated as linear or nonlinear in objective functions and/or constraints.

We test a linear reserve design model (Williams *et al.*, 2005) with four objectives: minimize reserve proximity, maximize reserve connectivity, maximize compactness, and minimize total cost (see Appendix B for full formulation). The test result is illustrated in Figure 1. The reserve design’s spatial constraints such as compactness and contiguity are significant features to our problem. However, the model leaves out the rest of unselected land parcels, i.e. the model does not concern configurations of the unselected areas, which is an important issue to our problem. Also, the model only allows for one type of land use.

The *land allocation* problem addresses the question of how to allocate an area of land for development or other purposes. In most cases, the goal of the problem is to maximize the quality of a particular allocation based on multiple criteria (Gilbert, 1985) such as the cost of land acquisition and development, the proximity of the development to desirable and undesirable land features, and the shape of the allocated land areas. Wright *et al* (1983) present a multiobjective integer programming formulation for the land acquisition problem. The basic model formulation is a three objective integer programming – maximizing acquired area and compactness and minimizing cost. Compactness, often regarded as a nonlinear factor, is represented by the sum of the lengths of external

**Figure 1.** Reserve design (Williams *et al*, 2005) test result

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Weight on distance = 0.5
Weight on connectivity = 0.5
Weight on compactness = 0.5
Weight on cost = 0

**Figure 2.** Land acquisition model (Wright *et al*, 1983) test result

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Weight on cost = 0.5
Weight on compactness = 0.5

The test result of the land acquisition problem based on Wright *et al* (1983)’s formulation applied to regular square grid parcels (with randomly generated acquisition cost) is shown in Figure 2 (see Appendix B for full formulation). Similar to the reserve design problem, the land acquisition model does not take equity concerns and spatial characteristics of the unselected land parcels into account. For our purpose, there exist desired spatial configurations for the unselected areas which can be regarded as passive use of our problem.

**Figure 2.** Land acquisition model (Wright *et al*, 1983) test result

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Weight on cost = 0.5
Weight on compactness = 0.5

The outer part of entire region is regarded as one big parcel (irregular shape), and the parcel number if 17.
The political districting problem seeks to identify a districting plan that avoids gerrymandering. Political districting is the process of partition a region into its subregions for the basis of electoral representation. Desirable conditions of resulting districts include: nearly equal populations, contiguity of parcels within a district, and geographical compactness of each district (Mehrotra, 1998). Constraints consist of allowable deviation of population of a district from the mean district population, shape compactness, exclusion distance between population unit centers, etc. Generally the objective is to minimize total districting cost, or minimize the maximum districting cost (bottle-neck approach).

In the sense that the centers of the districts can be viewed as facilities that are located to serve the population units in the district, political districting problem resembles facility location problems (Mehrotra, 1998). Indeed Hess (1965) modeled political redistricting problem as a location-allocation problem, but solved it by heuristic methods. Garfinkel (1970) solved the districting problem with exact methods, and Hojati (1996) proposed the use of Lagrangian relaxation to determine the centers of the districts and the use of sequence of capacitated transportation problems to resolve the splitting problem. In many cases the solution method consists of two phases: generating potential districts and optimization. Since the number of possible districts is exponentially large, Mehrotra (1998) used column generation method, and Garfinkel (1970) used a “tree search” algorithm whose procedure is to start at an arbitrary unit and adjoin contiguous units until the combined population becomes feasible.

The test result of a simple political model (Hess et al, 1965) by using AMPL/CPLEX is in Figure 3, below (see Appendix B for full formulation).

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5 Gerrymandering is a form of redistricting in which electoral district or constituency boundaries are manipulated for an electoral advantage.
In this model instance 16 parcels are partitioned into five smaller divisions such that land parcels in each division are connected and compact, and the population of each division is nearly equal. This type of method that divides a target region to clusters satisfying desired conditions is what we need to apply to our problem. However, the basic political districting problem does not reflect socio-economic characteristics of the neighborhoods, and does not distinguish different districts according to land use type.

There are multiple extensions to the political districting model, such as maximizing the number of districts where majority population exceeds the minority population. We are interested in an extension in which each district indicates a certain type of land use under the assumption that several districts can be assigned to the same land use type. Another extension of the districting problems useful for our purposes is related to a measure of social equity. In the political districting problem, the districts have nearly equal population size whereas in our problem, we wish to design districts such that the distribution of net social benefit is perceived as equitable.

To summarize modeling insight from this and previous sections, we consider the following issues as central to our math model:

1. The distance between water source and land parcels of passive use is minimal.
2. Parcels in the same district are contiguous.
3. Parcels in the same district are assigned to the same type of land use.
4. Each district is compact.
5. Deviation of net social benefit of each district should be limited such that each
district has fairly equalized net social benefit.

As a result, parcels assigned to human land use should be far from the water source, have
higher elevations, and have higher net social benefit values. Also, by minimizing
external borders we expect to maximize compactness of each district. External border is
defined as a border between two different districts. In our formulation, we assume that
the study area is a square grid composed of equally-sized parcels. Thus, we assume that
the border length is unity.

Index and Data

A neighborhood (parcel) is the smallest unit of land, and a district is a collection of these
neighborhoods (parcels). The entire study area is composed of several districts, each
with a designated land use.

\[ i = \text{Index of neighborhood} \ (i = 1, 2, ..., N) \]

\[ j = \text{Index of districts} \ (j = 1, 2, ..., D) \]

\[ k = \text{Index of the land use type} \ (k = \text{“H” human habitat, k = “P” passive use}) \]

\[ D = \text{The number of districts required} \]

\[ D_p = \text{The number of districts of passive land use} \]

\[ C = \text{Total funds available for redevelopment} \]

\[ \alpha = \text{percentage of allowable deviation of social benefit from average social benefit} \]

\[ d_i = \text{Euclidean distance from the nearest water source to a neighborhood} \ i \]

\[ e_i = \text{Elevation level of a neighborhood} \ i \]

\[ c_{ik} = \text{Cost of reconstruction in parcel} \ i \ \text{for land use} \ k \]

\[ s_i = \text{Net social benefit of neighborhood} \ i \]

\[ M = \text{’Big M’} \ (= \text{total number of parcels}) \]

\[ A_i = \text{Set of parcels which are adjacent to parcel} \ i \]

\[ w_1 = \text{Weight on the distance objective} \]

\[ w_2 = \text{Weight on the elevation objective} \]

\[ w_3 = \text{Weight on the connectivity objective} \]
$w_4 = \text{Weight on the net social benefit objective}$

**Decision Variables**

$x_{ijk} = 1$ if the $i^{th}$ neighborhood is assigned to the $j^{th}$ district, and land use $k$, 0 otherwise.

$B_j = 1$ if the $j^{th}$ district is to be developed for Passive use, 0 otherwise.

$P_{ij} = 1$ if $\sum_k x_{ijk} = 1$ and $\sum_k x_{ijk} = 0$, 0 otherwise. This indicates an external border.

$N_{ij} = 1$ if $\sum_k x_{ijk} = 1$ and $\sum_k x_{ijk} = 0$, 0 otherwise. This indicates an external border.

**Model**

**Minimize**

\[ w_1 \sum_{i=1}^{N} \sum_{j=1}^{D} d_i x_{ijp} - w_2 \sum_{i=1}^{N} \sum_{j=1}^{D} e_i x_{ijH} + w_3 \sum_{i=1}^{N} \sum_{k=1}^{D} \sum_{j=1}^{D} (P_{ij} + N_{ij}) + w_4 \sum_{i=1}^{N} \sum_{j=1}^{D} s_j x_{ijH} \]  

(1)

**Subject to**

\[ \sum_j \sum_k x_{ijk} = 1 \quad \forall i \]  

(2)

\[ \sum_i x_{ijp} \geq \sum_k \sum_k x_{ikh} - (1 - B_j) \times M \quad \forall j \]  

(3)

\[ \sum_i x_{ijH} \geq \sum_k \sum_k x_{ijk} - B_j \times M \quad \forall j \]  

(4)

\[ \sum_i s_j x_{ijH} \geq (1 - \alpha) \sum_{i=1}^{N} \sum_{j=1}^{D} s_j / D \quad \forall j \]  

(5)

\[ \sum_i s_j x_{ijH} \leq (1 + \alpha) \sum_{i=1}^{N} \sum_{j=1}^{D} s_j / D \quad \forall j \]  

(6)

\[ \sum_i x_{ijk} - \sum_k x_{ijk} - P_{ij} - N_{ij} = 0 \quad \forall i, \forall l \in A_i, \forall j \]  

(7)

\[ \sum_j \sum_k c_{jk} x_{ijk} \leq C \]  

(8)

\[ \sum_{j=1}^{D} B_j = D_p \]  

(9)

\[ \sum_i \sum_k x_{ijk} \geq 1 \quad \forall j \]  

(10)

$x_{ijk} : \text{binary} \quad \forall i, \forall j, \forall k$  

$P_{ij}, N_{ij} : \text{binary} \quad \forall i, \forall l \in A_i, \forall j$  

$B_j : \text{binary} \quad \forall j$  

(11)
The first term of objective function (1) minimizes the distance between the water source and passive land use parcel. The second term maximizes the elevation level of parcels to be developed for human habitat. The third term of objective function minimizes the total number of external borders of districts. This will result in districts that are as compact as possible and ensures that parcels in the same district are connected. The last term of objective maximizes the total net social benefit of human use land parcels. Under our definition of net social benefit, parcels with higher net social benefit value are preferred for human habitation.

Constraints (2) enforce that each land parcel (neighborhood) is assigned to only one district and only one land use type. Constraints (3) require that all land parcels that belong to the same district have to be assigned to the same land type. Constraints (5) require that the deviation of the risk aversion level of each district from the average risk aversion level should lie within an allowable range, defined by percentage $\alpha$. Constraints (6) define external border variables $P_{ij}$ and $N_{ij}$ used to enforce compactness. Constraint (7) restricts the total cost of reconstruction to the given budget. Constraint (8) defines the total number of districts of passive land use. Constraints (9) ensure that each district has at least one land parcel. Constraints (10) – (12) ensure that all decision variables are binary.

The size of our problem instances depends on the cardinality of decision variables and constraints. The number of decision variables is at most $ndk + d + 2n(n - 1)d$ where $k$ is the number of land use types, $n$ is the number of land parcels, and $d$ is the number of districts. The number of constraints is $n^2 - (n - 5)d + n + 2$. The number of districts and the number of land use types are typically much smaller than the number of land parcels ($d \ll n, k \ll n$). Thus, it would appear that the size of our model grows by $O(n^2)$. However, regular square grid parcels have at most 4 adjacent parcels. Thus, the cardinality of decision variables is at most $ndk + d + 8nd$, and the number of constraints is $n + 4nd + 5d + 2$. Therefore, with regular square grids, the problem size increases by $O(n)$. All of our computational experiments are based on 25 square grid parcels of the same size, five districts, and two types of land use. This problem size leads to 1055 binary decision variables and 451 constraints after pre-processing. If we increase the
number of parcels to 100 (10 by 10 square grids), the problem incident has 4605 binary decision variables and 1926 constraints.

Since all decision variables are binary, and the equity constraint (5) is known to be hard to solve in political districting literature, our model is likely to be difficult to solve as the problem size grows. The political districting problem is a class of the set partitioning problem (Balas and Padberg 1976), which is known to be NP-hard. Our problem adopts political districting and adds land use assignment, thus our model is NP-hard. Likewise, our problem is a class of spatial zoning problem, and the spatial zoning problem⁶ which is NP-hard (Wei and Chai, 2004) as well.

For a large problem size, a heuristic approach may provide reasonable trade-off between solution time and solution quality. The heuristic algorithm is the following:

**Index and Data for heuristic algorithm**

We use similar notation for heuristic to our mathematical model.

\[ i = \text{Index of neighborhood (} i = 1, 2, \ldots, N \text{)} \]
\[ j = \text{Index of districts (} j = 1, 2, \ldots, D \text{)} \]
\[ k = \text{Index of the land use type (} k = \text{“H” human habitat, } k = \text{“P” passive use)} \]
\[ D = \text{The number of districts required} \]
\[ \alpha = \text{percentage of allowable deviation of social benefit from the average net social benefit} \]
\[ \bar{d}_i = \text{Normalized Euclidean distance (ranging from 0 to 1) from the water source to a neighborhood } i, \text{ the higher the better for human use.} \]
\[ \bar{e}_i = \text{Normalized elevation level (ranging from 0 to 1) of a neighborhood } i, \text{ the higher the better for human use.} \]
\[ \bar{s}_i = \text{Normalized social net benefit value of neighborhood } i, \text{ the higher the better for human use.} \]

**Step 1: Normalization and obtain achievement level.**

---

⁶ Spatial zoning is defined as to draw lines for a boundary, which partition geographical zones with territory subject to some side constraints (Wei and Chai 2004, Bozkaya et al 2003, Wang and Bong 2001).
In this phase, we normalize all data and calculate achievement level for human use and passive use parcels. For human use, we prefer longer distance to the water source, higher elevation, and higher net social benefit value, thus we normalize the data at following:

\[
\text{Normalized elevation of parcel } i, \quad \overline{e}_i = \frac{e_i - e_{\min}}{e_{\max} - e_{\min}}
\]

and similarly for other attributes.

For passive use, parcels with lower elevation, close to water, lower net social benefit value will be assigned. Thus we conduct normalization as following:

\[
\text{Normalized elevation of parcel } i, \quad \underline{e}_i = \frac{e_{\max} - e_i}{e_{\max} - e_{\min}}
\]

and similarly for other attributes.

Achievement is obtained by simple summation of normalized data.

\[
\text{Achievement for human use, } \overline{a}_i = \overline{d}_i + \overline{e}_i + \overline{s}_i
\]

\[
\text{Achievement for passive use, } \underline{a}_i = \underline{d}_i + \underline{e}_i + \underline{s}_i
\]

Step 2: Compose Human use districts

Choose a parcel of the highest human use achievement, and combine its neighborhoods until it satisfies the stopping rule, which is the allowable range of overall net social benefit average.

\[
\text{Average} = \frac{\sum_{i=1}^{N} s_i}{D}
\]

where \( s_i \) is the unnormalized net social benefit value of parcel \( i \) and \( D \) is the total number of districts. If it yields an enclave\(^7\), backtrack the parcel which has been just added.

Step 3: Compose Passive use districts

---

\(^7\) Enclave (Nemhauser and Garfinkel, 1970): by adding a new parcel, the solution may result in an isolated parcel, or a district which cannot satisfy any constraints.
Combine rest of the parcels to construct passive use districts. Choose the parcel with the highest passive use achievement level and keep adding neighboring parcels until the district satisfies the stopping rule. The backtrack and stopping rules are the same as step 2.

Our heuristic has complexity \( O(n^3) \); with 25 parcels the running time is \( 5.40635 \times 10^{-2} \) seconds, thus the computation time is expected to be 3.46 seconds with 100 parcels, which is still fairly low.

V. Data

We use distance to the water source, elevation level, and net social benefit for the model. Recall that we use square grid parcels, thus the unit border length is assumed to be 1. For more realistic application with irregular grid or rectangular grids, we can simply multiply the real border length to our external border decision variables \((P_{ij}, N_{ij})\).

The geographic location of water sources is based on New Orleans map in which the northern part is Lake Ponchartrain, and southern part of the main city is Mississippi river. To simplify the problem we ignored the canals and eastern sea. Since we use unit border length, we also use simplified Euclidean distance between land parcels and water source. For example, parcel 1 through 5 are adjacent to the lake Ponchartrain, we use distance 1 for those parcels although the real distance from the center of each parcel to the water source varies between 0.7 and 1.6 miles.

As well as the distance data, elevation level of each parcel is based on the elevation map of Orleans parish (Figure 4). We match the elevation data to the proximate integers ranging from -4 to 4 as seen in Table 1. Although our test data do not use exact measures of elevation, we assigned the proximate levels on each parcel on the basis of mapping with the real elevation map such that the test result can give us insight for the solution to the real world problem.
Values for net social benefit cannot be extracted in any obvious way from administrative data. Under our definition of net social benefit, we have used the pre-Katrina population and income data, post-Katrina open school and open hospital data as our input to measure net social benefit. We normalized the population, income, number of school, and Euclidean distance from the neighborhood center to the operating hospital. Our test measure, consists of the unweighted sum of the four normalized input data, is validated by visual representation on a map. In Figure 5, the neighborhoods with higher measure are near the southern of the city, consistent with socio-economic characteristics of the city. Net social benefit values range between 0 and 3, and we discretize the values such that the value lies between 1 and 5. By reducing the data range, we can improve computational efficiency. Data summary is found in Table 1.

Figure 5. Estimated Net Social Benefit of each neighborhood in Orleans Parish

<table>
<thead>
<tr>
<th>Elevation</th>
<th>measured</th>
<th>value used</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4 ~ -1.25</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>-1.25 ~ -0.5</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-0.5 ~ 0</td>
<td>-2</td>
<td></td>
</tr>
<tr>
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<td>2 ~ 3</td>
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<td>3 ~ 4</td>
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<tr>
<td>4 to 8.5</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Lighter shades: the higher NSB measure, therefore preferred for human habitation.
Darker shades: the lower NSB measure, therefore preferred for passive land use.
Table 1. Data table

<table>
<thead>
<tr>
<th>Distance (mile)</th>
<th>Elevation</th>
<th>NSB</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>measured</td>
<td>proximate</td>
<td>measured</td>
<td>proximate</td>
</tr>
<tr>
<td>0.7~1.6</td>
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<td>-4 ~ -1.25</td>
<td>-4</td>
</tr>
<tr>
<td>1.7~3</td>
<td>2</td>
<td>-1.25 ~ -0.5</td>
<td>-3</td>
</tr>
<tr>
<td>3.0~4.5</td>
<td>3</td>
<td>-0.5 ~ 0</td>
<td>-2</td>
</tr>
<tr>
<td>4.0~5.0</td>
<td>4</td>
<td>0 ~ 0.5</td>
<td>-1</td>
</tr>
<tr>
<td>5.0~6.0</td>
<td>5</td>
<td>0.5 ~ 1</td>
<td>0</td>
</tr>
<tr>
<td>6.0~7.0</td>
<td>6</td>
<td>1 ~ 2</td>
<td>1</td>
</tr>
<tr>
<td>7.0~8.0</td>
<td>7</td>
<td>2 ~ 3</td>
<td>2</td>
</tr>
<tr>
<td>8.0~9.0</td>
<td>8</td>
<td>3 ~ 4</td>
<td>3</td>
</tr>
<tr>
<td>9.0~10.0</td>
<td>9</td>
<td>4 to 8.5</td>
<td>4</td>
</tr>
</tbody>
</table>

Discrete value ranging between 1 and 10

Our allowable deviation of net social benefit (α) is set to 25%. We believe that this value is likely to give enough difference of NSB between passive land use and human land use, while ensures that the differences in NSB between districts are not dramatically large; Mehrotra et al (1998) mention that population range (which is the similar concept of net social benefit in our problem) is extremely narrow, there is typically no feasible plan in which the population units are not split or, if there is a feasible plan, it is likely that the districts would not be compact. Also, a restrictive constraint increases the computational burden.

The number of districts is pre-defined for model simplicity. It is an arbitrary number that we can easily change. By running several tests each time with different number of districts we can see the different results. We use 5 districts in our test.

VI. Computational Results

We tested our model on a 5 by 5 square grid land parcels using AMPL/CPLEX 10.0.0. An initial test result, using values of unity for all objective function weights (Figure 6) shows that land parcels assigned to the same district are also assigned to the same land use, and those parcels are contiguous. Also, parcels in the higher elevation region (southern part of the target area) are assigned to human habitation, and each district is reasonably sized due to the equity constraint.
Different objective weights result in different solutions (Figure 7). When minimizing the distance between passive land use and water source (weight: 1, 0, 0, 0 in order) to assign land parcels nearby water to passive use, a minimal number of parcels are assigned to the passive land use; as shown in panel (a) in Figure 7. This result occurs because a small number of passive use parcels barely satisfying NSB deviation constraint will result in the minimal value of total distance to the water source if those parcels are adjacent to the water source. When we maximize the elevation of parcels for human land use (weight: 0, 1, 0, 0 in order), the solver selects highly elevated areas for human land use and all the rest of parcels are all for passive land use (panel (b) in Figure 7). Panel (c) in Figure 7 is the result from maximizing compactness (weight: 0, 0, 1, 0 in order). The solution does not account for elevation or distance, but all parcels in the same district are connected and the shape of each district is compact. When maximizing the total NSB of parcels for human land use (panel (d) in Figure 7, weight: 0, 0, 0, 1 in order), many parcels are assigned to the human land use. Since NSB objective does not account for the elevation or distance, it assigns low-lying land parcels that are close to water source (parcel 2 and 3) to the human land use. Except for panel (c), we obtain checkerboard pattern solutions.
Thus compactness objective function must be included in any reasonable, policy-relevant solution.

**Figure 7. Objective weights test results**

(a) Weight on distance objective

<table>
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<tr>
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<th>3</th>
<th>4</th>
<th>5</th>
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<tr>
<td>w^1 = 1, w^2 = w^3 = w^4 = 0</td>
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</table>

(b) Weight on elevation objective

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(c) Weight on compactness objective

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(d) Weight on NSB objective

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</table>

In addition to the above test results, we can see the different objectives are competing by value path graph\(^8\) (Figure 8); no the test result is dominated for across all objectives. Overall test 1 appears to be a promising candidate for a compromise solution.

\(^8\) Value path is a line drawn for each non-inferior alternative to indicate the level of achievement of each objective (Cohon, 1978).
All AMPL/CPLEX test results are summarized in Table 2. First, we tested our model with NSB data ranging between 1 and 10 without rounding RHS constant of NSB deviation constraint (constraint 4 and 5). With unity weights, it takes 24 seconds to solve. The second test is conducted with rounded RHS constant and scaled down NSB values ranging between 1 and 5. It dramatically reduced running time, the number of simplex iterations, and branch-and-bound nodes.

If we wish to relax optimality to reduce running time, then we can use heuristic approach that does not guarantee the optimality, but is capable of solving large combinatorial problems. Heuristic result shown in Figure 9 is similar to our exact IP solution from unity weights; overall the parcels in the upper side are assigned to the passive land use, and the ones in the lower part are assigned to the human land use. It returns solution in less than a second whereas exact IP runs around 10 seconds (with unity weights) in AMPL/CPLEX.
Figure 9. Heuristic test result

VII. Discussion

As discussed previously, our problem is NP-hard, thus with large instances it is hard to solve the problem by an exact unified mathematical model. Although the global optimum is not guaranteed, heuristic method can quickly solve the problem. In practice, there are many other factors that affect the planning decisions; it is likely that in real world problem, mathematically optimal solution could provide the overall framework but not the exactly best solution to implement. In fact, it is almost impossible to find “the best solution” for the complicated real world problem like New Orleans reconstruction plan. Thus heuristic approach can provide enough basis of the planning schema to planners without solving the problem in optimum. However, it is also promising that our exact optimization method can be used; as a NP-hard integer program, unexpectedly it yields practical amount of computational time for some weight combinations. These running times may be further improved by applying computational integer programming techniques such as identifying strong inequalities or facet-defining inequalities.

Note that it is also possible to break this problem into a two phase problem that makes the original problem more easily solvable; phase I generates feasible districts, and phase II selects optimal districts from the potential districts generated from phase I. The first phase is in many cases solved by heuristics approach - when we have a large number of neighborhood cells, we will need column generation technique to generate feasible districts - and the second phase is a single objective LP. To illustrate, the two steps are:
### Table 2. Computational Result

<table>
<thead>
<tr>
<th>Test</th>
<th>Run No.</th>
<th>Objective Weights</th>
<th>Run time (sec.)</th>
<th>Weighted Objective value</th>
<th>Component Objectives</th>
<th>Number of simplex Iteration</th>
<th>Number of B-B nodes</th>
<th>Number of Cuts</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>w₁ w₂ w₃ w₄</td>
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<td>Dist. Elev. Comp NSB</td>
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<td>21 13 28 39</td>
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1: Give more weight on feasibility than optimality
2: Cover cut option is likely to be used
3: Gomory fractional cut is likely to be used
4: GUB cut is likely to be used
5: Do not use any cuts

When no cuts used, the running time, the number of simplex iterations, and the branch-and-bound nodes are significantly reduced. It seems that our problem easily yields binary solution without adding cuts which is unlikely to happen in integer programming.
Phase I: generate feasible districts

Start at an arbitrary parcel and check which land use achievement level is higher for the parcel. Adjoin adjacent parcels where the same land use type is preferred until the total social net benefit becomes feasible. This implicit enumeration method is used in Garfinkel and Nemhauser (1970) to generate feasible political districts.

Phase II: choosing M districts among a set of generated districts, $J$

$D_j = 1$ if the $j^{th}$ district is selected, 0 otherwise

$S_j =$ Social net benefit of district $j$ from the redevelopment

$a_{ij} = 1$ if neighborhood $i$ is included in district $j$

Maximize $\sum_{j \in J} S_j D_j$

Subject to $\sum_{j \in J} D_j = M$

$\sum_{j \in J} a_{ij} D_j = 1 \quad \forall i$

Implementation of this method is a subject for future research.

As noted earlier, the fourth attempt of New Orleans redevelopment plan was announced in the end of March 2007. According to the plan, 17 targeted recovery zones encompass public assets in key business corridors in order to accelerate recovery and attract investors. Edward Blakely\(^9\) said “the development will spur activity from investors. When one area starts to do well, investors will want to invest nearby. This will allow the city to redevelop wisely and will help residents make smart choices about where to rebuild.” Instead of reassignment of land use, the city plan suggests homeowners to elevate their houses, and focus on developing some population centers. The plan takes the Orleans’ post-Katrina status and spatial structure only in terms of elevation into account.

VIII. Conclusion

This research is motivated by New Orleans’ redevelopment planning in the aftermath of hurricane Katrina. Reconstruction planning for New Orleans, the most urbanized area in

\(^9\) Dr. Edward Blakely, Executive Director of Recovery Management for the City of New Orleans.
Louisiana, needs to address several issues: spatial location, especially distance to water source, elevation of the land parcels, socio-economic level, and selection of parcels that are contiguous and compact.

Our model is a multi-objective integer programming subject to equity and spatial constraints, which adopts the political districting problem. The problem is NP-hard, thus it is hard to solve by exact method as the problem instance increases. As an alternative method, heuristic algorithm yields competitive results to the exact optimization method. Although the exact optimization model is computationally intensive for known NP-hard problems, our model is promising because the computational cost dramatically decreases by scaling parameters.

Computational results indicate that alternative development strategies constitute a non-dominated set. With unity weights, the solutions from the exact optimization method and heuristic are competitive. Test results from both methods tend assign the upper parts of New Orleans to passive land use, and the lower parts to the human land use. This result is consistent with our assumption that low-lying land parcels and parcels nearby water are better off for passive land use, and parcels with high elevation and high NSB values are better off for human habitat. As a result, some of the wealthy neighborhoods are assigned to passive land use; although it seems adverse to the expectation, “area of higher economic status should be redeveloped for human use”, the result does make sense for several reasons. The neighborhood has low elevation and is adjacent to water source. Also, it has no school opened after Katrina, is far from the operating hospitals, and has low population. This is evidence that our model incorporates important social equity considerations.

We do not expect that planners to use our model solutions directly. However, we hope that this study provides important intuition and guidance to real-world decisionmakers attempting to provide assistance to damaged neighborhoods and displaced residents.

There are a number of promising extensions to this work as well. For example, our model may change significantly if we allow neighborhoods to have irregular shapes, if we allow more than two land use options, and if we concern other types of risk as well as flood. In addition, a follow up study may develop a two-phase method to solve the planning problem, or identify facet-defining inequalities for the problem that accelerate computational efficiency.
References


C. Hartman and G. D. Squires, “There is no such thing as a natural disaster – Race, Class, and Hurricane Katrina” *Routledge* New York (2006)


Appendix A: New Orleans and New Orleans Metropolitan Area

For U.S. Census purposes, this New Orleans MSA includes seven parishes: Jefferson, Orleans, Plaquemines, St. Bernard, St. Tammany, St. Charles, and St. John the Baptist.

The Louisiana state legislature created a commission, the Regional Planning Commission, to be responsible for the planning and development of the New Orleans metropolitan area. The parishes covered by the commission are: Jefferson, Orleans, Plaquemines, St. Bernard, and St. Tammany.

The Center for Cultural and Eco-Tourism at the University of Louisiana at Lafayette has designated the New Orleans metropolitan area as the **Greater New Orleans** region. This region includes four parishes: Jefferson, Orleans, Plaquemines and St. Bernard.

Satellite image of the New Orleans metro area (7 parishes): Wikipedia – New Orleans metropolitan area. The area surrounded by red line is Orleans parish, the city of New Orleans.
Appendix B: Details of Mathematical Models

**Reserve design model** refer to Williams et al (2005)

- Minimize reserve proximity: distance between land parcels to be selected
- Maximize reserve connectivity: contiguity of land parcels to be selected
- Maximize compactness: minimize the boundary length of the reserve
- Minimize total cost: minimize the total cost of selecting (or preserve, acquire) land parcels

\[ b_i = \text{the boundary length of site } i \]
\[ b_{ij} = \text{the length of the shared boundary between adjacent sites } i \text{ and } j \]
\[ d_{ij} = \text{distance between candidate sites } i \text{ and } j \]
\[ M = \text{total number of sites to be acquired} \]
\[ w_1 = \text{weight on the sites proximity} \]
\[ w_2 = \text{weight on the reserve connectivity} \]
\[ w_3 = \text{weight on the compactness of the reserve} \]
\[ w_4 = \text{weight on the total cost of acquisition} \]
\[ c_i = \text{cost of acquiring site } i \]
\[ x_i = 1 \text{ if site } i \text{ is selected, 0 otherwise} \]
\[ u_{ij} = 1 \text{ if candidate site } i \text{ and } j \text{ are both selected, 0 otherwise} \]

Minimize

\[ w_1 \sum_{i=1}^{M} \sum_{j>i} d_{ij} u_{ij} - w_2 \sum_{i=1}^{M} \sum_{k>i} u_{ik} + w_3 \left( \sum_{i=1}^{M} b_i x_i - 2 \sum_{i=1}^{M} \sum_{k>i} b_{ik} u_{ik} \right) + w_4 \sum_{i=1}^{M} c_i x_i \]

Subject to

\[ \sum_{i=1}^{M} x_i = M \]
\[ u_{ij} \geq x_i + x_j - 1 \]
\[ x_i, u_{ij} = \text{binary} \]

**Land allocation / acquisition model** refer to Wright et al (1983)

- Minimize land acquisition cost
- Minimize total external borders

\[ c_i = \text{the cost of acquiring cell } i \]
\[ S_{ij} = \text{the length of the border between cell } i \text{ and } j \]
\[ w_b = \text{the weight on the cost objective} \]
\[ w_c = \text{the weight on the compactness objective} \]
\[ T_i = \text{the set of cells adjacent to cell } i \]
\[ n = \text{the number of cells in the feasible area} \]
\( x_i = 1 \) if cell \( i \) is acquired, 0 otherwise

\( P_{ij} = 1 \) if \( x_i = 1 \) and \( x_j = 0 \), 0 otherwise

\( N_{ij} = 1 \) if \( x_j = 1 \) and \( x_i = 0 \), 0 otherwise

Minimize \[
    w_b \sum_{i=1}^{n} c_i x_i + w_c \sum_{i=1}^{n} \sum_{j \neq i} S_{ij} (P_{ij} + N_{ij})
\]

Subject to
\[
    \sum_{i=1}^{n} x_i = M \\
    x_i - x_j - P_{ij} + N_{ij} = 0 \quad \forall i, j \in T_i \\
    x_i, P_{ij}, N_{ij} = \text{binary} \quad \forall i, j \in T_i
\]

**Political Districting model** refer to Hess et al. (1965)

- Minimize moment of inertia

The following model creates five districts composed of 16 population units. The allowable population deviation is \( \alpha = 0.25 \) (25\% from the average population of districts). Assume that only population unit centers will be district centers.

\( P_i = \) population of the \( i \)th population unit

\( d_{ij} = \) distance between centers of population units \( i \) and \( j \)

\( \alpha = \) percentage of allowable population deviation from average population

\( M = \) the number of districts

\( x_{ij} = 1 \) if the \( i \)th population unit is assigned to the \( j \)th center

Minimize \[
    \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}^2 P_{ij} x_{ij}
\]

Subject to
\[
    \sum_{j=1}^{n} x_{ij} = 1 \quad \forall i \in \{1, 2, \ldots, n\} \\
    \sum_{i=1}^{n} x_{ij} = M \quad \forall j \in \{1, 2, \ldots, n\} \\
    \sum_{i=1}^{n} P_i x_{ij} \geq (1 - \alpha)(\sum_{i=1}^{n} P_i / M) \times x_{ij} \quad \forall j \in \{1, 2, \ldots, n\} \\
    \sum_{i=1}^{n} P_i x_{ij} \leq (1 + \alpha)(\sum_{i=1}^{n} P_i / M) \times x_{ij} \quad \forall j \in \{1, 2, \ldots, n\}
\]
Appendix C: Population of displaced Katrina’s victims (October 2005)

Source: FEMA Census Bureau, Great GNODC