

Explaining Fashion Cycles: Imitators Chasing Innovators in Product Space

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Abstract

This paper considers the problem of a fashion trend-setter confronting an imitator who can produce the same product at lower cost. A one-dimensional product space is considered, which is an abstraction of the key attribute of some consumer good.

Three broad strategies can be optimal for the fashion-leader: (1) Never innovate; milk profits from the initially advantageous position but ultimately concede the market without a fight. (2) Innovate once but only once, which just temporarily defers conceding the market. (3) Cycle infinitely around product space, never letting the imitator catch up and capture the market. Sometimes the cycles start immediately; sometimes the innovator should wait for a time before beginning the cycles.

The optimal solution exhibits strong state-dependency, with so-called Skiba curves separating regions in state space where various of these strategies are optimal. There are even instances of intersecting Skiba curves. In most cases, analytical expressions can be stated that characterize these Skiba curves.

1 Introduction

This paper considers the problem of a fashion trend-setter that has to deal with an imitator who can produce the same product at a lower cost. A one-dimensional product space is considered, which is an abstraction of the key attribute of some consumer good. An obvious example is the width of neckties, which we all have observed to vary over time. Other examples might include

the extent to which accessories are flashy or understated, the width of labels on sport coats, or the length of skirts. The trend-setter defines what is fashionable, and off-label brands imitate them.

This paper suggests a novel explanation for the existence of fashion cycles, namely movement around a 'product space' that is strategic on the part of the fashion-setter and imitative on the part of low-cost competitors. In reality the product design space is of very high dimension, leaving lots of room for complex trajectories that never settle down to a single steady state. We will show, however, that such complexity is not essential to this story. Even in a one-dimensional abstraction of that space, the optimal solution may involve continual adaptation and imitation. Furthermore, a side benefit of sticking with a one-dimensional product space is that we can write explicit expressions describing the thresholds between initial conditions where different strategies are optimal.

It turns out that the structure of the optimal solution depends on how much it costs to develop new designs and on the initial positions of both firms in the product space. For low design costs it is optimal for the firms to cycle around the product space indefinitely, with a Skiba curve separating the two possible directions for changing the design initially.

For intermediate values of the cost, two other Skiba curves circumscribe an area in the middle of the product space where it is optimal for the fashion setter not to innovate, at least initially. In that case either the imitator simply catches up and conquers the whole market or the fashion setter changes its design later.

If the investment cost of making new designs is still larger, two other Skiba curves arise, which separate a policy of "never changing the design" from a policy where the market leader makes an initial major design change but no subsequent changes.

The paper is organized as follows. Section 2 reviews some relevant literature. The model is presented in Section 3. A short overview of the solution structure is provided in Section 4. Section 5 analyzes the solution structure when the costs of making new designs are so low that periodically changing designs forever is optimal. In Section 6 these costs are large enough that it is not optimal to change the design more than once. Section 7 considers parameter constellations where, depending on the initial situation, either periodic design changes or making no new designs at all can be optimal.

2 Literature Review

Firms engage in at least two kinds of product design innovation: technological innovation and stylistic or fashion innovation (Schweizer, 2003). The former improves the product. E.g., computers today are faster than they were ten years ago. Fashion innovation differentiates what's new from current models without improving functionality. For women's clothes, red may be "in" this year and blue may be "out", but that does not mean red is intrinsically better than blue. Certainly the color red is not a new invention per se. Furthermore, it is likely that in a few years blue will be in and red will be out, and a few years

after that, red will be in again. The same happens for the width of men's ties, the length of skirts, and the popularity of one material relative to another.

In short, we observe that consumers are willing to pay more for one good (the "in" good) than another, functionally equivalent product (the one that is "out"). Economists have long been fascinated by this counter-intuitive behavior, dubbed "Veblen effects" in honor of Thorstein Veblen's seminal inquiries into conspicuous consumption (Bagwell and Bernheim, 1996). Typical explanations involve "status" or "prestige" goods conferring utility on their consumers by allowing implicit association with other high-status consumers of that good. If a good is so expensive that only the rich can afford it, then onlookers can infer that anyone they see consuming it must be rich. Allowing others to make that inference may bring various benefits to the conspicuous consumer. A variety of economic models have been developed under which it is optimal for consumers to behave in this way (e.g., Bikhchandani et al., 1992; Coelho and McClure, 1993; Bagwell & Bernheim, 1996; Frijters, 1998; Corneo and Jeanne, 1999; Bianchi, 2002).

Here we take this consumer behavior as given, rather than trying to "explain" it within a rational actor framework, and instead ask how firms might manage fashion innovation in order to exploit this behavior in order to maximize profits. There is already a large management science/operations research literature providing practical guidance to fashion goods producers, but it does not treat the rate of fashion innovation as the decision variable of interest. Rather, these papers address manufacturing (Degraeve and Vandebroek, 1998; Jain and

Paul, 2001), supply chain management (Donohue, 2000; Mantrala & Rao, 2001; Milner & Kouvelis, 2002), inventory policy (Fisher et al., 2001), pricing (Zhao and Zheng, 2000), and other management issues that arise in the context of a given fashion innovation's product life cycle.

Likewise there is a literature advising firms how quickly to make technological innovations that improve product functionality (e.g., Paulson Gjerde et al., 2002), but little has been written about how to manage the rate of fashion innovation.

A partial exception is Swann's (2001) case study of the evolution of two prestige cars, the Rolls Royce and the Ferrari. However, that is more of an interesting descriptive analysis of a particular case than an effort to derive prescriptive insights from a general model.

The closest analog in the literature to the current investigation is Pesendorfer's (1995) innovative paper. As in our model, Pesendorfer's fashion producer dynamically optimizes the timing of fashion innovations and, finds, as do we, that the optimal solution could involve introduction of new fashions at fixed, regular intervals whose period varies inversely with the cost of innovation. There are three significant differences, however. First, Pesendorfer explicitly models the behavior of two discrete types of individuals and the producer's decision about how to vary the price of a given fashion over time. In that sense, Pesendorfer's focus is on creating a rational-actor model of fashion that includes the incentives of producers not just consumers, whereas, again, we just take consumer's taste for fashion (meaning products differentiated from low-cost al-

ternatives) as a given.

Second, Pesendorfer thinks of innovations as discrete. At some fixed unit cost, the innovator can create a new design that is completely differentiated from the current design, and renders the current design instantly and completely obsolete. However, it seems more realistic to think of a product design space. Innovation implies moving one's product within that space, and one could move a little (minor innovation) or a long way (major innovation). One or the other might turn out to be optimal, depending on the particular circumstances, but the model should recognize that the producer has that choice, rather than assuming arbitrarily that all innovation must necessarily be draconian.

We address only a single-dimensional product space, which is clearly an abstraction. Even a simple item of clothing has multiple attributes (color, fabric, length, texture, etc.). Still, we view allowing even a one-dimensional continuum to be an advance. Related to this, we view the cost of innovation as increasing in the "amount of innovation". Consider test marketing, for example. Understanding how consumers will react to modest variations might be relatively easy, but accurately predicting the response to a radical change might require more market research. Likewise, it might take more advertising to persuade people that what initially seems like a very extreme departure from current trends will in fact become *de rigeur*. Indeed, Barnett and Freeman (2001) found, albeit in the context of technological not fashion innovation, that firm mortality rates increase with the simultaneous introduction of multiple significant innovations.

The third and most important difference between our model and Pesendor-

fer's pertains to the existence and behavior of other producers. Pesendorfer considers the effects of competition, but focuses on a monopolistic producer. Furthermore, even in the competition case, Pesendorfer (1995, 773) did not "*allow imitation of successful designs. Imitation would give designers an additional incentive to create new fashions periodically. Clearly imitation is an important force behind the creation of new designs. However, through the creation of brand names, designers can at least partially insulate themselves from competition with potential imitators. In this paper I consider the case in which the designer has well-defined property rights over his innovations.*"

Pesendorfer's no-imitation case is no doubt of interest, but so is allowing imitation because "knocking off" expensive designers is pervasive and because protecting intellectual property rights concerning fashion goods can be difficult, at least in the US. (Some European countries may have stronger protections.) Fashion innovations are, by the definitions used here, ineligible for patent protection because they are not a new invention that advances beyond the prior art in a non-obvious way. Something like reintroducing the color mauve in 2003, when it was popular in the 1980s but fell out of favor in the 1990s clearly does not meet that test. Likewise copyright protection cannot be afforded in the US to "useful articles", so it can only protect design elements that can be identified separately from and can exist independently of the utilitarian aspects of the article. Practically, that means that fashion designs in apparel (as opposed to accessories) are hard to copyright. Finally, although defending a fashion trademark against counterfeiting is relatively straightforward in the

courts, defending trade dress against imitation is more difficult. (The Lanham Act differentiates between "trademarks", which are words, emblems, logos, or symbols used to identify goods and distinguish them from those sold by others and "trade dress", which refers to the product's overall image or appearance including shape, size, color, packaging, and marketing.)

To give a concrete example, Abercrombie & Fitch sued American Eagle Outfitters in 1998 for "intentional and systematic copying of its brand, images and business practices, including its merchandise, marketing and catalog" (Seiling, 1998). However, both the lower court and the Pennsylvania Sixth Circuit Court of Appeals sided with American Eagle Outfitters because the clothing designs for which Abercrombie & Fitch sought protection were functional as a matter of law and therefore not protectable under trade dress (Catalog Age, 2002). Abercrombie & Fitch then recently filed suit seeking just to prevent American Eagle Outfitters from using the number '22' on its clothing, arguing that it had a common law trademark on that number (Associated Press, 2003). That suit has yet to be resolved, but even if Abercrombie & Fitch wins, it would only affect a minor aspect of American Eagle Outfitters' alleged imitation.

In our model we assume there is a single innovative "fashion czar" that defines what is fashionable within the product space. This is an outcome Pe-sendorfer found to be among the plausible competitive equilibria. Essentially if all consumers believe that only the fashion czar is capable of creating fashion, then this will be the equilibrium outcome. The fashion czar's product is imitated by low-cost producers who are not strategic about their fashion inno-

vation. That is, the fashion czar might invest in costly activities that support innovation such as "cool hunting" (gathering intelligence about trend-setting consumers' preferences, see, e.g., Gladwell, 1997), "depth test" marketing preliminary designs with bellwether groups (e.g., Fisher and Rajaram, 2000), or advertising heavily to mold expectations about what will be "in" (Pastine & Pastine, 2002). The imitators, in contrast, follow the simpler, low-cost strategy of continually adapting their designs to conform to those of the fashion czar, whatever those designs are.

Note two differences with some articles in the literature. First, some models of fashion cycles are based on innovation and imitation by consumers who are conformists or non-conformists with regard to purchasing decisions (e.g., Matsuyama, 1992). Here it is producers who innovate or imitate. Second, portions of the fashion cycle literature assume that unit production costs are identical for "in" and "out" products since they are functionally identical; in our model it is allowable and perhaps even more reasonable to think of the fashion czar as having higher production costs.

Thus we imagine a market populated by heterogeneous firms. One firm optimizes, at some nontrivial information processing cost; others follow heuristic strategies that are cheaper to implement. This structure is akin to that of Conlisk (1980) and Sethi & Franke (1995), but our model is continuous time and the producers' decisions pertain to product design, specifically where they position their products in some product space.

3 The Model

In the model X represents the decision maker's position in some consumer product space, and Y represents the position of a competitor in that same space. The notion of a product space here is a one-dimensional abstraction of the key attribute of some consumer good. E.g., it could be width of neckties. The design house (e.g. Polo Ralph Lauren or Boss) defines what is fashionable with high mark-up ties. Off-label brands imitate them. The design (tie width) is constantly changing as the high mark-up labels seek to distinguish themselves from the low cost providers.

The imitator's corporate strategy is simply to imitate the market leader (X), so Y always chases X . Pesendorfer (1995) suggests that one can think of Y not just as a single follower, but rather as a group of followers, which motivates the absence of strategic behavior on their part. Not only do they lack market research capability and other prerequisites to strategic behavior, but they may also each be relatively small, making it hard to amortize fixed costs of such strategic infrastructure. For this reason our problem is formulated as an optimal control problem, not a dynamic game.

Thus the system dynamics for this model would be simply:

$$\dot{X} = u, \tag{1}$$

$$\dot{Y} = X - Y, \tag{2}$$

where u is the control variable.

The product space is constrained by zero and one (neckties of infinite width make no sense), so that

$$0 \leq X \leq 1. \tag{3}$$

Since Y just follows X , expression (3) implies that Y is also effectively constrained to be between zero and one without needing to make this explicit.

The fashion leader's objective function balances the cost of fashion innovation against the benefits of sales, where sales are highest if the fashion-leader's product is well-differentiated in product space from the low-cost imitator's product. For simplicity we assume that profits grow as the square of the distance between the innovator's and imitator's products. Note sales and profits in this model do not depend on the absolute location of either firm in product space since fashion is not useful per se, except to differentiate (Pesendorfer, 1995). Extensions in which consumers care not only about differentiation but also about the absolute position in product space could be an interesting topic for further research. The greater the rate of innovation, the more costly that innovation is. In particular, it is assumed here that the cost of innovation is linear in the rate of innovation. Hence, the fashion-setter's objective function is:

$$\max \int_0^\infty e^{-rt} \left[\frac{1}{2} (Y - X)^2 - c|u| \right] dt. \tag{4}$$

and the decision maker seeks to optimize expression (4), subject to the system dynamics (1)-(2) and the state constraint (3).

As an alternative to expression (2) we could have chosen the formulation

$$\dot{Y} = k(X - Y),$$

with $k > 0$ measuring the speed of convergence. However, by an appropriate time transformation it can be shown that increasing k has the same effect as jointly increasing the switching cost c and decreasing the discount rate r . Hence, nothing is lost by normalizing k equal to 1, which has the advantage that the number of parameters in the model is reduced.

4 Properties of Optimal Solutions

Before beginning detailed analysis, it is useful to make some observations about the nature of optimal solutions to this problem. Most are self-evident or require only a brief explanation, but collectively they help delineate the space within which one must search for optimal solutions.

Proposition 1 *If it is optimal to exercise control, it is optimal to jump all the way to a boundary ($X = 0$ or $X = 1$).*

This follows from the observation that the cost of moving is linear in the distance moved, but the benefit is convex. An immediate implication is that once the fashion leader has reached a boundary, the leader will subsequently always be at one boundary or the other.

Proposition 2 *Once the fashion leader has reached a boundary, and hence*

will always be at a boundary, then by symmetry the problem becomes a one-dimensional dynamic program whose state is the distance D the imitator is away from that boundary, with value function V^* which satisfies

$$V^*(D) = \max\{D^2 dt + e^{-rdt} V^*(De^{-dt}), -c + e^{-rdt} V^*(1 - D)\}$$

Proposition 3 *Assume the fashion leader is at a boundary ($X = 0$ or 1). Let D_0 be some distance such that when the imitator is that distance away, the fashion setter prefers jumping to the opposite boundary over staying in place. Then for all $D < D_0$, the fashion setter would also rather jump to the opposite boundary than stay in place.*

Proof. This is because the cost of moving is the same while the revenue is higher when $D < D_0$, i.e., $V(D, \text{jump}) > V(D_0, \text{jump})$. On the other hand, clearly, $V(D, \text{stay}) < V(D_0, \text{stay})$, while by definition $V(D_0, \text{stay}) < V(D_0, \text{jump})$. ■

Corollary 4 *If it is ever optimal to jump away from a boundary, it is optimal to continue to jump forever.*

This follows from Proposition 3 because all jumps leave the fashion leader at the boundary, and eventually the shadow approaches that boundary arbitrarily close.

Corollary 5 *If it is optimal to jump more than once, it is optimal to jump forever.*

This follows from Proposition 1 (all jumps, including the first, are to a boundary) and Corollary 4.

Corollary 6 *When the fashion leader is at a boundary, the optimal strategy is fully characterized by a single distance parameter D_0 . If the imitator's distance from the boundary $D \leq D_0$, then the fashion leader should jump immediately. Otherwise, the fashion leader should wait until D decreases to D_0 and then jump.*

Corollary 7 *Only seven strategies are candidates for optimality:*

- 1) *Never moving*
- 2) *Jumping once to $X = 0$*
- 3) *Jumping once to $X = 1$*
- 4) *Jumping forever, with the first jump to $X = 0$*
- 5) *Jumping forever, with the first jump to $X = 1$*
- 6) *Waiting for some time and then jumping forever, with the first jump to $X = 0$*
- 7) *Waiting for some time and then jumping forever, with the first jump to $X = 1$.*

In the next sections we show that all these seven strategies actually occur for some parameter values.

Proposition 8 *Strategy pair #2/#3 and quadruple #4/#5/#6/#7 are incompatible in the following sense. For any given set of parameters, if there exist initial conditions such that any of strategies #4, #5, #6 or #7 is optimal, then there do not exist initial conditions such that strategies #2 or #3 are optimal.*

To see this, note that if either Strategy #4, #5, #6 or #7 are optimal for some set of initial conditions, then there must exist a D_0 such that $V(D_0, \text{jump}) >$

$V(D_0, \textit{stay})$. Since if the fashion leader were to jump only once, eventually D would become less than D_0 , making it no longer optimal to stay.

As will be illustrated below, all other combinations of strategies can co-exist. That is, for all other combinations of strategies, there exist parameter values such that any one of those strategies can be optimal depending on the initial conditions. What is striking is that analytic expression can be written fully characterizing most of the boundaries (so-called DNS thresholds) separating the regions where each of the alternative strategies is optimal. These boundaries are found by equating the value functions computed under each candidate optimal strategy. It is to the computation of those value functions that the discussion turns next.

5 Solution structure for low unit cost

To begin the analysis suppose that the market leader's product is initially at one of the boundaries of product space and, without losing generality, suppose it is at the lower end, $X = 0$, and $Y < 1/2$. (If initially $X = 0$ and $Y > 1/2$ then any innovation would be both costly and revenue-reducing, so clearly the fashion-setter should do nothing at least until $Y < 1/2$.) If the cost of making new designs is low, then when the imitator gets close enough to 0, the decision maker's position in the product space will move from zero to one. Later, when the imitator comes sufficiently close to one, the trend-setter will jump from one back to zero as fast as possible. Since the control variable appears linearly in

the optimization problem and is unbounded, these movements take the form of discrete jumps. In this way the solution structure depicted in Figure 1 is obtained.

This section contains two subsections. In the first subsection the properties of the cycle in Figure 1 are investigated. By doing this we obtain an upper bound on the unit cost for which this solution structure is in fact optimal. The aim of the second subsection is to find all points (X, Y) at which the decision maker is indifferent between jumping to zero and jumping to one. It turns out that all these points are situated on a curve, which in the optimal control literature is called a Skiba-curve. This literature was initiated by Skiba (1978), who in a one state optimal control model detected a threshold (the *Skiba point*) at which the decision maker is indifferent between either converging to a positive steady state or converging to zero. Haunschmied et al. (2003) extended this analysis to a two state optimal control model so, as in the present paper, due to the extra dimension, the Skiba point becomes a Skiba curve. For other recent research concerning Skiba points and Skiba surfaces the reader is directed e.g. to Wagener (2005a, b) and Deissenberg et al. (2004).

5.1 Properties of the Cycle

On the cycle it holds that either $X = 1$ or $X = 0$ (see Figure 1). Let Y_0 be the position of the imitator in the product space at which the decision maker is indifferent between staying at zero or jumping from zero to one, and, analogously, Y_1 is the imitator's position for which the market leader is indifferent between

staying at one or jumping from one to zero. Defining T_1 to be the length of the time interval at which $X = 1$, and T_0 is the time interval length at which $X = 0$, it can be obtained from expression (2) that

$$\begin{aligned} X &= 1, Y = 1 - (1 - Y_0)e^{-t} & \text{for } 0 < t < T_1, \\ X &= 0, Y = Y_1 e^{T_1 - t} & \text{for } T_1 < t < T_1 + T_0. \end{aligned}$$

Only the relative positions in the product space matter, which implies that

$$\begin{aligned} T_1 &= T_0 = T, \\ Y_1 &= 1 - Y_0. \end{aligned}$$

Since Y has the same value at the end of the interval where $X = 1$, and at the beginning of the interval where $X = 0$, it holds that:

$$e^{-T} = Y_0 (1 + e^{-T}). \quad (5)$$

Next, we determine Y_0 by choosing that value of Y_0 that maximizes the objective. Due to the one-to-one correspondence implied by (5), this also gives the cycle length $2T$. . Evaluating the objective on the initial interval $[0, T]$ (starting with $Y(0) = Y_0$ and $X(0) = 1$) gives:

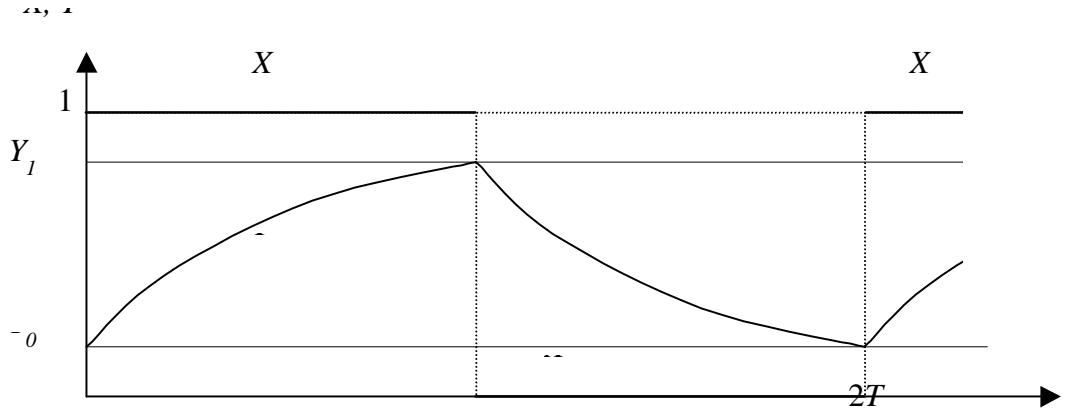


Figure 1: The optimal periodic solution structure.

$$\int_0^T e^{-rt} \left(\frac{1}{2}(1-Y)^2 \right) dt - e^{-rT}c = (1-Y_0)^2 \frac{1-e^{-T(r+2)}}{2(r+2)} - e^{-rT}c.$$

For reasons of symmetry, excluding discounting, the objective has the same value on the second interval $[T, 2T]$ (starting with $Y(T) = 1 - Y_0$ and $X(T) = 0$) as on the first interval, and this also holds for all consecutive intervals of time length T . Therefore, the objective value becomes:

$$\begin{aligned} V &= \sum_{n=0}^{\infty} e^{-nrT} \left((1-Y_0)^2 \frac{1-e^{-T(r+2)}}{2(r+2)} - e^{-rT}c \right) \\ &= \frac{1}{2(r+2)} \frac{(1-Y_0)^{r+2} - Y_0^{r+2}}{(1-Y_0)^r - Y_0^r} - \frac{Y_0^r}{(1-Y_0)^r - Y_0^r} c \end{aligned} \quad (6)$$

The first order condition eventually leads to

$$c = \frac{1}{r(r+2)} \left(\frac{Y_0^{r+2}}{(1-Y_0)^{r-1}} - \frac{(1-Y_0)^{r+2}}{Y_0^{r-1}} + \frac{r - 2Y_0(r-1) - 6Y_0^2 + 4Y_0^3}{2} \right). \quad (7)$$

This equation implicitly determines Y_0 as a function of the parameters r and c , which is depicted in Figure 2. From this figure it can be concluded that the market leader will not change the design very often (Y_0 is low) if the cost of changing the design, c , is large. The same holds for the relation between Y_0 and the discount rate, because a large discount rate implies that the decision maker is more influenced by the immediate costs of design change. Furthermore, the figure shows that for large discount rates, Y_0 depends heavily on the rate, while for smaller discount rates Y_0 is insensitive to changes in the discount rate. In fact, in this figure the curve for $r = 0.001$ could not be distinguished from that for $r = 0.01$.

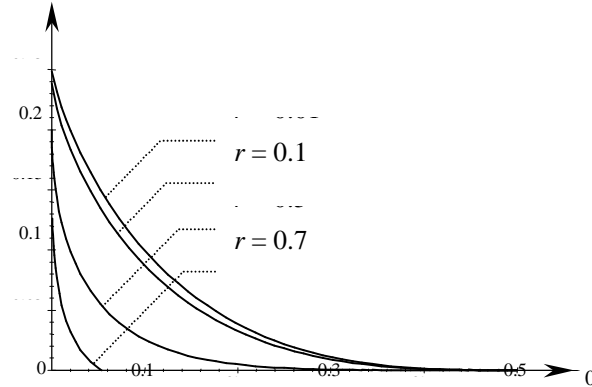


Figure 2: Optimal Switching Threshold Y_0 as a function of c and r .

Clearly it holds that $Y_0 \leq 1/2$ for all $c > 0$. For very small values of c the following result is established.

Proposition 9 *i) When changing the design is costless, i.e. $c = 0$, the threshold Y_0 equals $1/2$.*

ii) However, for very low values of c , Y_0 approaches $1/2$ only for $r \leq 0.5$, i.e.,

$$\begin{aligned}\lim_{c \rightarrow 0} Y_0(c) &= 0.5 \quad \text{for } r \leq 0.5, \\ \lim_{c \rightarrow 0} Y_0(c) &< 0.5 \quad \text{for } r > 0.5.\end{aligned}$$

Proof. From (7) it can be obtained that $Y_0 = 1/2$ for $c = 0$. Furthermore, due to this same expression it can be shown that for $Y_0 = 1/2$ it holds that $\frac{dc}{dY_0} = 0$, $\frac{d^2c}{dY_0^2} = 0$, and $\frac{d^3c}{dY_0^3} = 16(r - \frac{1}{2})$. Thus, if $r > 0.5$ the c -curve in Figure 2 lies below the Y_0 -axis for values of Y_0 less than but close to $1/2$. Since $c = 1/2(r + 2) > 0$ for $Y_0 = 0$, this proves that $\lim_{c \rightarrow 0} Y_0(c) < 0.5$ for $r > 0.5$.

■

Result i) is intuitively plausible since – if changing the design is costless – the market leader’s strategy is simply always to maximize the distance between X and Y , which leads to chattering around $1/2$. It is clear that $Y_0 = 0.5$ implies that the cycle length equals zero. Hence, the proposition implies that

$$\begin{aligned}\lim_{c \rightarrow 0} T(c) &= 0 \quad \text{for } r \leq 0.5, \\ \lim_{c \rightarrow 0} T(c) &> 0 \quad \text{for } r > 0.5.\end{aligned}$$

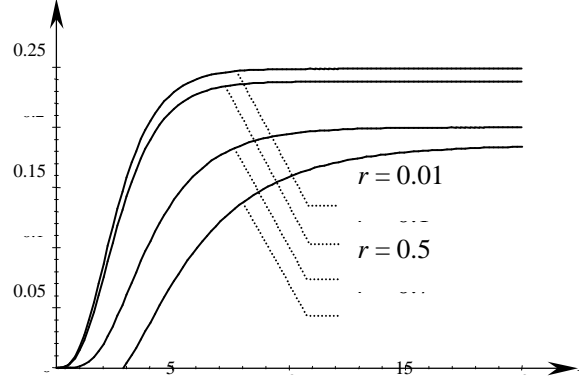


Figure 3: Optimal Interval Length T .

Result ii) is illustrated in Figure 2 for $r = 0.7$, where $\lim_{c \rightarrow 0} Y_0(c) = 0.053$. Apparently, for very large discount rates, immediate switching is not optimal even when the cost of design change is almost zero. For the sake of illustration, optimal interval lengths T for various parameter values are depicted in Figure 3.

The following result provides an upper bound on the cost of changing the design above which it is not optimal to have a solution structure as depicted in Figure 1.

Proposition 10 *Exactly for*

$$c > \frac{1}{2(r+2)} \quad (8)$$

Y_0 does not exist and for $c = \frac{1}{2(r+2)}$ it holds that $Y_0 = 0$.

Proof. By letting $Y_0 \rightarrow 0$ in (7), we obtain that $c = \frac{1}{2(r+2)}$. From this same

expression it can be derived that a non-negative value for Y_0 does not exist for $c > \frac{1}{2(r+2)}$. ■

For $c \geq \frac{1}{2(r+2)}$ the cost of changing the design is so expensive that the same design is kept forever when the decision maker finds itself at one of the boundaries of the product space ($X = 0$ or $X = 1$). One simply stays there while the imitator's product becomes more and more similar. This implies that the decision maker's revenue decreases over time.

In the example with $r = 0.7$, $r = 0.5$, $r = 0.1$, and $r = 0.01$, the maximum value of c for which periodic design change is optimal is $c = 0.1852$, $c = 0.2$, $c = 0.2381$, and $c = 0.2488$, respectively. If the discount rate is large the decision maker is reluctant to incur immediate costs. Therefore, the upper bound on c above which a solution with frequent design change does not occur, goes down as r increases.

Note that Proposition 10 does not imply that designs will not be changed at all for $c > \frac{1}{2(r+2)}$. If initially the market leader's product is in the interior of the product space ($0 < X < 1$), an initial design change such that X jumps to one of the boundaries and then stays there may still be optimal. This possibility will be explored in Section 6.

5.2 Skiba Curve

The aim of this subsection is to find a curve in the (X, Y) -plane on which the fashion trendsetter is indifferent between choosing the design $X = 0$ or $X = 1$. By definition the outside points of this curve are $(0, Y_0)$ and $(1, 1 - Y_0)$. It is

also clear that $(1/2, 1/2)$ must be part of the Skiba curve.

To find an analytical expression for the whole curve we consider an arbitrary point (\bar{X}, \bar{Y}) , with $Y_0 < \bar{Y} < 1 - Y_0$. Then we evaluate the objective values for an immediate upward jump to 1 and an immediate downward jump to 0. Those points (\bar{X}, \bar{Y}) , for which both values are equal, belong to the Skiba curve.

First we consider an immediate upward jump from \bar{X} to 1. On the initial time interval, say $0 < t < \bar{t}$, Y increases from \bar{Y} to $1 - Y_0$. Then, from that moment on the solution structure depicted in Figure 1 applies, the objective value of which is given by V (see (6)). This implies that the value of the objective, $V^{up}(\bar{X}, \bar{Y})$, corresponding to "jumping upward" is

$$V^{up}(\bar{X}, \bar{Y}) = -c(1 - \bar{X}) + \int_0^{\bar{t}} e^{-rt} \left(\frac{1}{2}(1 - Y)^2 \right) dt - e^{-r\bar{t}}c + e^{-r\bar{t}}V$$

which can be rewritten as

$$V^{up}(\bar{X}, \bar{Y}) = -c(1 - \bar{X}) + \left(\frac{Y_0}{1 - \bar{Y}} \right)^r (V - c) + \frac{1}{2}(1 - \bar{Y})^2 \frac{1 - \left(\frac{Y_0}{1 - \bar{Y}} \right)^{r+2}}{r + 2}. \quad (9)$$

Analogously, we consider an immediate downward jump from \bar{X} to 0 and evaluate the objective value

$$V^{down}(\bar{X}, \bar{Y}) = -c\bar{X} + \left(\frac{Y_0}{\bar{Y}} \right)^r (V - c) + \frac{1}{2}\bar{Y}^2 \frac{1 - \left(\frac{Y_0}{\bar{Y}} \right)^{r+2}}{r + 2}. \quad (10)$$

To obtain the Skiba curve, we equate the objective values of jumping upwards

and downward. In the appendix it is obtained that this gives:

$$\begin{aligned} \bar{X} = & -\frac{1}{4(r+2)c} \left[(1-\bar{Y})^2 \left(1 - \left(\frac{Y_0}{1-\bar{Y}} \right)^{r+2} \right) - \bar{Y}^2 \left(1 - \left(\frac{Y_0}{\bar{Y}} \right)^{r+2} \right) \right] \\ & + \frac{1}{2} \left(\left(\frac{1}{\bar{Y}} \right)^r - \left(\frac{1}{1-\bar{Y}} \right)^r \right) \frac{Y_0^r}{(1-Y_0)^r - Y_0^r} \left(\frac{(1-Y_0)^{r+2} - Y_0^{r+2}}{2(r+2)c} - (1-Y_0)^r \right) + \frac{1}{2}. \end{aligned} \quad (11)$$

This is the Skiba curve, although we note that there is no explicit expression for Y_0 . Instead, it is implicitly given by (7). For the parameter values

$$r = 0.1 \quad \text{and} \quad c = 0.1, \quad (12)$$

the Skiba curve is depicted in Figure 4.

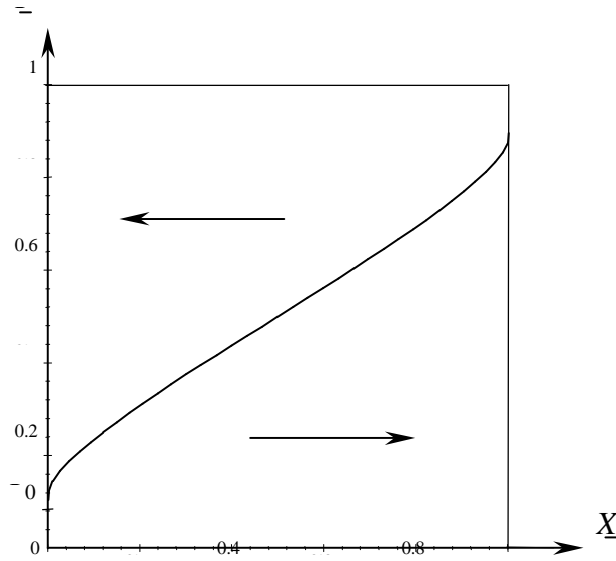


Figure 4: Skiba curve for $r = 0.1$ and $c = 0.1$.

6 Solution structure for large unit cost

From Proposition 10 the cyclical solution structure of Figure 1 cannot be optimal when the cost of changing the design is large, i.e. when $c > \frac{1}{2(r+2)}$. This implies that then it can never be optimal to change the design in such a way that X jumps from a level lower than Y to a level that is higher than Y , or vice versa. After excluding such design changes, three candidate policies are left, namely:

- starting from a situation where $\bar{X} > \bar{Y}$, jump up to $\bar{X} = 1$ but make no subsequent design changes in the future. The value of the objective that results from this policy is

$$V^{1up}(\bar{X}, \bar{Y}) = -c(1 - \bar{X}) + \frac{1}{2}(1 - \bar{Y})^2 \frac{1}{r+2}.$$

- starting from a situation where $\bar{X} < \bar{Y}$, jump down to $\bar{X} = 0$ but make no subsequent design changes in the future. Then the value of the objective is

$$V^{1down}(\bar{X}, \bar{Y}) = -c\bar{X} + \frac{1}{2}\bar{Y}^2 \frac{1}{r+2}.$$

- stay at \bar{X} , which gives

$$V^{stay}(\bar{X}, \bar{Y}) = \frac{1}{2}(\bar{Y} - \bar{X})^2 \frac{1}{r+2}.$$

The fashion trend-setter is indifferent between making only an initial design change leading to an upward jump of X and making no change at all, when

$V^{1up}(\bar{X}, \bar{Y})$ equals $V^{stay}(\bar{X}, \bar{Y})$. This leads to the following Skiba-curve:

$$\bar{Y} = \frac{1}{2}\bar{X} - \left(c(r+2) - \frac{1}{2}\right).$$

This is an upward sloping line, which lies below the 45^0 line, because the policy with the initial upward jump can only occur if $\bar{X} > \bar{Y}$. The Skiba curve only occurs in the relevant region if $\bar{X} < 1$ for $\bar{Y} = 0$, which leads to the conclusion that this Skiba curve only exists in case

$$c < \frac{1}{r+2}.$$

Being indifferent between an initial downward jump or refraining from any design change, thus equating $V^{1down}(\bar{X}, \bar{Y})$ and $V^{stay}(\bar{X}, \bar{Y})$, gives the following Skiba-curve:

$$\bar{Y} = \frac{1}{2}\bar{X} + c(r+2).$$

This is an upward sloping line above the 45^0 line. This curve is only relevant if $Y < 1$ for $X = 0$, which again gives $c < \frac{1}{r+2}$.

The results are summarized in the following proposition.

Proposition 11 (a) Consider the c -region $\frac{1}{2(r+2)} < c < \frac{1}{r+2}$. Then the optimal policy is

- $\bar{X} < 2\bar{Y} - 2c(r+2)$: make an initial design change equivalent to a downward jump to $X = 0$ in the one dimensional product space. Then refrain from doing any changes afterwards.

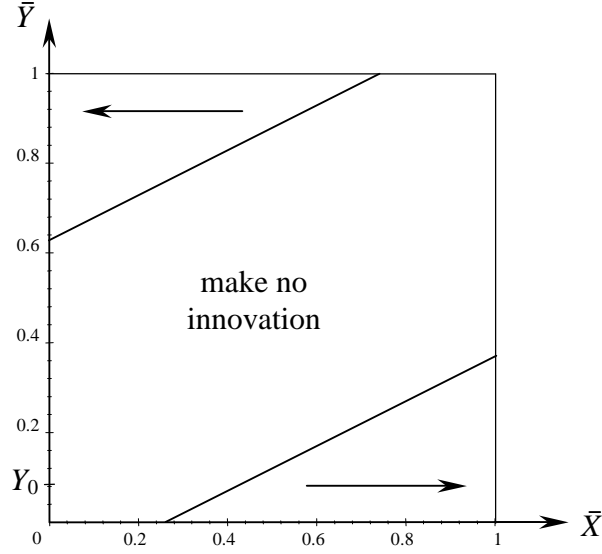


Figure 5: Skiba curves for $r = 0.1$ and $c = 0.3$.

- $2\bar{Y} - 2c(r + 2) < \bar{X} < 2\bar{Y} + 2c(r + 2) - 1$: *make no design changes.*
- $\bar{X} > 2\bar{Y} + 2c(r + 2) - 1$: *make an initial design change equivalent to an upward jump to $X = 1$. Afterwards make no design changes.*

(b) Consider the c -region $c > \frac{1}{r+2}$. Then the optimal policy is to make no design changes at all.

The next figure illustrates case (a). After choosing $r = 0.1$ we must have $0.2381 < c < 0.4762$ to be in the relevant parameter region. The figure is drawn for $c = 0.3$.

7 Intermediate c

In the hairline case where $c = \frac{1}{2(r+2)}$, it holds that $Y_0 = 0$, and the Skiba curve (11), that separates the two policies of initially jumping up or down followed by periodic design changes as depicted in Figure 1, becomes $\bar{X} = \bar{Y}$. This hairline case is presented in Figure 6.

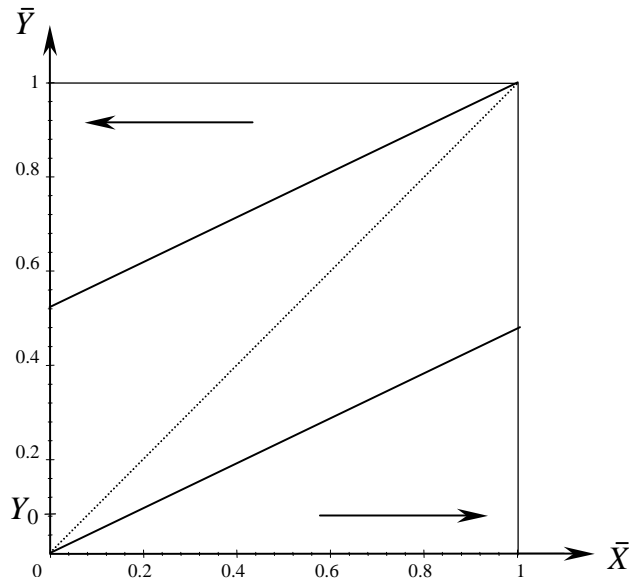


Figure 6: Hairline case $c = \frac{1}{2(r+2)}$ for $r = 0.1$.

It should be noted that the Skiba curve $\bar{X} = \bar{Y}$, denoted by (11), is not really relevant since here the unit cost is too large for periodic design changes to be optimal. Instead, it is better to refrain from doing any design changes when $X = Y$.

This raises the question whether for values of the unit cost c a little bit below $\frac{1}{2(r+2)}$ (implying that Y_0 is close to zero) making no changes at all could

be better than jumping to the cycle for some initial values of \bar{X} and \bar{Y} . After all, in Section 3 we only compared the objective values resulting from periodic design changes after initial upward and downward jumps, without checking their absolute values. In case these objective values are negative, then a policy of making no design changes at all would be preferable.

The point where it is least attractive to jump to the cycle is $(1/2, 1/2)$. This implies that if a policy of making no new designs would be optimal anywhere, it would certainly be optimal for $\bar{X} = 0.5$ and $\bar{Y} = 0.5$. In Figure 7 we plot the difference in the objective values for jumping to the cycle and staying: $V^{up} - V^{stay}$, where $Y_0 = 0.01$. This figure shows that indeed for this Y_0 staying at $\bar{X} = \bar{Y} = 0.5$ is optimal for an interval that includes r -values between 0.2 and 0.4, while this does not occur for $r = 0.1$. The thin line proves that the parameters are still in the relevant region, i.e., $\frac{1}{2(r+2)} - c > 0$.

To determine the size of the region where making no design changes is optimal (so far we only know that it includes $\bar{X} = .5$ and $\bar{Y} = .5$) we have to determine two other Skiba curves. The first one includes those points where jumping up followed by periodic design changes gives the same objective value as making no changes. On the second one the fashion trend-setter is indifferent between a policy of jumping down followed by periodic design changes, and making no changes in the design. A numerical example in which these curves occur is depicted in Figure 8.

In Figure 8 two other regions occur, where it is in fact optimal to have an initial period of making no design change followed by a jump to the cycle. Here

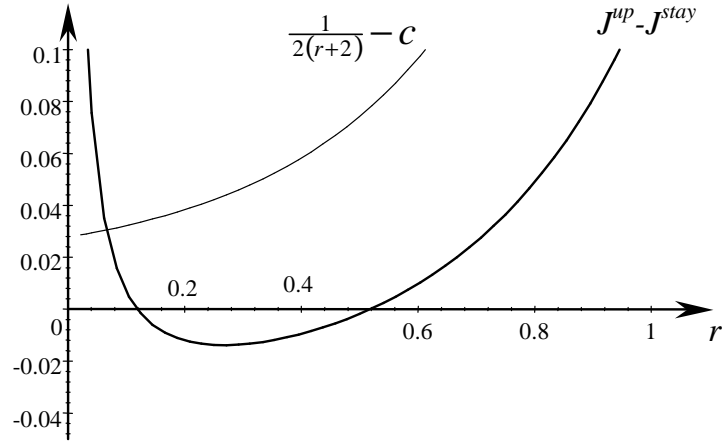


Figure 7: Difference in objective values for jumping to the cycle and staying, when $X = Y = 0.5$ and $Y_0 = 0.1$.

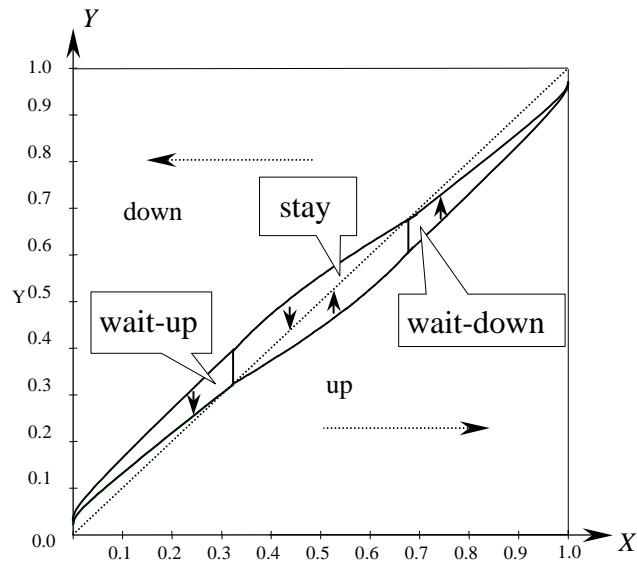


Figure 8: Eight Skiba curves for the case that $Y_0 = 0.01$, $r = 0.2$, and $c = 0.189$.

the idea is that when the first jump is upward (downward) it is only optimal to make this design change after the imitator has moved in the downward (upward) direction for a sufficiently long time. Only then does jumping create a large enough difference between the two designs for the trend-setter to make enough profits to offset the cost of making the jump. Within the region "wait-up" ("wait-down") the upward (downward) movement takes place at the moment that Y reaches the lower (upper) boundary of this region. In Appendix B we provide some details on the computation of these Skiba curves.

8 Conclusions

Fashions change and even cycle. A variety of models have been advanced to explain why. Most have focused on consumers' tastes and behavior. Pesendorfer (1995) introduced perhaps the best-known model that explicitly considers optimal dynamic strategies for suppliers of fashion goods. It focuses on the monopoly case. However, it is not always easy to protect intellectual property claims concerning fashion (as opposed to technical) innovations. Indeed, two basic elements of the fashion industry are constant innovation by high-end designer labels and low-cost "knock-off" brands striving to offer products that look like those of the trend-setters.

Here we introduce a model whose solution describes how a high-end trend-setter ought to respond to competition from one or more low-cost imitators when the product space is bounded and consumers value one design over another

only to the extent that it is distinguishable from the low-cost products. That is, consumers have no intrinsic preference for one design over another, and it is not possible for innovators to improve a product along some unbounded directional dimension, always leading followers in that particular direction (e.g., constantly by making faster and faster microchips).

The optimal strategy depends on the parameter values and, in many cases, there is state-dependency. However, when the costs of innovation are low enough, the trend-setter should innovate indefinitely even though the product space is bounded. I.e., it is optimal to create fashion cycles. Because there is no notion of one side of product space being intrinsically better or worse than the other, the optimal initial direction of innovation depends on the innovator and imitator's initial positions in product space. In particular, a two-dimensional Skiba curve separates regions in state space within which it is optimal for the innovator to begin by moving "left" or "right" in product space.

Not surprisingly, when the costs of innovation are high enough, the optimal strategy involves no innovation. The trend-setter simply milks the profits available because of its initial product differentiation, but is eventually overtaken by imitators who by virtue of their lower cost structure take over the entire market. Sometimes it is optimal for the trend-setter to extend this transient leadership with a single innovation.

For intermediate costs there are initial positions such that the innovator is indifferent between embarking on a long-term strategy of innovating indefinitely and one of these alternate strategies. Again, the collection of these indifference

points constitute Skiba curves. Indeed, there are places in state space where several different Skiba curves meet. Furthermore, in most instances it is possible to write explicit analytical expressions characterizing these two-dimensional Skiba curves and to explore how they depend on various model parameters.

Given our one-D abstraction of product space, some aspects of the optimal solution are artificial, such as the idea that fashion bounces forever between the same two points. In reality, although may be simple cycling in one dimensional projections of the higher dimensional product space (e.g., a color can be in, then out of fashion, then back in again), the true product space is of much higher-dimension. Translating the insights of our stylized model back into that richer, more realistic image of the variety possible in fashion goods, we would obtain the following prediction. We would expect the fashion leader to make bold moves (equivalent to jumping from one boundary to another) in directions that maximally differentiate it from the followers, while still remaining within the realm of what is "feasible" in customers' minds. That is in fact not a bad characterization of what is done at fashion shows.

9 Appendix A: Derivation of the Skiba Curve of Section 3

From (9) and (10) it is obtained that equating the objective values of jumping upwards and downward gives:

$$\begin{aligned}
& -c\bar{X} + \left(\frac{Y_0}{\bar{Y}}\right)^r (V - c) + \frac{1}{2}\bar{Y}^2 \frac{1 - \left(\frac{Y_0}{\bar{Y}}\right)^{r+2}}{r+2} \\
= & -c(1 - \bar{X}) + \left(\frac{Y_0}{1 - \bar{Y}}\right)^r (V - c) + \frac{1}{2}(1 - \bar{Y})^2 \frac{1 - \left(\frac{Y_0}{1 - \bar{Y}}\right)^{r+2}}{r+2} \\
\Leftrightarrow & c(1 - \bar{X}) - c\bar{X} + \left(\left(\frac{Y_0}{\bar{Y}}\right)^r - \left(\frac{Y_0}{1 - \bar{Y}}\right)^r\right) (V - c) \\
= & \frac{1}{2(r+2)} \left[(1 - \bar{Y})^2 \left(1 - \left(\frac{Y_0}{1 - \bar{Y}}\right)^{r+2}\right) - \bar{Y}^2 \left(1 - \left(\frac{Y_0}{\bar{Y}}\right)^{r+2}\right) \right] \\
\Leftrightarrow & c(1 - 2\bar{X}) + \left(\left(\frac{1}{\bar{Y}}\right)^r - \left(\frac{1}{1 - \bar{Y}}\right)^r\right) (V - c) Y_0^r \\
= & \frac{1}{2(r+2)} \left[(1 - \bar{Y})^2 \left(1 - \left(\frac{Y_0}{1 - \bar{Y}}\right)^{r+2}\right) - \bar{Y}^2 \left(1 - \left(\frac{Y_0}{\bar{Y}}\right)^{r+2}\right) \right]
\end{aligned}$$

Plugging in V from (6) results in:

$$\begin{aligned}
& c(1 - 2\bar{X}) + \left(\left(\frac{1}{\bar{Y}} \right)^r - \left(\frac{1}{1 - \bar{Y}} \right)^r \right) (V - c) Y_0^r \\
= & \frac{1}{2(r+2)} \left[(1 - \bar{Y})^2 \left(1 - \left(\frac{Y_0}{1 - \bar{Y}} \right)^{r+2} \right) - \bar{Y}^2 \left(1 - \left(\frac{Y_0}{\bar{Y}} \right)^{r+2} \right) \right] \\
\iff & c(1 - 2\bar{X}) \\
& + \left(\left(\frac{1}{\bar{Y}} \right)^r - \left(\frac{1}{1 - \bar{Y}} \right)^r \right) \left(\frac{(1 - Y_0)^{r+2} - Y_0^{r+2}}{(1 - Y_0)^r - Y_0^r} \frac{1}{2(r+2)} - \frac{Y_0^r}{(1 - Y_0)^r - Y_0^r} c - c \right) Y_0^r \\
= & \frac{1}{2(r+2)} \left[(1 - \bar{Y})^2 \left(1 - \left(\frac{Y_0}{1 - \bar{Y}} \right)^{r+2} \right) - \bar{Y}^2 \left(1 - \left(\frac{Y_0}{\bar{Y}} \right)^{r+2} \right) \right] \\
\iff & c(1 - 2\bar{X}) \\
& + \left(\left(\frac{1}{\bar{Y}} \right)^r - \left(\frac{1}{1 - \bar{Y}} \right)^r \right) \frac{1}{(1 - Y_0)^r - Y_0^r} \left(\frac{(1 - Y_0)^{r+2} - Y_0^{r+2}}{2(r+2)} - (1 - Y_0)^r c \right) Y_0^r \\
= & \frac{1}{2(r+2)} \left[(1 - \bar{Y})^2 \left(1 - \left(\frac{Y_0}{1 - \bar{Y}} \right)^{r+2} \right) - \bar{Y}^2 \left(1 - \left(\frac{Y_0}{\bar{Y}} \right)^{r+2} \right) \right] \\
\iff & \bar{X} = -\frac{1}{4(r+2)c} \left[(1 - \bar{Y})^2 \left(1 - \left(\frac{Y_0}{1 - \bar{Y}} \right)^{r+2} \right) - \bar{Y}^2 \left(1 - \left(\frac{Y_0}{\bar{Y}} \right)^{r+2} \right) \right] \\
& + \frac{1}{2} \left(\left(\frac{1}{\bar{Y}} \right)^r - \left(\frac{1}{1 - \bar{Y}} \right)^r \right) \frac{Y_0^r}{(1 - Y_0)^r - Y_0^r} \left(\frac{(1 - Y_0)^{r+2} - Y_0^{r+2}}{2(r+2)c} - (1 - Y_0)^r \right) + \frac{1}{2},
\end{aligned}$$

which is the Skiba curve given by expression (11). QED

10 Appendix B: Technical Details of Section 5

We now discuss some technical details that relate to Figure 8. Ignoring for the moment the regions "wait-up" and "wait-down", we arrive at Figure 9.

Note that at point A the curve $V^{stay} = V^{up}$ crosses the 45° line $X = Y$. This means that to the left of point A, the curve $V^{stay} = V^{up}$ is not relevant anymore. Consider e.g. point B. Here $V^{stay} = V^{up}$ but both policies "stay

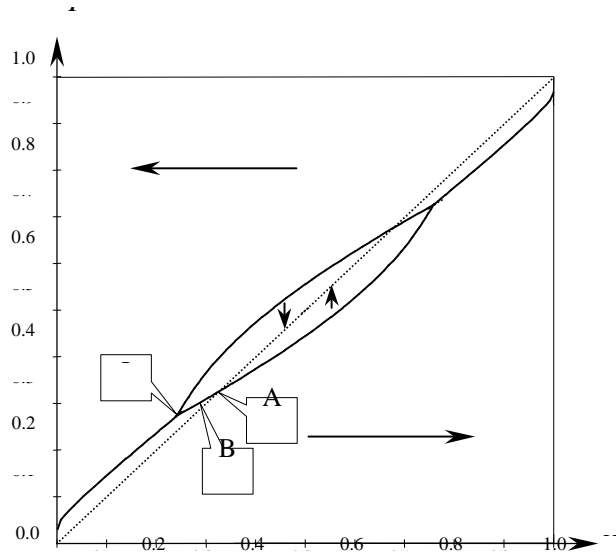


Figure 9: Figure 8 without regions "wait-up" and "wait-down".

forever" and "jump up and follow cycle" are not optimal anymore. The reason is that from B onwards, staying at the current value of X one immediately enters the region where "jumping up" is better than "staying". This implies that a jump taking place at any point in time after leaving B is better than staying forever or jumping immediately. This means that to the left of A and around the "naive triple point" T another policy has to be considered, namely "wait until Y has reached \tilde{Y} and then jump up". The value of this policy is

$$V^{wait-up} = \int_0^{\tilde{t}} e^{-rt} \frac{1}{2} (X - Y)^2 dt + e^{-r\tilde{t}} V^{up}(X, \tilde{Y}), \quad (13)$$

where \tilde{t} is given by the time that Y reaches \tilde{Y} , i.e.,

$$\tilde{t} = \ln \left(\frac{Y^{start} - X}{\tilde{Y} - X} \right) \quad (14)$$

Since $Y = X - (X - Y^{start}) e^{-t}$, the integral in (13) equals

$$\int_0^{\tilde{t}} e^{-rt} \frac{1}{2} (X - Y)^2 dt = \frac{(X - Y^{start})^2}{2(r+2)} \left[1 - \left(\frac{\tilde{Y} - X}{Y^{start} - X} \right)^{r+2} \right]$$

and (13) becomes

$$V^{wait-up} = \frac{(X - Y^{start})^2}{2(r+2)} + \frac{1}{(Y^{start} - X)^r} \left[-\frac{(\tilde{Y} - X)^{r+2}}{2(r+2)} + (\tilde{Y} - X)^r V^{up}(X, \tilde{Y}) \right]. \quad (15)$$

Since the term in brackets (depending on \tilde{Y}) does not depend on Y^{start} , the maximization of $V^{wait-up}$ w.r.t. \tilde{Y} does not depend on Y^{start} . Clearly this value \tilde{Y} will be below the A-B-T line in Figure 9. At point T, i.e., for $Y = 0.27431$ and $X = 0.24217$ in Figure 9 we numerically obtain $\tilde{Y} = 0.2475$ which means that \tilde{Y} is closer to the 45° line than to T.

It should be noted that the difference $V^{wait-up} - V^{up} > 0$ can be interpreted as the option value of waiting.

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