Sequential Entry, Switching Costs and Strategic Pricing

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Abstract

This paper examines the pricing decisions of the firms when they enter the market sequentially. The incumbent enjoys the first mover advantage through switching costs. Hence, the users who buy from the incumbent incur a cost to switch to the entrant. In equilibrium both firms differentiate maximally, and we show that forward looking incumbent anticipates the incoming competition and can strategically overprice its product in first period to avoid competition with the entrant later. The outcome is such that, in the presence of sufficient switching costs, both incumbent and entrant offer high prices. Therefore, consumer welfare is reduced considerably. We extend the model by introducing forward looking sophisticated customer. We show that sophisticated customers can anticipate this behavior and force competition in next period but incumbent can increase its first period price even more, although higher switching costs for this to be sustainable.

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1 Introduction

One of the key decisions for a firm entering the market is how to price and position its product. It is important because positioning decision is typically irreversible whether it is the physical positioning of the store or the perceptual positioning of the brands. Pricing decisions are critical when the firms enter the market sequentially and the incumbent enjoys first mover advantage through switching costs. Incumbent has to decide how to price the product in present period, while anticipating the competition in future. It is therefore not surprising that product positioning, first mover advantage and pricing have received a lot of attention in marketing literature (e.g. see Wind 1990, Urban and Hausar 1993, Dolan 1993, Tyagi 2000).

Consider a situation where an incumbent firm has to price its product when the consumers have to incur switching costs. How should it price its product? The answer is not as obvious. First, the firm has to anticipate the entrant’s product positioning and pricing. Second, firm needs to account for forward looking customers who may recognize the switching costs embedded in the product. Prior research has explored many of these factors in isolation. One major goal of this paper is to model all these factors together. This, in turn, allows us to highlight many new interesting results that have both managerial and academic relevance. For example, traditionally, in the presence of switching costs, entrant is expected to price its products lower than the well established player\(^1\). While this indeed is true in many instances, one major finding of our paper is that we show the situation where both incumbent and entrant can offer high prices if the incumbent behaves strategically. We show that this is not the mere effect of entrant trying to fill a niche in the market but a strategic “over-pricing” by incumbent in first period to soften price competition later. Moreover,

\(^1\)The results hold whether the products are differentiated or not
unlike previous work (Klemperer 1986), this “over-pricing” is not to compete with entrant in future but to avoid competition. There are instances of such high pricing. For example, consider the entry of Xbox by Microsoft in the video game market. Sony’s PlayStation has been a well established high price, high quality brand in this market. It has a loyal consumer base with a large number of games already written for it. Switching from one brand to the other is costly due to the incompatibility in software, high level of brand loyalty, and different console designs. Given such a well established player, one would have expected Xbox to offer lower price. But when Xbox was released in USA, Europe and even Japan, it was priced even higher than PlayStation in all these markets (Sloovic 2002)\textsuperscript{2}. Such higher prices have persisted for a long time though PlayStation has a bigger market share.

We present a model in two period setting such that an Incumbent firm enters the market first and covers some market. In the second period, entrant firm enters and chooses the optimal position as well as its price and both firms compete. We show that the presence of switching costs introduces discontinuity in the demand function of the entrant. This, in turn, allows the incumbent to strategically price the product and cover the market in first period such that it can soften the competition in the next period. We argue that switching costs play a very critical role for the existence of such an equilibrium. If they are too low then the only outcome is what a standard model predicts; i.e competition in second period with entrant offering low price. We then extend our model by introducing forward looking, sophisticated customers. Interestingly, we observe both penetration pricing and overpricing strategies by the incumbent depending on the switching costs. When the switching costs are low, incumbent offers penetration pricing in the first period. On the other hand, when the switching costs are high, incumbent can act strategically and overprice its product.

\textsuperscript{2}Note that Xbox is not considered necessarily a higher quality product than PlayStation
The paper is organized as follows. Given the large body of work on entry and pricing decisions, we provide an overview of relevant prior work in section 2. Section 3 presents our model with myopic consumers. We extend our model to incorporate forward-looking sophisticated consumers in section 5. Finally, we discuss some limitations our work, point to future work and and provide concluding remarks in section 6. All proof are relegated to the Appendix.

2 Prior Literature

Ever since Hotelling’s (1929) seminal work, product differentiation, pricing and entry issues have been studied in great detail in both economics and marketing literature. D’Aspremont et al (1979) show that in the presence of quadratic transport costs, maximum differentiation is indeed optimal. Hauser and Shugan (1983) examine how an incumbent responds to competitive new entry. Kumar and Sudarshan (1988) and Hauser (1988) generalize this model and show that maximum differentiation is the optimal strategy even in multi-attribute product space. Carpenter (1989) also looks at two-attribute model where customers have ideal points that describe their most preferred level for each attribute. Ansari et al (1994) establish the equilibrium when preferences are non-uniformly distributed and show that maximum differentiation is not necessarily optimal. In our paper, product is differentiated on one attribute and we show that maximal differentiation is the optimal strategy but the main focus of our paper is how sequential entry and switching costs can lead to a non obvious outcome.

Significant work on switching cost is done by Klemperer. He also outlines some categories of switching costs and their causes including “brand loyalty”. It has been

\footnote{See Klemperer (1995) for an excellent review of switching cost literature}
observed that in general, when market structure is duopolistic in both period in the presence of switching costs, both firms lower their prices in first period to attract customers and gain market share and increase their prices in second period to milk the established market, Klemperer (1987b). Wernerfelt (1991) presents a theory of brand loyalty (a form of switching cost) and shows that how higher brand loyalty leads to higher prices. The entrant’s entry strategy and incumbent advantage has also been studied by Carpenter and Nakamoto (1990) where they show that depending on preference asymmetry both high and low positioning strategies are viable. Tyagi (2000) shows that the first mover tends to occupy the most attractive location but only if the entrant does not have significant cost advantage.

In our paper, we highlight the importance of switching costs when products are differentiated and we show that overpricing by incumbent is a very strategic behavior. This allows it to soften price competition later. Klemperer’s (1987) shows similar results but the products are not differentiated in that paper and such behavior by incumbent is to compete with entrant later due to entry of new customers. In our paper, such behavior by incumbent is to soften competition. While higher market share and hence penetration pricing is a strategic response in many cases (Wernerfelt 1986; Dolan and Jeuland 1981) to deter new entry, we show that in some cases when entry is unavoidable, overpricing can be an optimal response too.

3 Model

We use the Salop’s circular city model to avoid the end point problem of Hotelling’s linear city model. It also allows us to ignore the incumbent’s location decision. Users

\footnote{In case of a linear city model, incumbent will locate at the most attractive location; i.e. the center location (Tyagi 2000). In circular city all locations are equally attractive. We will show that entrant would like to maximally differentiate.}
are distributed uniformly in physical or perceptual distance for a particular attribute over the circle of two unit circumference\footnote{There is no loss of generality in taking two units instead of one unit. This allows us to compare it with Hotelling’s linear city model}. Firm 1, which is the incumbent, enters the market in the first period. It offers a price $p_0$ and captures some market share. In the next period, period 2, firm 2, which is the entrant, enters the market and both firms compete in prices. First, we assume that users are myopic and do not consider the consequences of their current period action. We will generalize this assumption in the next section.

The utility of a user is specified as $u = R - p_0 - x^2$ where $R$ is the reservation price. The users at distance $x$ from the firm incur disutility $tx^2$. Any convex function is sufficient for further analysis but we use quadratic function as it has been extensively used in literature (Tyagi 2000). To keep the mathematics simple and to focus on the main issues of switching costs, we will assume that $t$ is one unit. We also assume that $R$ is high enough that each consumer makes a purchase when both firms are in the market\footnote{when only firm 1 is in the market, $R$ need not be so high that everyone makes the purchase. In fact, it depends on the pricing $p_0$ of firm 1. We will show that it is in incumbent’s interest to not cover the whole market}. The users who buy incumbent’s product in period 1, incur a switching cost $s$ if they switch to the entrant in period 2. There are many examples when users incur switching cost when switching to other brands or firms. It could be incompatibility with existing equipment, transaction costs, learning costs or some psychological or non-economic cost like “brand loyalty”.

Without loss of generality we assume that the marginal cost of producing a product or service is zero. The game is played in the following stages. (i) \textit{Stage I} (period 1): Incumbent enters the market, chooses its location and offers a price $p_0$. (ii) \textit{Stage II} (period 2): Entrant enters the market and chooses its location. (iii) \textit{Stage III} (period 2): Both firms offer prices.
In the spirit of sub-game perfect equilibrium, we will start with *stage III* in second period, period 2. Therefore, we will take the first period price of incumbent \( p_0 \), and the location of entrant as given and solve for the second period Nash-Equilibrium. The analysis simplifies considerably by showing that the best strategy for entrant is to maximally differentiate irrespective of any switching cost. We first show in Appendix that entrant maximally differentiates.

Suppose that incumbent offers price \( p_1 \) and entrant offers price \( p_2 \) in second period. Our analysis differs from other papers in that due to sequential entry and switching costs, there are two regions of interest here. The firms can choose its prices such that (i) Entrant attracts only those customers who did not buy incumbent’s product in first period. We call this *non-compete* strategy. (ii) Entrant also attracts some customers who bought the product from the incumbent in first period. We call this *compete* strategy.

### 3.1 Myopic Customers

As seen from fig-1, incumbent enters the market and offers price \( p_0 \) and covers the market \( 2x_0 = 2\sqrt{R-p_0} \). In the next period, entrant enters the market and locates diametrically opposite the incumbent. Entrant may price its product or services such that it follows either *non-compete* strategy or *compete* strategy depending on the relative profits it can make. Similarly, incumbent may try to force the entrant to choose either *non-compete* or *compete* strategy depending on profitability of its respective strategies. Equilibria in *compete* strategy has been well studied and not the focus of the paper. We will explore the *non-compete* region and establish both mixed strategy and pure strategy equilibrium. We will specifically show that it is in incumbent’s interest to not deliberately cover the market in first period by strategically pricing its
products steeply. Note that the incumbent would have offered monopoly pricing if there were no competition in second period. It strategically leaves just enough market for the entrant so that it does not compete with it in the second period.

To be able analyze this structure, we will first focus on second period equilibrium, i.e. the equilibrium prices offered by incumbent \( (p_1) \) and entrant \( (p_2) \). But it is immediately clear that second period equilibrium is influenced by first period price \( (p_0) \). Therefore, we first define different regions based on \( p_0 \) and outline the second period equilibrium in those regions. Now, the game will be complete because for all \( (p_0) \), the second period outcome has been defined. Incumbent will choose the optimal \( p_0 \) (which in turn depends on switching costs) which will lead to an optimal \( p_1 \) and \( p_2 \) in the next period.

### 3.2 Second Period Equilibrium

Both Incumbent and entrant have two strategies available, compete and non-compete. Consider the entrant’s decision, given certain \( p_0 \) and \( p_1 \). Entrant can either set compete prices or non-compete prices. If it plays compete strategy then, it attracts the old buyers of the incumbent. Consider a user at distance \( x \) from incumbent who is indifferent between incumbent and entrant then

\[
R - p_1 - x^2 = R - p_2 - (1 - x)^2 - s
\]

\[
x = \frac{1 - p_2 + p_s + s}{2}
\]

Entrant’s demand is obviously \( 2(1 - x) \). Therefore, its price and profits are

\[
p^c_2 = \frac{1 + p_1 - s}{2}
\]
\[ \Pi_E^C = \left( \frac{1 + p_1 - s}{2} \right)^2 \]  

(2)

When it plays \textit{non-compete} strategy, then by definition, it only attracts the buyers who did not purchase from incumbent in first period. Therefore, its demand is \(1 - x_0 = 1 - \sqrt{R - p_0}\). It should offer a price \(p_2\) such that the buyer located at \(x_0\) is just indifferent. Therefore, for a given \(p_1\), it will offer \(p_2\) such that

\[
R - x_0^2 - p_1 = R - p_2 - (1 - x_0)^2
\]

\[
\Rightarrow p_2^N = 2x_0 + p_1 - 1
\]

\[
\Rightarrow p_2^N = 2\sqrt{R - p_0} + p_1 - 1
\]

(3)

The profit of the entrant can be specified as

\[
\Pi_E^N = (1 - x_0)p_2^N
\]

\[
\Pi_E^N = (1 - \sqrt{R - p_0})(2\sqrt{R - p_0} + p_1 - 1)
\]

(4)

Now, consider the incumbent’s best response function. For a given, \(p_0\) and \(p_2\), incumbent can also offer two strategies: compete, non-compete. When it offers compete strategy, it competes with entrant to preserve its first period buyers. Therefore it is clear from eq(1) that incumbent’s optimal price and profit is given by

\[
p_1^C = \frac{1 + p_2 + s}{2}
\]

\[
\Pi_I^C = \left( \frac{1 + p_2 + s}{2} \right)^2
\]

(5)

When incumbent offers \textit{non-compete} strategy, it only attract its first period buyers. Therefore, its demand is simply \(2x_0 = 2\sqrt{R - p_0}\). To get this demand, it
should offer a price $p_1$ such that the buyer located at $x_0$ is just indifferent from buying from him or the entrant. Therefore, for a given $p_2$, it will offer $p_1$ such that

$$R - x_0^2 - p_1 = R - p_2 - (1 - x_0)^2 - s$$

$$\Rightarrow p_1^N = -2x_0 + p_2 + s + 1$$

$$\Rightarrow p_1^N = -2\sqrt{R - p_0} + p_2 + s + 1$$

$$\Pi_I^N = 2(\sqrt{R - p_0})(p_2 + s + 1 - 2\sqrt{R - p_0}) \tag{6}$$

Note in previous two strategies, the market share for the incumbent is either less than or equal to first period market. Incumbent, by the virtue of switching cost $s$, is milking its first period base. But, there exists another strategy for the incumbent such that it wants to expand its market beyond $x_0$ in the second period. In other words, for very high $p_2$, incumbent can offer low $p_1 < p_0$ such that its second period market share is higher than the first period share. Since incumbent is attracting new buyers, its demand is given by

$$R - p_1 - x^2 = R - p_2 - (1 - x)^2$$

$$x = \frac{1 - p_1 + p_2}{2}$$

Therefore, the optimal price and profits of the incumbent is given by

$$p_1^c = \frac{1 + p_2}{2} \tag{7}$$

$$\Pi_I^c = \left(\frac{1 + p_2}{2}\right)^2 \tag{8}$$

A similar strategy for the entrant also exists where entrant competes with incumbent but only for the new users. For low values of $p_1$, entrant may have to
compete to just get some new users. The price and profit for the entrant are simply

\[
p_2^E = \frac{1 + p_1}{2}
\]

(9)

\[
\Pi_E^E = \left(\frac{1 + p_1}{2}\right)^2
\]

(10)

Now, we can characterize the reaction functions of the incumbent and entrant completely. At any given \(p_1\), entrant can offer either \(p_2^C\), \(p_2^C\), or \(p_2^N\) as outlined above. Similarly, for any given \(p_2\), incumbent can offer \(p_1^C\), \(p_1^C\), or \(p_1^N\) depending on its profits. The key to note is how \(p_0\) shifts these reaction functions and hence change the market structure. Before, we proceed, we plot the reaction functions for some numerical values of \(s = 0.5\), \(R = 2.4\). We pick three different values of \(p_0\) and show how the equilibrium changes with \(p_0\). We take \(p_0 = p_m = \frac{2F}{3} = 1.6\), \(p_0 = 1.8\) and \(p_0 = 1.95\).

In all three figures, \(p_1\) is on x-axis and \(p_2\) is on y-axis. Solid lines are for the incumbent’s reaction function and dotted lines are entrant. As seen from Figure 2a,
there is clearly a pure strategy equilibrium where both firm are competing. But the key to note is the discontinuity in the entrant’s reaction function. For low values of \( p_1 \), entrant does not want to compete and offers non-compete prices. For some \( p_1 \) which we denote as \( \bar{p} \), entrant is indifferent between non-compete and compete strategy. For any \( p_1 > \bar{p} \), entrant wants to offer compete prices.

The impact of \( p_0 \) can be assessed through its impact on \( \bar{p} \). In figure 2b, when \( p_0 = 1.8 \), there is no pure strategy equilibrium. Note that \( \bar{p} \) is higher than in figure 2a. Incumbent can set higher \( p_0 \) and essentially force a mixed strategy equilibrium where entrant mixes between non-compete and compete prices. Interestingly, unlike previous studies, where the firms usually offer lower prices in the first period to lock-in buyers, this shows that sometimes offering higher than monopoly prices in first period can also be optimal.

Incumbent’s reaction function also has kinks at two places. The first segment is the compete price offered by incumbent for low \( p_2 \). As \( p_2 \) increases, the incumbent finds is more profitable to play non-compete strategy as evident from the second segment. Finally, once, \( p_1 = p_0 \), incumbent does not want to increase its price anymore.\(^7\) In figure-2c, we observe the discontinuity in incumbent’s reaction function as well. For very high \( p_2 \), incumbent is better off offering compete price \( p_1^* \). Entrant is offering such high prices that incumbent wants to attract some new users in second period. Again, we see a mixed strategy equilibrium where incumbent is mixing between non-compete and expand prices and entrant is offering non-compete price.

With this intuition, we are now ready to formalize the second period equilibria. As we can see, there are three distinct region of \( p_0 \) which leads to three different market structure and profits. Incumbent, by virtue of its first mover advantage, can

\(^7\)Since, \( p_0 \) is already above monopoly price and entrant is not competing, raising \( p_1 \) beyond \( p_0 \) will reduce the profits
choose an optimal \( p_0 \)

Consider a region when \( p_0 < p_0^A \) where \( p_0^A = R - \frac{1}{144} \left( -6 + s - 3 \sqrt{4s - \frac{r^2}{3}} \right)^2 \). In this region, as we see in figure 2a, there is compete equilibrium in the second period and both firms compete. The equilibrium prices and profits are given by the following proposition.

**Proposition 1** For some \( p_0 < p_0^A \), there exists a pure strategy equilibrium in prices such that entrant captures some first period buyers of the incumbent. In equilibrium incumbent offers \( p_1^* = \frac{3+s}{3} \) and entrant offers \( p_2^* = \frac{3-s}{3} \).

Since incumbent is the first mover and users incur switching costs to switch to entrant, it is able to offer higher prices than the entrant in equilibrium. Note that the incumbent knows that it will loose some of the buyers to the entrant. It is also interesting to note that first period prices have no impact on second period equilibrium\(^8\). The profits made by incumbent and entrant are \( \pi_I = \left( \frac{3+s}{3} \right)^2 \) and \( \pi_E = \left( \frac{3-s}{3} \right)^2 \) respectively. Calculation of \( p_0^A \) is straight-forward. Note the discontinuity in entrant’s fuction at \( \overline{p} \). \( \overline{p} \) is simply the price \( p_1 \) when entrant is indifferent between playing compete and non-compete strategies. We can get the \( \overline{p} \) by comparing \( \Pi^C_E \) and \( \Pi^N_E \). Thus, \( \overline{p} \) can be calculated as

\[
\overline{p} = 3 + s - 4x_0 + \sqrt{8s(1 - x_0)} \tag{11}
\]

where \( x_0 = \sqrt{R - p_0} \). Note that in pure strategy the incumbent sets the price \( p_1^* = \frac{3+s}{3} \). Once \( \overline{p} > p_1^* \), there will only exist a mixed strategy. Since \( \overline{p} \) is a function of \( p_0 \), and increasing in \( p_0 \), for some \( p_0, \overline{p} > p_1^* \).

\(^8\)as long as incumbent covers enough market. By definition, the second period market share of incumbent is less than first period.
Consider the case when $p_0 > p^A$. As we explained above, once $p_0 > p^A$, there exists only mixed strategy equilibrium.

Proposition 2 For $p_0 > p^A$, there exists a mixed strategy equilibrium, where entrant mixes between compete ($p_2 = \frac{3x_0}{3 - 2x_0}$) and non compete strategy ($p_2 = 2x_0 + \bar{p} - 1$) and incumbent offers $\bar{p}$.

The key point to note is that even though incumbent offers $\bar{p}$, it is either playing compete or non-compete strategy. In other, although $\bar{p}$ is the best response of incumbent for some mixing strategy of entrant, depending upon $p_0$, the price $\bar{p}$ is either a non-compete or a compete price. The kink in incumbent’s reaction function in figure 2b clarifies this. For some smaller value of $p_0$ and hence $\bar{p}$, incumbent is in compete region and therefore its profits are given by eq(5).

$$\Pi_f^C = \left(\frac{1 + p_2 + s}{2}\right)^2$$

where $p_2 = 2\bar{p} - 1 - s$. Note that by definition, in compete region $\bar{p} = \frac{1+p_2+s}{2}$. Therefore, $\Pi_f^C = \bar{p}^2$. On the other hand, for larger values of $\bar{p}$, incumbent is in non-compete region and therefore its profits are given by eq(6).

$$\Pi_f^N = 2(\sqrt{R - p_0})\bar{p}$$

These two equations define the region of $p_0$ where $\bar{p}$ is either compete response or non-compete response of the incumbent. By equating these two, we define the region $p_0^B = R - \left(\frac{9+s+2\sqrt{9s-2s^2}}{321}\right)^3$. Therefore, for $p_0^A < p_0 < p_0^B$, incumbent offers $\bar{p}$ and makes the profit $\Pi_f^C$. For $p_0 > p_0^B$, incumbent again offers $\bar{p}$ but makes the profit $\Pi_f^N$.

There is finally one more region to specify to complete our discussion of second
period equilibrium. Note that in figure-2c, once $\overline{p} = p_0$, entrant always wants to play non-compete strategy. But now incumbent wants to mix between non-compete and expand strategy as evident from the discontinuity in figure-2c. When incumbent deviates from $\overline{p}$ to $p_f^*$, the price and profits are given by equation (8).

We can calculate $p_2$ and subsequently $\overline{p}$ at which the incumbent would like to deviate. Note that for incumbent to deviate to $p_f^*$, when entrant offers $p_2$, it has to be that $\Pi_f > \Pi_f^N$. In other words,

$$\left(\frac{1 + p_2}{2}\right)^2 \geq 2(\sqrt{R - p_0})(1 + s + p_2 - 2x_0)$$

where $x_0 = \sqrt{R - p_0}$. This leads to $p_2 = 4x_0 - 1 - \sqrt{8sx_0}$. Or, at $p_1 = \overline{p} = 2x_0 + s + \sqrt{8sx_0}$ incumbent will deviate to $p_f^*$. It is clear that $p_1 = 2x_0 + s + \sqrt{8sx_0}$ is increasing in $p_0$ and $s$. From figure-2c, it is evident from the vertical segment in the incumbent’s reaction function that incumbent would not like to raise $p_1 = \overline{p}$ beyond $p_0$. The reason is quite intuitive. Incumbent has no incentive to increase $p_1 = \overline{p}$ beyond $p_0$ because at this $p_1$, entrant is offering only non-compete prices and not offering any competition to the incumbent. Since entrant is not offering any competition and $p_0$ is already more than monopoly price, the profit will reduce if $p_1$ increases beyond $p_0$. Therefore, optimal $p_1$ is $\min(p_0, \overline{p})$.

Therefore, incumbent mixes between $p_1 = \min(p_0, \overline{p})$ and $p_f^*$. Does a pure strategy equilibrium possible when both incumbent and entrant offer non-compete prices? The answer apparently is yes. For pure strategy equilibrium to exist, the incumbent should not deviate to $p_f^*$. In other words, incumbent offers $p_1 = \overline{p} = p_0$ and entrant offers non-compete $p_2 = 2\sqrt{R - p_0} + p_0 - 1$. For incumbent to not deviate,

$$\left(\frac{2\sqrt{R - p_0} + p_0}{4}\right)^2 \leq 2\sqrt{R - p_0^2}$$

14
\[ \Rightarrow \hat{p}_0 \leq 2\sqrt{R + 1} - 2 \]

Therefore, in this limit of \( p_0 \), we will observe a mixed strategy equilibrium. Once \( p_0 < 2\sqrt{R + 1} - 2 \), there will be a pure strategy equilibrium such that both players offer a non-compete prices. Under what condition will \( p_0 < 2\sqrt{R + 1} - 2 \)? We know that incumbent offers \( p_1 = \bar{p} = p_0 \) in second period. Solving \( p_1 = \bar{p} = p_0 \) gives us the optimal \( p_0 \) when a pure strategy equilibrium exists. Therefore,

\[ \bar{p} = p_0 \]

\[ \Rightarrow 3 + s - 4x_0 + \sqrt{8s(1 - x_0)} = p_0 \]

\[ \Rightarrow 3 + s - 4\sqrt{R - p_0} + \sqrt{8s(1 - \sqrt{R - p_0})} = p_0 \]

We define the solution of the inequality above to be \( p_0^{**} \). The close form solutions are difficult to get but it can be verified from implicit function theorem that \( \frac{\partial p_0^{**}}{\partial s} < 0 \) and that at \( s = 0, p_0^{**} = 4\sqrt{R + 1} - 5 > \hat{p}_0 \). In other words, when \( s \) is low, for some \( p_0 > p_0^B \), we will observe a mixed strategy equilibrium. As \( s \) increases, \( p_0^{**} \) will fall and at some \( s = s_{\text{high}} \), it will be equal to \( \hat{p}_0 = 2\sqrt{R + 1} - 2 \). At this point, we will see a pure strategy equilibrium where both players play a non-compete strategies. We formalize this intuition in the following proposition

**Proposition 3** For some \( p_0^B < p_0 \leq p_0^{**} \) and \( s < s_{\text{high}} \), there exists a mixed strategy equilibrium such that incumbent mixes between a non-compete price \( p_1 = \min(p_0, \bar{p}) \) and an expand price \( p_1^* = \frac{2\sqrt{R - p_0} + p_0}{2} \). Entrant offers non-compete price \( p_2 = 2\sqrt{R - p_0} + p_1 - 1 \). For some \( s \geq s_{\text{high}} \), there exists a pure strategy equilibrium where both incumbent and entrant offer a non-compete price such that \( p_1 = p_0 \) and entrant offers \( p_2 = 2\sqrt{R - p_0} + p_0 - 1 \).
The most interesting point to note is the fact that for some switching costs $s$, incumbent can offer a high initial price $p_0$ and avoid the competition later. Moreover, for high enough switching costs, we get a pure strategy equilibrium where both incumbent and entrant are playing non-compete pricing.

3.3 First Period Equilibrium

The previous section characterized the complete equilibrium of the second period. Now, incumbent has to choose an optimal $p_0$. Note that we know that for $p_0 < p_0^A$, there is a compete equilibrium as specified in proposition 1 and the incumbent makes $(\frac{3s}{\delta})^2$. For some $p_0^A < p_0 < p_0^B$, there is a mixed strategy equilibrium as specified in proposition 2 and incumbent’s profit is simply $p^2$. For some $p_0^B < p_0 < p^*_0$, the equilibrium is still characterized by 2 but incumbent’s profits are somewhat different and given by $2\left(\sqrt{R - p_0}\right)p$. Finally, once $p_0 = p^*_0$, the equilibrium is specified by proposition 3 and incumbent makes $2\left(\sqrt{R - p_0}\right)p_0$. Going beyond $p^*_0$ is clearly sub-optimal.

What is the optimal $p^*_0$? The answer depends on the second and first period profits. One of the key contributions of this paper is to be able to define these regions such that incumbent can strategically raise its first period price and force any of the regions above. But, by raising its price in the first period, it forgoes profits. Therefore, such strategic behavior is profitable only if second period profits gains can recoup this loss. We argue in the following section that depending on the switching costs, it is indeed profitable for incumbent to act strategically and overprice its products in first period.

Note that incumbent makes $(\frac{3s}{\delta})^2$, when it is competing. Therefore, the strategic pricing will be profitable only if by raising prices, incumbent can be made more
money than compete strategy. Therefore, gains in the profits are simply

\[ G = \Pi_I(p_0, p_1) - \left( \frac{3 + s}{3} \right)^2 \]

On the other hand, the loss in the profit due higher \( p_0 \) is

\[ L = 2 \left( \sqrt{R - p_0} \right) p_0 - 2 \left( \sqrt{R - p_m} \right) p_m \]

Without strategic \( p_o \), incumbent would offer monopoly price in first period. First note that \( \Pi_I(p_0, p_1) \) is either \( \bar{p}^2 \) or \( 2 \left( \sqrt{R - p_0} \right) \bar{p} \) depending on \( p_0 \). But as \( p_0 \) increases \( \bar{p} \) increases as well, leading to higher second period profits for incumbent because incumbent in moving from compete regime to non-compete regime. Therefore, as \( p_0 \) increases gains \( G \) increases but the Loss \( L \) increases as well. Therefore, the incumbent is solving the following optimization problem as long as it is profitable to do, i.e; as long as \( G + L > 0 \).

\[
\begin{align*}
\text{Max}_{p_0} & \quad G + L \\
\text{sub} & \quad \bar{p} \leq p_0
\end{align*}
\]

Or,

\[
\begin{align*}
\text{Max}_{p_0} & \quad \left( 2\sqrt{R - p_0} \right) \bar{p} - \left( \frac{3 + s}{3} \right)^2 + 2 \left( \sqrt{R - p_0} \right) p_0 - 2 \left( \sqrt{R - p_m} \right) p_m \\
\text{sub} & \quad \bar{p} \leq p_0
\end{align*}
\]

In the appendix, we show that for some \( s > s_{low} \), incumbent would like to offer \( p_0 > p_m \). In other words, incumbent wants to overprice its product in the first period. Formally,
**Proposition 4** For some $s < s_{low}$, incumbent offers a monopoly price $p_0 = \frac{2R}{3}$ in the first period and firms compete in second period. For some $s_{low} < s < s_{high}$, incumbent overprices its product in first period and there is a mixed strategy equilibrium as specified in proposition 3. Finally, for some $s > s_{high}$, there exists a pure strategy equilibrium where incumbent overprices its product and offers $p_1 = p_0 = p^*$ and entrant offers $p_2 = 2\sqrt{R - p_0} - p_1 + 1$.

Since we do not have a close form solution for $p_0^*$ and hence $s_{low}$ and $s_{high}$, we provide a numerical example to clarify our intuition. In the following table (table-1), first column is the reservation price $R$. Second column is the minimum switching costs $s_{low}$ required to sustain overpricing strategy by the incumbent. Third column is the optimal price $p_0^*$, at that switching cost. Fourth column is the switching cost $s_{high}$ required to sustain pure strategy non compete equilibria. Fifth column is the optimal $p_0^*$ at that switching cost. Final column is simply the monopoly price $p^m$.

<table>
<thead>
<tr>
<th>$R$</th>
<th>$s_{low}$</th>
<th>$p_0^*(s_{low})$</th>
<th>$s_{high}$</th>
<th>$p_0^*(s_{high})$</th>
<th>$p^m = \frac{2R}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.60</td>
<td>0.010</td>
<td>1.34</td>
<td>0.11</td>
<td>1.22</td>
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<td>1.53</td>
<td>0.25</td>
<td>1.34</td>
<td>1.20</td>
</tr>
<tr>
<td>2.00</td>
<td>0.040</td>
<td>1.71</td>
<td>0.43</td>
<td>1.46</td>
<td>1.33</td>
</tr>
<tr>
<td>2.20</td>
<td>0.062</td>
<td>1.88</td>
<td>0.67</td>
<td>1.57</td>
<td>1.46</td>
</tr>
<tr>
<td>2.40</td>
<td>0.090</td>
<td>2.06</td>
<td>0.97</td>
<td>1.68</td>
<td>1.60</td>
</tr>
<tr>
<td>2.60</td>
<td>0.124</td>
<td>2.24</td>
<td>1.34</td>
<td>1.79</td>
<td>1.73</td>
</tr>
<tr>
<td>2.80</td>
<td>0.164</td>
<td>2.59</td>
<td>1.84</td>
<td>1.89</td>
<td>1.86</td>
</tr>
</tbody>
</table>

**Table 1: Critical Switching Cost**

Few interesting observations are worth noting. As switching cost increases, the first period price $p_0$ necessary to sustain collusive prices declines. Intuitively, higher switching costs make it easier for incumbent to force non compete prices. For very large switching cost $s$, incumbent could potentially offer monopoly prices in both
periods. As \( R \) increases, it is harder to sustain pure strategy equilibrium. Again, higher \( R \) means more and more market is covered in first period. Incumbent finds it increasingly difficult to offer enough market share to entrant to sustain pure strategy equilibrium. But note that mixed strategy equilibrium is readily achieved. It is worth pointing out that even in mixed strategy equilibrium, incumbent generally mixes between \( p_1 = p_0 \) and \( p_1 = \pi \) \(^9\).

To calculate the firm profits prices, we use the following numerical example to provide a clearer insight. For \( R = 2 \) the monopoly price by the incumbent in first period is \( 2R/3 = 1.33 \). As seen from the table, \( s > 0.43 \) for pure strategy equilibrium to be sustainable. Let \( s = 0.45 \). If incumbent was not strategic then it would offer monopoly price in the first period and firms will compete in second period. The monopoly profits are given by \( 2\sqrt{R-p^m}p^m = 2.17 \). In the second period, incumbent offers \( p_1 = \frac{3+s}{s} = 1.15 \) and entrant offers \( p_2 = \frac{3-s}{s} = 0.85 \). The incumbent’s second period profit is \( (\frac{3+s}{3})^2 = 1.32 \) for the total profit of \( \Pi_I = 3.5 \). The entrant’s profits are given by \( (\frac{3-s}{3})^2 = 0.72 \). If the incumbent acts strategically and overprices its product in first period then it offers \( p_0^i = 1.45 \) in both periods and entrant offers \( 2\sqrt{R-p_0} + p_0 - 1 = 1.93 \) in second period. Now incumbent forgoes some profits in first period by making only \( 2\sqrt{R-p_0p_0} = 2.14 \) but gains a lot in the second period by again making \( 2\sqrt{R-p_0p_0} = 2.14 \) for the total of 4.28. Entrant also makes the profit of 1.02. It is immediately clear that both incumbent and entrant offer high prices and make higher profits. Clearly, consumer welfare reduces considerably.

\(^9\)For low value of switching cost, \( p_1 = \pi \) will be less that \( p_0 \).
4 Sophisticated Consumer

In the previous section, our analysis was based on the “myopic” consumer. The consumer were not forward looking and did not take into account the impact of their first period actions on the next period firm strategies. In this section, we extend our model by incorporating forward looking or “sophisticated” consumers.

4.1 Compete Region

Consider the compete equilibrium we established in the previous section where incumbent offers monopoly price \( p_m = \frac{2R}{3} \) in the first period and covers \( \sqrt{R - p_0} \) market. Consider a customer who is located at a distance \( \sqrt{R - p_0} - \epsilon \) and buys from the incumbent in first period. Clearly her surplus is zero. In period two, the same customer buys from the entrant but pays a switching cost \( s \) to switch. If the customer were forward looking then it is immediate obvious that it could choose to not buy from the incumbent in the first period. This allows her to not pay any switching cost later. But this also means that the incumbent’s demand is reduced and hence the first period pricing will change as well. Consider a customer at a distance \( d \) from the incumbent who is indifferent to buying or not buying from the incumbent in first period when it buys from the entrant in second period. For her to be indifferent, the following must hold:

\[
0 + R - (1 - d)^2 - \frac{3 - s}{3} = R - d^2 - p_m + R - (1 - d)^2 - \frac{3 - s}{3} - s
\]

where \( lhs \) is the the utility of the customer when she does not buy from incumbent in first period and \( rhs \) is utility when she buys from the incumbent in first period. This gives us the optimal value of \( d \) or demand for the incumbent when it offer monopoly
price

\[ d = \sqrt{\frac{R}{3} - s} \]

Note that this demand is less compared to myopic case where the first period demand in case of monopoly pricing is \( \sqrt{\frac{R}{3}} \). Therefore, optimal price offered by the incumbent is no more \( p_m = \frac{2R}{3} \). The following proposition formalizes the optimal first period pricing in case of sophisticated consumer.

**Proposition 5** In case of forward looking sophisticated customer, incumbent offers a penetration price of \( p_0 = \frac{2(R-s)}{3} \) in first period. In the second period, both the incumbent and entrant compete and offer \( \frac{R+s}{3} \) and \( \frac{R-s}{3} \) as competitive pricing.

This is an interesting result which corroborates many empirical instances of firm offering low pricing to attract customers. Note that higher the switching costs, the lower the price incumbent wants to offer to be able to attract a larger market share. In what follows, we will show that for certain switching cost \( s < s^* \), incumbent offers penetration pricing and for \( s > s^* \), it offers high pricing.

### 4.2 Non-Compete Region

Now consider the non-compete equilibria we established in previous section. Incumbent could offer \( p_0(s^*) \) and entrant will be forced to stay in non-compete region. But consumers pay high price in both periods reducing their welfare. Can consumers change the strategy of the firms and force them to compete in second period by behaving strategically in first period? The answer apparently is yes.

Consider the customer who is located at a distance \( \sqrt{R - pU + \epsilon} \) and does not buy from the incumbent in first period. If it buys the product in period 1 from
incumbent then it incurs a small negative disutility. Since entrant is just indifferent between compete and non-compete region, that consumer can make the entrant now choose compete region. In case of competition, both firms reduce their second period prices according to proposition 1. This leads to significantly higher surplus for the user in second period. In contrast, the same user would have got \( \epsilon \) surplus in second period if there were no competition and zero surplus in first period. Clearly, the user by strategically buying in first period can force competition in second period. But can incumbent now act differently to alleviate the impact of competition and force *non-compete* behavior? The answer is again yes. Incumbent could again act strategically by increasing its first period price \( p_0 \) to force the *non-compete* behavior. This trade-off between incumbent wanting the *non-compete* equilibrium and consumers opting for *compete* characterizes the dynamics of this game.

Consider a user located a distance \( x \) from \( \sqrt{R - p_0} \) who did not buy the product in period 1. Suppose that all users in the range \( \sqrt{R - p_0} + x \) to \( \sqrt{R - p_0} \) buy the product in period 1 and incur some negative utility. The following figure clarifies our exposition.

From the previous section, we know that if entrant chooses to compete then its profits are given by eq ?? such that \( \pi_E(p_0) = \frac{(1+p_0-x)^2}{4} \). Similarly, if the entrant chooses to stay in ”non-compete” region then the profits are given by eq ?? ?. Note that entrant’s demand in this case is \( 1 - \sqrt{R - p_0} - x \) and not \( 1 - \sqrt{R - p_0} \). Therefore, entrant’s price will be \( p_2 = R - (1 - \sqrt{R - p_0} - x)^2 \). Therefore, the profit in ”non-compete” region for entrant is

\[
\pi_{{Non-Compete}}^E = 2 \left( 1 - \sqrt{R - p_0} - x \right) \left( R - (1 - \sqrt{R - p_0} - x)^2 \right)
\]
For entrant to be able to compete, following should be true

$$\frac{(1 + p_0 - s)^2}{4} > 2 \left( 1 - \sqrt{R - p_0} - x \right) \left( R - (1 - \sqrt{R - p_0} - x)^2 \right)$$  \hspace{1cm} (12)

For some \( x = x^* \) this inequality will hold. But given such \( x^* \), the consumer (who is located at \( x^* \)) should find it beneficial to force competition. In other words, the disutility it incurs in first period should be justified in second period. The total two period utility when firms compete is\(^{10}\)

\[
U_c = (R - p_0 - (\sqrt{R - p_0} + x)^2) + \left( R - s - \left( \frac{3 - s}{3} \right) - (1 - \sqrt{R - p_0} - x)^2 \right) \\
U_e = (-2x\sqrt{R - p_0} - x^2) + \left( R - s - \left( \frac{3 - s}{3} \right) - (1 - \sqrt{R - p_0} - x)^2 \right)
\]

First term is the utility of buying from incumbent in first period (it is negative) and the second term is the utility of buying from entrant in second period. In contrast, if user does not force competition, and chooses to stay in ”non-compete” region then the utility it gets is given by proposition (4)

\[
U_{nc} = R - \left( 1 - x - \sqrt{R - p_0(s^*)} \right)^2 - (2\sqrt{R - p_0(s^*)} + p_0(s^*) - 1)
\]

Again, for some \( x = x^{**} \), the following condition will hold.

\[
U_c > U_{nc} \hspace{1cm} (13)
\]

Since we do not have close form solutions for \( p^*(s) \), equation 15 does not have any

\(^{10}\)Note that not all users in the segment \([\sqrt{R - p_0}, x^*]\) will buy from the entrant as our equation suggests. Some of them will continue to buy from incumbent even in the second period. But those prices still will be competitive prices. The lower bound on the utility \( U_c \) comes at \( x = x^* \). If it is profitable for the user located at \( x^* \) to force competition then it is true for all users in \([\sqrt{R - p_0}, x^*]\).
close form solution in \( x \). Similarly, eq (14) also does not have any close form solution for \( x \). Therefore we will define some properties of the solutions of equation (14) and (15) and use those properties to establish equilibrium.

First consider the solution \( x^* \) of eq(14). Note that higher \( x \) reduces customer incentive to force competition. In other words, as the marginal user is further away from incumbent, it incurs larger disutility in first period, which in turn, will act as an disincentive to buy in first period. Therefore, given the presence of sophisticated customers, incumbent would like to offer \( p_0 \) such that it increases \( x \), which in turn may lead to non-compete equilibrium. It can be verified from eq(14) that for a fixed \( p_0 \) the right hand side of eq(14) is decreasing in \( x \). Clearly, as \( x \) increases, entrant is left with less market in non-compete region and it prefers to compete. For a fixed \( x \), it is also verifiable that both sides of equation are increasing in \( p_0 \). As incumbent increases its price, entrant’s profit in compete region, as well as in non-compete region increases. But from the previous section, we know that for high enough \( p_0 \), the non-compete region profit dominates and entrant chooses to stay in non-compete region. Therefore, for a fixed \( x \), as incumbent increases its price, the entrant finds it profitable to stay in non-compete region. Clearly, to counter this effect, \( x \) would like to increase itself, such that entrant chooses to compete. The same idea applies to \( s \). As the switching costs increase, entrant finds it hard to compete. Therefore, \( x \) again has to increase to compensate for the loss in profit due to switching cost. The following lemma formalizes this intuition.

**Lemma 1** \( x^* \) is increasing in \( p_0 \) and \( s \).

Therefore for a given \( p_0 \) and \( s \), there exists a \( x^* \) such that sophisticated customers can force competition. Clearly, both incumbent and customers will play this game such that either it becomes unprofitable for the incumbent to raise its price, or
customer to increase \( x \).

Now consider the solution of eq 15. We denote it by \( x^{**} \). First note that at \( x = 0 \), \( U_c \) is higher than \( U_{nc} \), otherwise, not a single customer would like to force competition. For a fixed \( p_0 \), \( U_c \) is decreasing in \( x \). It is intuitively obvious. As \( x \) increases, the negative utility of buying from incumbent increases. Therefore the benefits of forcing competition decreases. On the other hand, \( U_{nc} \) is increasing in \( x \). Again, intuitively it is clear that as \( x \) increases, customer is closer to entrant, and therefore, its surplus increases with higher \( x \). Therefore, for a fixed \( p_0 \), there exists a \( x^{**} \) such that \( U_c = U_{nc} \). Any \( x > x^{**} \) would make customer prefer “non-compete” region. How does \( x^{**} \) change with \( p_0 \)? By implicit function theorem, we get the following

\[
\frac{\partial x}{\partial p_0} = -\frac{1 - \sqrt{R - p_0} - x}{2\sqrt{R - p_0}(\sqrt{R - p_0} + x)}
\]

This is clearly negative. Therefore, for a higher \( p_0 \), \( x^{**} \) decreases. Similarly, when \( s \) increases, the entrant’s incentive to compete decreases. Therefore, \( U_c \) will fall. Therefore \( U_{nc} \) should fall as well which is possible only by decrease in \( x^{**} \).

**Lemma 2** \( x^{**} \) is decreasing in \( p_0 \) and \( s \).

Note that beyond \( x^{**} \), the consumer does not want to force competition and at least \( x^* \) is required for entrant to compete. Therefore, a consumer can force entrant into competition only if \( x^* < x^{**} \). Note from two lemmas above that for a higher \( p_0 \), this condition becomes hard to satisfy. In other words, incumbent can set high \( p_0 \) and avoid competition in next period.

Let’s consider the strategy for incumbent. In table - 1, we outlined the minimum \( p_0 \) needed for a given \( s \) such that incumbent can force non-compete equilibrium. Let’s
us denote that \( p_0 \) with \( p_0^{\text{min}} \). When incumbent increases the price from \( p_0^{\text{min}} \), it moves further away from monopoly price and its profits are eroded. Also note that for \( p_0^{\text{min}} \), \( x^* < x^{**} \). If that is not so then switching costs are so high that entrant can never be forced into competition.

This suggests that incumbent would like to increase its price \( p_0 \) from \( p_0^{\text{min}} \) and force \( x^* > x^{**} \) such that entrant stays in non-compete region in the presence of sophisticated consumers. But it can not increase its price beyond \( p_0^{\text{max}} \) as we will show below. Therefore, if for a given \( p_0 \in [p_0^{\text{min}}, p_0^{\text{max}}] \), it can force \( x^* > x^{**} \), then it is assured of a non-compete equilibrium, otherwise it has no option but to compete with the entrant. In case of competition, it offers monopoly price in first period and competitive pricing in second period according to proposition 1.

The maximum \( p_0^{\text{max}} \) offered by incumbent can be realized by solving the following equation

\[
4 \left( \sqrt{R - p_0} \right) p_0 = 2 \left( \sqrt{R - p_0^{\text{mn}}} \right) p_0^{\text{mn}} + \left( \frac{3 + s}{3} \right)^2
\]

where \( p_0^{\text{mn}} = \frac{2R}{3} - s \) is the penetration price in case of competition. Note that for low values of \( p_0 \), the \( \text{lhs} \) of the equation will dominate \( \text{rhs} \) because incumbent is always better as a monopoly than competition. As \( p_0 \) increases, \( \text{lhs} \) will start monotonically decreasing while \( \text{rhs} \) is a constant. Therefore, there exists a \( p_0^{\text{max}} \) such that the incumbent would not like to increase \( p_0 \) beyond that. This means that incumbent can increase its price until \( p_0^{\text{max}} \) and force the condition \( x^* > x^{**} \) such that entrant stays in ”non-compete” region. If for \( p_0^{\text{max}} \), \( x^* < x^{**} \), then incumbent has no option but to be in ”compete” region. Given these strategies of consumers and incumbent the equilibrium of this game is outlined below.
Proposition 6 For a given $s$, in the presence of sophisticated consumers, if there exists a pure strategy equilibrium in non-compete region then $p_0^* >= p_0^{min}$ and $p_2 = (2 \sqrt{R - p_0} + p_0 - 1)$.

This proposition suggests that in the presence of sophisticated customers price offered by incumbent increases compared to myopic consumer case. Therefore, the price offered by entrant decreases in non-compete region. Note that this equilibrium may not exist for low switching costs as we show below and incumbent has no alternative but to compete.

We will continue with the same numerical example as in previous section to get some insights. Let $R = 2$, $s = 0.45$. Then $p^m = \frac{2R}{3} = 1.33$. In the absence of sophisticated customers, we know that in equilibrium, incumbent will set $p_0^{min} = 1.46$ (from table -1). Given these parameters, we want to explore the equilibrium in the presence of sophisticated customers. First note that $p_0^{max} = 1.68$. Therefore incumbent will be willing to set its price $p_0$ between $[1.46 \quad 1.68]$, if it can force non-compete region. For non-compete region the condition of $x^* > x^{**}$ should be satisfied. In our example, $x^* = 0$ as expected and $x^{**} = 0.345$ which is too high. Therefore, in the presence of sophisticated consumers the previously outlined equilibrium does not hold.

Let incumbent increase its price to $p_0 = p_0^{max} = 1.68$. Now $x^* = 0.106$ and $x^{**} = 0.347$. Again, since $x^* < x^{**}$, only equilibrium possible is compete equilibrium. Therefore, for the same switching costs, incumbent can not force non-compete region and is forced to compete. If switching costs $s$ increase now, then from lemma 2 and 3, we know that $x^{**}$ will decrease and $x^*$ will increase such that eventually $x^* > x^{**}$. Therefore, there exists such $s^{**}$, that allows incumbent to offer $p_0 = p_0^{max}$ and force non-compete region. Once $s^{**}$ increases further, incumbent can lower $p_0$ from $p_0^{max}$.
We provide the numerical critical value of $s^{**}$ in the presence of sophisticated customers, that will enable incumbent to force non-compete equilibrium. The following table (table-2) outlines the critical switching cost such that incumbent offers $p_0^{max}$ and force non-compete region.

<table>
<thead>
<tr>
<th>$R$</th>
<th>$s^{**}$</th>
<th>$p_0^{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.60</td>
<td>0.29</td>
<td>1.35</td>
</tr>
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<td>0.47</td>
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<td>2.60</td>
<td>1.32</td>
<td>2.39</td>
</tr>
<tr>
<td>2.80</td>
<td>1.55</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Table 2: Critical Switching Cost

Compared to table -1 these switching costs are generally higher as one would expect. In the presence of sophisticated customers, higher switching costs are needed for the incumbent to sustain ”non-compete” equilibrium. At the same time, incumbent is also offering higher prices. Therefore, incumbent offers higher prices and leaves more market for entrant to induce it into ”non-compete” region. Note that entrant now offers lower prices than before.

Again starting with previous example. Let $R = 2$ and $s = 0.66$. Now incumbent offers $p_0 = 1.68$ and entrant offers $p_2 = 1.81$. Incumbent makes the profit of 3.8 while entrant makes the profit of 1.57. Note that just the threat of consumers behaving strategically forces incumbent to offer high prices if the switching costs are high enough. Otherwise, it offers penetration pricing and competes in next period.
5 Appendix: Proofs

Entrant’s Location Let’s assume that the entrant locates a distance \((1 - b)\) from the incumbent and offer \(p_2\) and incumbent offers \(p_1\). From the left side of the segment the following equality has to be satisfied where \(x\) is demand for the incumbent.

\[
R - x^2 - p_1 = R - (1 - x - b)^2 - p_2 - s
\]

Similarly, from the right side of the segment the following equality has to be satisfied where \(y\) is demand for the incumbent.

\[
R - y^2 - p_1 = R - (1 - y + b)^2 - p_2 - s
\]

The total demand for incumbent is \(x + y = \frac{1 - p_1 + p_2 - b^2 + s}{1 - b^2 - s}\) and for entrant \(2 - x - y = \frac{1 - p_1 - p_2 - b^2 - s}{1 - b^2 - s}\). Maximizing the profits and solving the two equations lead to \(p_1^* = \frac{3 + s - b^2}{3}\) and \(p_2^* = \frac{3 - s - b^2}{3}\). Substituting these prices back into the profit equation leads to \(\pi_I = \frac{(3 + s - b^2)^2}{9(1 - b^2)}\) and \(\pi_E = \frac{(3 - s - b^2)^2}{9(1 - b^2)}\). It is clear that as \(b\) increases, the profits reduce for both incumbent and entrant. Therefore, the optimal strategy is for the entrant to locate at \(b = 0\); i.e. maximally differentiate.

**Proposition 1** From eq 1, the demand for incumbent in second period is \(2x = 2\left(\frac{1 - p_1 + p_2 + s}{2}\right)\). Hence the incumbent maximizes

\[
\pi_I(p_1, p_2) = (1 - p_1 + p_2 + s)p_1
\]

\[
2p_1 = 1 + p_2 + s
\]
Similarly, the entrant’s demand is $2(1 - x)$. Therefore, the entrant is maximizing

$$
\pi_E(p_1, p_2) = (1 + p_1 - p_2 - s) p_2 \\
2p_2 = 1 + p_1 - s
$$

Solving these two equations leads to $p_1^* = \frac{3 + s}{3}$ and $p_2^* = \frac{3 - s}{3}$.

\[\blacksquare\]

**Proposition** ?? We know that

$$
\Pi_I(p_0) = G + L = 2 \left( \sqrt{R - p_0} \right) \overline{p} - \left( \frac{3 + s}{3} \right)^2 + 2 \left( \sqrt{R - p_0} p_0 - 2 \left( \sqrt{R - p_m} \right) p_m
$$

with the constraint such that $\overline{p} \leq p_0$. Taking the first order condition

$$
\frac{\partial \Pi_I}{\partial p_0} = 2 \left( \sqrt{R - p_0} \right) \frac{\partial \overline{p}}{\partial p_0} - \frac{\overline{p}}{\sqrt{R - p_0}} - \frac{p_0}{\sqrt{R - p_0}} + 2\sqrt{R - p_0} \\
\frac{\partial \Pi_I}{\partial p_0} = 2 \left( \sqrt{R - p_0} \right) \left( \frac{\partial \overline{p}}{\partial p_0} + 1 \right) - \frac{1}{\sqrt{R - p_0}}(p_0 + \overline{p})
$$

(14)

Note that

$$
\frac{\partial \overline{p}}{\partial p_0} = \frac{1}{\sqrt{R - p_0}} \left( 2 + \frac{\sqrt{s}}{\sqrt{2(1 - \sqrt{R - p_0})}} \right)
$$

Sustituting this leads to

$$
\frac{\partial \Pi_I}{\partial p_0} = 4 + \frac{\sqrt{2s}}{\sqrt{(1 - \sqrt{R - p_0})}} + 2\sqrt{R - p_0} - \frac{1}{\sqrt{R - p_0}}(p_0 + \overline{p}) = 0
$$

where $\overline{p} = 3 + s - 4\sqrt{R - p_0} + \sqrt{8s(1 - \sqrt{R - p_0})}$. Solving for optimal $p_0^*$ is obviously difficult but it is easy to show that optimal $p_0 > p_m = \frac{2R}{3}$. In other words, incumbent
would like to overprice its product. It can be easily verified that the left hand side of
the above equation is decreasing in \( p_0 \) and increasing in \( s \). At \( s = 0 \) and \( p_0 = p_m = \frac{2R}{3} \),
we can rewrite the equation as

\[
\frac{\partial \Pi_I}{\partial p_0} = 4 + 2 \sqrt{\frac{R}{3} - \frac{3}{R}} \left( \frac{2R}{3} + 3 - 4 \sqrt{\frac{R}{3}} \right) \\
\frac{\partial \Pi_I}{\partial p_0} = 8 - \frac{3\sqrt{3}}{\sqrt{R}}
\]

which is clearly positive. Therefore, for any \( s \), optimal \( p_0 \) has to be higher than \( p_m \)
to make \( \frac{\partial \Pi_I}{\partial p_0} = 0 \). Note that \( \frac{\partial \Pi_I}{\partial p_0} > 0 \). Eventually for some \( s \) and \( p_0 \), \( \overline{p} = p_0 \) and our
constraint will kick in and ensure that optimal \( p_0^* \) is simply the solution of \( \overline{p} = p_0 \). In
such case, we will observe \( p_0 = p_1 = p_0^* \).

We now show that existence of \( s_{low} \). First note that \( \Pi_I(p_0^*) > 0 \) for incumbent
to pursue the policy of overpricing. At \( s = 0 \), and with some simplifications, we can
rewrite \( \Pi_I \) as

\[
\Pi_I(p_0) = G + L = 2 \left( \sqrt{R - p_0} \right) (6 + 2p_0) - 1 - 8(R - p_0) - 2 \left( \sqrt{R - p_m} \right) p_m
\]

The following figure highlights that there exist no \( p_0 \) at \( s = 0 \), which makes \( \Pi_I(p_0) > 0 \).

From the previous section, we know that \( \frac{\partial \Pi_I}{\partial s} > 0 \). Therefore, for some \( s = s_{low} \),
\( \Pi_I(p_0^*) > 0 \) and incumbent would find it profitable to overprice its product.


References


31


