

Cognitive inconsistencies and non-symmetric friendship ^{*}

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Abstract

Non-reciprocated relationships, such as all workers knowing the president of the company but only a few of the workers being known by the president, and non-symmetric relationships, such as workers thinking that they know the president and thinking that the president does not know them, are endemic to most social situations. While such inconsistencies may be expected in relationships such as giving advice and lending money, they are rarely expected to occur in seemingly symmetric relationships such as friendship. Nevertheless, they do. We suggest that research in this area has been hampered by the confused language used for describing 'symmetries' and 'non-symmetries'. We present a framework for thinking about these relations that clearly distinguishes cognitive inconsistencies and non-symmetric and non-reciprocated relations. Then, we employ this framework and constructural theory to suggest that owing to cognitive inconsistencies, any interaction-based relationship, including friendship, can potentially be non-symmetric. We examine a series of hypotheses concerning interaction and interaction-based behaviors that derive from this theory using friendship relations. We find that we are able to predict both who is friends with whom, non-symmetry in friendship, and non-reciprocities in the expectation for and recall of friendship.

0. Introduction

The study of friendships has been a focus of researchers for several decades (Moreno 1934; Newcomb 1961; Bell 1981; Fischer 1982; Hallinan and Williams

^{*} This work was supported in part by a grant provided by the NSF under grant No. IST-8607303.

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1987; Wellman 1988; Krackhardt 1992). Part of this work has uncovered structural consequences of friendships, but a good portion of this research has explored the more basic question of what constitutes a friendship link. Throughout this research a common thread has been the assumption that friendships are inherently symmetric. This assumption, however, is suspect. Ego biases (Kumbasar 1993, 1994) and other factors may affect the degree of symmetry in these networks. Further, research in this area has been hampered by the lack of a consistent vocabulary for describing non-symmetries. The use of an inconsistent vocabulary has resulted in researchers ignoring fundamental cognitive differences in the basis for non-symmetry in a dyadic relation such as friendship. In this paper, we present a framework for discussing cognitive inconsistencies and non-symmetry with respect to relationships. We then demonstrate that there is a systematic cognitive basis for the existence of non-symmetry in friendship.

1. A framework for discussing cognitive inconsistency and dyadic non-symmetry

In discussing the relations between individuals, it is important to distinguish between the tie at the sociometric level and the tie at the full ordered pairwise/paired comparison (Krackhardt style data) level. At the sociometric level we are concerned with ego's self report of his or her relations with others. At the sociometric level the relation is of the form R_{ij} such that R_{ij} represents Person i sends Relation R to Person j . In contrast, when full ordered pairwise/paired comparisons are used we are concerned with each person's report on the relations between all other pairs of individuals, whether or not ego is actually one of the members of the pair. Thus, at the Krackhardt style data level the relation is of the form R_{ijk} such that $R[ijk]$ represents that Person k perceives that Person i sends Relation R to Person j (Krackhardt 1987a).

In either type of data inconsistencies can arise. Further, procedures for compressing Krackhardt style cognitive-social network data into sociometric data may induce certain types of inconsistencies. In the case of sociometric data, inconsistencies exist just in the case $R_{ij} \neq R_{ji}$. Whether these inconsistencies result from non-symmetries or non-reciprocities at the cognitive level depends on how the data was collected, as will be seen. However, for Krackhardt style data, there are many different types of inconsistencies. In Krackhardt style data inconsistencies, specifically cognitive inconsistencies, occur for a pair of individuals i and j , where R_{iji} does not equal any of the following: R_{ijj} ; R_{jii} ; R_{jij} .

These various types of inconsistencies have been confused. This confusion exists at two levels. First, words that describe different types of inconsistency are often used interchangeably: e.g. asymmetry¹ and non-reciprocity. For example, Holland

¹ In contrast, and in keeping with graph theory, we reserve the term asymmetry to refer to a property of a matrix. A matrix is asymmetric in the case where for every (i, j) dyad either $(R_{ij} = 1 \text{ and } R_{ji} = 0)$ or $(R_{ij} = R_{ji} = 0)$.

and Leinhardt (1979: 66) note that the central question they are trying to answer is whether there is "... anything in sociometric data besides... a tendency for choices to be reciprocated." They answer this question by looking at the u-m, a, n distribution and note that (1979: 72) "from the triad census t we can obtain ... the number of mutual, m, asymmetric, a, and null, n, dyads in the di-graph." Second, theoretical discussions often move back and forth between the sociometric and Krackhardt style data level without denoting the level at which the argument holds.

Before proceeding, it is worth separating out the different types of inconsistencies that can arise at the Krackhardt style data level. We are concerned only about those inconsistencies in which the perceiver is directly involved. Consequently, there are only four 'relational primitives' for any (i, j) pair. In terms of friendship, these primitives are:

- (1) R_{iji} : whether i perceives that self (i) considers other (j) a friend;
- (2) R_{ijj} : whether j perceives that other (i) considers self (j) a friend;
- (3) R_{jii} : whether i perceives that other (j) considers self (i) a friend;
- (4) R_{jjj} : whether j perceives that self (j) considers other (i) a friend.

For any (i, j) dyad, then, there are the following 16 outcomes (where, e.g., $R_{ijk} = 1$ implies that k perceives that i considers j a friend; $R_{ijk} = 0$ implies that k perceives that i does not consider j a friend):

	R_{jji}	R_{ijj}	R_{jii}	R_{jjj}
A	0	0	0	0
B	0	0	0	1
C	0	0	1	0
D	0	0	1	1
E	0	1	0	0
F	0	1	0	1
G	0	1	1	0
H	0	1	1	1
I	1	0	0	0
J	1	0	0	1
K	1	0	1	0
L	1	0	1	1
M	1	1	0	0
N	1	1	0	1
O	1	1	1	0
P	1	1	1	1

Of these outcomes, only the A and P outcomes are completely consistent; the remaining 14 possible outcomes show some sign of inconsistency or disagreement about whether i and j are friends. Inconsistencies occur any time any two of the

relational primitives above disagree. There are six possible disagreements (four primitives taken two at a time), each representing one of four possible types of disagreements.

Primitive inequality	Type of disagreement
$R_{iji} \neq R_{ijj}$	Non-confirmation
$R_{iji} \neq R_{jii}$	Non-symmetry
$R_{iji} \neq R_{jij}$	Non-reciprocity
$R_{ijj} \neq R_{jii}$	Reflected non-reciprocity
$R_{ijj} \neq R_{jij}$	Non-symmetry
$R_{jii} \neq R_{jij}$	Non-confirmation

Non-confirmations reflect a lack of agreement between the two parties about the existence of a tie from i to j (or vice versa). Non-symmetries stem from one perceiver's inconsistency in his or her belief that the tie is reciprocated (resulting in a non-symmetric matrix within his or her cognitive map of the structure). Non-reciprocities (or reflected non-reciprocities) represent a difference between the two parties each sending (or receiving) a relation to (or from) the other.

Depending on how sociometric data are collected, then one of these inconsistencies may underlie that data. For example, if the sociometric network is the result of asking each individual to state for each other individual whom i has relation R to, then resultant inconsistencies in the sociometric data are due to $R_{iji} \neq R_{jij}$. That is, inconsistencies in the sociometric data are due to non-reciprocities in the corresponding locations in the Krackhardt style data.

Taking account of these different bases for inconsistencies at the cognitive level, i.e. in Krackhardt style data, can actually help clarify various discussions in the literature. We argue that Heider would be most concerned about non-symmetries, Davis would be most concerned about non-reciprocities, and non-confirmations are often the methodological concerns of researchers who wish to uncover actual relations from one party to the next (e.g. Krackhardt 1990).

Let us consider these first two claims in more detail. Heider's formulation of inconsistency is not based on actual balance, but rather based on people's perceptions of the reciprocated liking: i.e. on non-symmetry. That is, according to a strict interpretation of Heider's theory, it is not the case that if Person 1 likes Person 2, and Person 2 likes Person 1 that balance exists; rather, balance exists if Person 1 thinks that Person 1 likes Person 2 and Person 1 thinks that Person 2 likes Person 1. In this case, an inconsistency (or lack of balance) exists in the case where $R_{iji} \neq R_{jii}$. This type of inconsistency is actually a case of non-symmetry in Krackhardt style data. On the other hand, Davis's idea of balance is more tied to the traditional notion of reciprocity: Person 1 likes Person 2, and Person 2 likes Person 1. In this case, the person doing the 'liking' decides whether the relationship exists. That is, the relationship tie from Person 1 to Person 2 is determined by Person 1's perception, whereas the relationship tie from Person 2 to Person 1 is determined

by Person 2's perception. In Davis's view, an inconsistency occurs when $R_{iji} \neq R_{jij}$. This is actually a case of non-reciprocity in Krackhardt style data.

The implications and source of the inconsistencies in Krackhardt style data are different. For example, a Heiderian inconsistency or imbalance is observed in outcome J (1 0 0 1) above: Person i sees self as considering Person j a friend, but does not see Person j reciprocating that friendship; similarly, Person j sees self as considering Person i a friend, but does not see Person i reciprocating that friendship. This outcome is not inconsistent from the Davis perspective, however, since both parties see themselves as friends of the other.

As we noted earlier, it is often the goal of a research agenda to translate these primitives using a set of combining rules into a two-dimensional relational matrix (Krackhardt 1987a). For example, one rule might be that $R_{ij} = 1$ iff $R_{iji} = 1$ (this is the one used by Davis and most researchers in their studies). Another might be that $R_{ij} = 1$ iff $R_{iji} = 1$ and $R_{ijj} = 1$ (Krackhardt 1990; Krackhardt and Kilduff 1990). These rules may be encoded in terms of the set of mutually exclusive outcomes of the four primitives given above. For example, the former rule above may be rewritten as

$R_{ij} = 1$ iff Outcome is from set (I J K L M N O P), else 0,

whereas, the latter can be written as

$R_{ij} = 1$ iff Outcome is from set (M N O P), else 0.

If we restrict ourselves to binary outcomes of $R_{ij} = 0$ or 1, we can enumerate all possible rules from this process by counting all possible subsets of the outcome set above. This number is equal to $\binom{16}{1} + \binom{16}{2} + \binom{16}{3} \cdots + \binom{16}{15}$, for a total of 65 534 rules that could be used to reduce the primitives to a R_{ij} matrix.

Which of the 65 thousand rules should be used is determined by the theory one is exploring. Frequently, researchers employ rules without being clear why they have chosen that particular rule. Sometimes rules are chosen because only one or two of the four primitives is available in the data collected. For example, in the Davis data sets only R_{iji} and R_{jij} are available. Thus for Davis, $R_{ij} = 1$ iff $R_{iji} = 1 \wedge R_{jij} = 1$ iff $R_{jij} = 1$. In this case, non-symmetry at the sociometric level occurs just in case non-reciprocity exists at the Krackhardt style data level. It is important to note that in the study of friendship most researchers collect sociometric data in a fashion similar to Davis (Krackhardt 1987a). That is, most collect data of the form R_{iji} and R_{jij} . Thus, for most researchers the non-symmetries observed at the sociometric level are actually non-reciprocities at the Krackhardt style data level. In writing up their results, these researchers typically use either or both terms, non-symmetry (or asymmetry) and non-reciprocity. In contrast, in the current study, we have data on all four primitives and will set forth several different rules testing different models of inconsistency.

2. Revisiting the nature of friendship

A common thread in the research on friendship has been the assumption that friendships were inherently symmetric in nature. Such assumptions were based on

an appeal to common sense and experience (e.g. Bell 1981), on empirical observations (e.g. Newcomb 1961), and on theory (e.g. Heider 1958; Davis 1968). These latter two scholars demonstrate the pervasiveness of this idea, as they come to the same conclusion from two very different perspectives. Davis, the sociologist, notes that friendship entails time spent together in the same proximity; since time and proximity are physically constrained to be symmetric, then the friendship tends toward symmetry (he further predicts transitivity with the same logic). Heider, the psychologist, makes the same prediction from a very different base. He argues that lack of symmetry in 'liking' produces imbalance and discomfort, and therefore "... it tends to become symmetrical; i.e., a balanced state exists if both (p L o) and (o L p) are true" (p. 205). Indeed, it is not infrequently that data are collected that presume symmetry; i.e. the researchers collect data in a way that does not permit non-symmetric relations to be recorded (e.g. Freeman et al. 1988).

The problem with this tendency toward symmetry is that too many non-symmetries are noted in the real world. A review of 1000 sociometric matrices by Davis and Leinhardt (1972) forced them to alter their model to include a preponderance of non-symmetric ties (these are actually non-reciprocated at the Krackhardt style data level.) Hallinan (1978) found a sizable number of non-symmetric/non-reciprocated friendship links among school children. Moreover, she found many of these links persisted, despite her best efforts to explain them away as temporary aberrations.

To reconcile these discrepancies, several scholars have appealed to psychology, suggesting that while non-symmetry may exist in the actual world (sociometric level), symmetry is the dominant model in the mind (Krackhardt style data level). Perhaps one of the most convincing lines of work that point to inherent symmetry in friendship relations was started by DeSoto (1960) and extended by Freeman (1992). DeSoto asked subjects to 'learn' a set of relationships among a group of four hypothetical people. He demonstrated that subjects took about 50% longer to 'learn' an observed interaction that was non-symmetric (and transitive) than one that was symmetric (and transitive). Freeman (1992) replicated these studies precisely and discovered that subjects made errors in their learning trials by filling in relationships that would make them symmetric.

In his perhaps most important extension of this work, Freeman (1992) argues that people have a strong tendency to view the structure of friendships around them as being symmetric and transitive, yielding an ultrametric structure. While he notes that 'real' friendship structures are not often ultrametric, the persistence in the literature is due in large part to the fact that people, including scientists, cognitively alter their perception of the social environment to minimize non-symmetries (and intransitivities). In other words, according to Freeman, individuals are cognitively forcing R_{ij} to be equal to R_{ji} . That is, individuals insist on seeing symmetries even when they do not exist.

While this work is compelling in empirical support and in logic, we point to two issues that are left unresolved. First, what the DeSoto/Freeman studies have definitively shown is that it is easier to learn friendship structures that

are symmetric and transitive.² That does not mean that people cannot learn non-symmetric ones, nor that they refuse to admit that they exist. In fact, in both the Freeman and DeSoto studies, people did learn the non-symmetric relations – it just took longer. In the real world of social living, people have hours, weeks, years to get to learn the structures in which they are embedded. Despite the DeSoto/Freeman findings, it is reasonable to expect that they will in fact learn and perceive non-symmetric friendship relations.

Second, the Freeman and DeSoto experiments were conducted in the laboratory. There were no other cues of history or complexity to assist the subjects in seeing anything but what is cognitively simpler, that is a symmetric and transitive order. It could be argued that the real world is much messier, and non-symmetries might be apparent to the individual perceiver if given an opportunity to report it.

We propose that non-symmetric and non-reciprocated friendships are an important and omnipresent part of the social world (sociometric level), even in the minds of people who are experiencing it (Krackhardt style data level). We propose that Carley's constructural theory (1990, 1991, forthcoming) can be used to predict when non-symmetries at the sociometric level will be present. Using this model, we derive predictions regarding the existence, initiation, and recall of friendship relations. We then test these predictions using cognitive social structure data collected by Krackhardt and Kilduff (1990) on friendship in a distribution firm referred to as PACDIS.

We suggest that friendship is a perceived relationship occurring as a byproduct of individual interaction and communication. Individuals can differ in their perception of whom they interact with and in their perceptions of who interacts with them; i.e. R_{iji} may not equal R_{jij} , and R_{iji} may not equal R_{ijj} . Friendship can similarly be perceived as either symmetric or non-symmetric. Indeed the English language is fraught with terms that indicate that people recognize non-symmetries in affective relationships: e.g. obsequious and unrequited love. This contrasts with the view of friendship as an exchange relationship in which, ultimately, friendships will tend toward symmetry. For example, Hallinan (1978: 194) proposed a four-stage exchange model of friendship formation in which ultimately all friendships are symmetric:

- (1) individual i decides to seek out individual j as a friend;
- (2) individual i behaves in a way that offers friendship to j ;
- (3) individual j recognizes the offer of friendship from i ;
- (4) individual j accepts the offer and reciprocates with friendship.

Hallinan notes that during Stages 2 and 3, the friendship between i and j is non-symmetric, i.e. i extends friendship to j but j does not extend friendship (yet) to i . We note that this non-symmetry at the sociometric level is due to non-reciprocity at the Krackhardt style data level (i.e. $R_{iji} = 1$ and $R_{jij} = 0$). While

² It should be noted that DeSoto and Freeman both demonstrated that "influence" relations were more easily learned when they were non-symmetric.

Hallinan's model posits that non-symmetries exist, it also suggests that the non-symmetry is only a temporary condition, and that in the near future the non-symmetry will disappear: either j will accept the friendship offer of i or i will withdraw the offer.

Hallinan's (1978: 207) data supported her contention that, at least among sixth graders, non-symmetric friendships were less stable than symmetric ones. In later work with her colleagues, she found some support for the tendency for friendships to be reciprocated (Hallinan and Williams 1987; Hallinan and Kubitschek 1988). Of more interest to us here, however, is the less-emphasized fact that there were a substantial number of non-symmetric friendships observed. For example, Hallinan and Williams (1987) found that, on average, 53% of the friendships in their sample were not reciprocated. Further, while non-symmetric relations may have been less stable than symmetric ones in Hallinan's study (1978), they were hardly negligible, lasting virtually half as long as the symmetric friendships. Hallinan's major explanation for the existence of these non-symmetries was that they are likely to occur when a lower status person makes a friendship overture to a higher status person (1978: 194). While this may explain why the non-symmetry is initiated, it does not explain its substantial duration.

In her later work, Hallinan has argued that status non-symmetry can result in stable friendship non-symmetries: "... There is also a tendency toward non-symmetry in friendship choices. Those who are more popular are typically the group members who are held in higher esteem by their peers because they possess some respected characteristic, talent, skill or resource. The result is the emergence of a status hierarchy in friendship relationships that is characterized by a tendency toward non-symmetry in social networks." (Hallinan and Kubitschek 1988: 83). Indeed, they found that 38% of the A-B friendships in their triad study were not reciprocated, but their variable POPULARITY did not significantly explain the results as predicted.

In contrast to this exchange perspective, we take a constructural perspective – a sociocognitive perspective in which friendship is a perceived relationship derived by the individual from his or her cognitive structure. The constructural perspective (Carley 1990, 1991, forthcoming; Kaufer and Carley 1993) provides an integrated and dynamic view of cognition, interaction, friendship, and social structure in which social structure through interaction affects cognition (and friendship), and cognition motivates interaction and hence alters social structure (and friendship). This perspective combines the view that individual's cognitive structures evolve in response to their position in the social structure (House 1977; Stryker 1977; Cartwright 1979; Boyd and Richerson 1989; Friedkin 1990a, 1990b; Morgan and Schwalbe 1990) with the view that evaluation of the cognitive structure provides the basis for action and interaction (and friendship) (Emerson 1962; Mead 1962; Garfinkel 1968; Granovetter 1973; Collins 1986; Marsden 1988; Turner 1988). The constructural perspective treats the social network and the distribution of resources as dynamic entities which are continually adjusted (or constructed) as individuals interact and exchange information. Under this perspective, individuals evaluate and determine their position in the social network by determining for

each other individual in the society how similar they are to that specific individual relative to all others in the society in terms of knowledge and opportunities for contact. According to constructural theory, non-symmetric relationships naturally occur as similarity is evaluated in a relative fashion. Because constructural theory directly relates Krackhardt style ties to sociometric ties in an integrated and dynamic fashion, it appears to an ideal perspective from which to examine non-symmetries in interaction and interaction-based relationships such as friendship.

Previous studies have examined different aspects of the constructural perspective. In most of these studies opportunities for contact are assumed to be uniform (Carley 1990, 1991; Kaufer and Carley 1993) and only the role of relative-shared knowledge in affecting interaction is considered. For example, Carley (1990) found that in an environment in which the opportunities for contact were uniform, the constructural perspective was a better predictor of change in interaction over time than either balance theory or exchange theory (Carley 1990). This study, however, considered only symmetric interaction. Predictions about non-symmetric relationships, such as friendship, can be derived from constructuralism (Carley 1991). Such predictions, however, have not been tested. Further, constructuralism has not been tested in a society where the opportunities for contact were not uniform.

In this paper, we employ Carley's 1995 constructural model, rather than Carley's 1991 formulation, as it specifies the role of opportunities for contact in affecting interaction. Using this model, we derive a set of hypotheses about who interacts with whom, and who sends the interaction to whom. We augment this model with a single assumption about the nature of perception and then derive a hypothesis about who recalls interacting with whom. We then test these hypotheses using socio-cognitive data previously collected by Krackhardt and Kilduff (1990) in which there are non-symmetries in an interaction-based activity, friendship, and differences in opportunities for contact. We find that our ability to predict interaction-based activity (in this case friendship), as well as its initiation and recall, is enhanced by taking into account the relative nature of individual cognition. We conclude by suggesting that the constructural perspective provides a unified framework for explaining what interaction-based activities occur, who sends them, and who recalls them.

3. Constructural perspective

Constructural theory draws together cognitive and social explanations of behavior (Carley 1990, 1991; Kaufer and Carley 1993). The cognitive tenets are that people have a certain amount of knowledge. Individuals with a large overlap in that knowledge will tend to interact, exchange information, and learn from each other. As individuals learn more information their tendency to interact with all members of the social system can potentially change. We elaborate below that portion of this theory that directly impinges on the current study. The reader should keep in mind that friendship, as a byproduct of interaction, will behave as does interaction.

Within the social system there are I individuals, such that individuals $ij \dots I$ are members of that social grouping. Each individual has the probability of interacting with another denoted by $P_{ij}(t)$. This is the probability that individual i chooses³ to interact with individual j provided that all individuals in the social system are available for interaction (i.e. not currently interacting). According to Carley's 1995 formulation, the probability that individual i chooses to interact with individual j at time t , given that all individuals are available for interaction, is a function of both how much knowledge i shares with j and how many opportunities i has to contact j relative to i 's shared knowledge and contact opportunities with everyone else. In order to clarify this discussion, we need to specify a measure of shared knowledge and a measure of opportunities for contact.

3.1. Shared knowledge

Within the social system there are a certain number of pieces of information⁴, denoted by K , that the individuals in that social grouping can learn. The number of pieces of information, K , can be thought of as the information that is currently available – the union across all members of the social system. Let us denote that individual i knows information k at time t by $F_{ik}(t) = 1$, else 0. Two individuals will be said to share a piece of information when they both know that piece of information. The level of shared knowledge, represented as $SK_{ij}(t)$, is simply the proportion of all possible pieces of information that the two individuals share at any particular time.⁵

$$SK_{ij}(t) = \frac{\sum_{k=1}^K F_{ik}(t) \wedge F_{jk}(t)}{K} \quad (1)$$

3.2. Opportunities for contact

The opportunities for contact between two individuals (represented as OC_{ij}) can be thought of as social, organizational, or physical constraints on how much time two individuals can interact and how much time they must interact. We define the opportunities for contact between two individuals in terms of their opportunities for contacting each other relative to their opportunities for contacting all others. Individuals have opportunities for contact when they are co-present at the same event or in the same location or are expected to (and do) interact, given the

³The use of the term chosen is deliberate. It denotes the view that all actions are the result of a decision, though not necessarily a conscious one, made by the individual given his or her current mental model. Cognition is seen to mediate between 'physical reality' (in this case opportunities for contact) and individual action. For a more thorough treatment of this view see Carley and Newell (1994).

⁴The term information is used in its broadest sense to include anything that can be symbolically represented and communicated.

⁵The symbol \wedge represents the logical 'and'.

social or organizational structure. Individuals working in the same building have more opportunities for contact than do individuals working in different buildings. Individuals near each other in the organizational hierarchy have more opportunities for contact (e.g. because they go to the same meetings and read each other's reports) than do those far apart. The opportunities for contact are a function not only of this 'co-presence', but also of the number of people present. Thus, two individuals who work in the same building where one thousand others work are less likely to interact than they would be if they worked in a building with one hundred others. Indeed, a possible operationalization is simply the fraction of hours in the day that two individuals spend in each others company weighted by the number of people simultaneously present. Alternatively, we can utilize a set of events, where E is the number of events, and for each event denote whether the individual is present: $E_{ie} = 1$ if i is present at event e . Then we can define opportunities for contact as

$$OC_{ij}(t) = \left(\frac{\sum_{e=1}^E \frac{E_{ie}(t) \wedge E_{je}(t)}{1}}{\sum_{h=1}^E E_{ie}(t) \wedge E_{he}(t)} \right) / E \quad (2)$$

Unlike shared knowledge, opportunities for contact between individuals are, at least in the short run, constant over time. The set of possible knowledge, K , is huge relative to the portion of it known by any individual. This makes it easy for individuals to learn new information and so change their shared knowledge. In contrast, contact opportunities are more stable as they are set by a variety of physical and institutional factors such as organizational design, task assignment, committee structures, what church one goes to, what sport teams and clubs one belongs to and one's pre-defined calendar of events.

3.3. Relative similarity – knowledge and opportunities for contact

Individuals will share information if they have contact opportunities; they will share at least the piece of information that they are co-present (e.g. in the same room together or work together). In contrast, just because individuals share information they are not guaranteed to interact as they may never have the opportunity to do so. For example, an Australian and an American who have both read Chaucer and taken algebra share information, but owing to geographical constraints may never have the opportunity to interact. Thus the effects of knowledge and opportunities for contact cannot be totally decoupled, but neither are they identical. While opportunities for contact are sufficient for interaction, shared knowledge is not. While individuals may interact simply if they have the opportunity, regardless of how much knowledge they have, individuals cannot interact unless they have the opportunity.

The fact that opportunities for contact (OC_{ij}) and shared knowledge ($SK_{ij}(t)$) cannot be completely decoupled in their determination of the probability ($P_{ij}(t)$)

that one individual chooses to interact with another is represented in Equation 3 by the multiplication of the terms. The fact that opportunities for contact alone, but not shared knowledge, is sufficient for interaction is represented in Eq. (3) by the additive effect of only OC_{ij} . Thus, the probability of interaction can be represented as

$$P_{ij}(t) = \frac{OC_{ij} + (OC_{ij} \times SK_{ij}(t))}{\sum_{h=1}^I OC_{ih} + (OC_{ih} \times SK_{ih}(t))} = \frac{OC_{ij}(1 + SK_{ij}(t))}{\sum_{h=1}^I OC_{ih}(1 + SK_{ih}(t))} \quad (3)$$

Shared knowledge affects the probability to interact in a very simple fashion. The more knowledge that one individual shares with another relative to what he or she shares with everyone else, the more likely he or she is to choose to interact with the other individual, all else held constant. Opportunities for contact affect the probability that one individual interacts with another in two ways. First, the opportunities for contact of one individual with another can engender a certain likelihood of interaction with another individual, regardless of how much information they initially share (additive term). A student and teacher, or employer and employee, or members of the same church have a certain 'prescribed' level of interaction to which they will adhere, in the absence of other prevailing factors. Second, for pairs of individuals who share the same amount of information, those who have fewer contact opportunities (such as students at different schools) are less likely to interact (multiplicative term).

3.4. Actual interaction

Whether two individuals actually interact is a function of both their probability of interaction and whether they are available for interaction. At any given time period, if one individual wishes to interact with another, and the other is already preoccupied with interacting with someone else, then they do not interact during that time period. While the details of this mechanism are not important to our discussion (see Carley 1990, 1991), the implications are. The main implication is that the probability of interaction and actual interaction are correlated, but not perfectly. Data from numerous studies concerned with the relation between the perception of specific interaction and actual interaction support this point (Burt and Bitner 1981; Hammer 1985). Freeman *et al.* (1987a,b) demonstrated that while the individual's perception of interaction was not a good estimate of specific interaction, it was a good estimate of long term average. Thus, while an individual might be unlikely to recall who was actually at an event, the recall may be a good predictor of who typically attends such events.

3.5. Fundamental non-symmetries

According to this theory, probabilities to interact are not necessarily symmetric (Carley 1990, 1991): i.e. it may be the case that $P_{ij}(t) \neq P_{ji}(t)$. Non-symmetries

occur simply because (1) each individual evaluates his or her probability to interact with each other individual *relative* to his or her relationship to all other individuals in the social system, and (2) the two individuals in question have *different relationships* to others in the social grouping (i.e. their ego networks differ). Relative evaluation given different ego networks produces non-symmetric behavior. To facilitate the following discussion, let us consider a particular restatement of Eq. (3) in terms of overall similarity. Let us define the similarity between i and j as $Sim_{ij}(t) = OC_{ij}(1 + SK_{ij}(t))$. The individual's similarity to 'all others' can be defined as $Sim_{i-others}(t) = \sum_{h=1}^I OC_{ih}(1 + SK_{ih}(t))$ s.t. $h \neq j$. Then, Eq. (3) can be restated as

$$P_{ij}(t) = \frac{Sim_{ij}(t)}{Sim_{ij}(t) + Sim_{i-others}(t)} \quad (4)$$

Probabilities to interact are not necessarily symmetric because the probability that individual i chooses to interact with j is a function not only of their shared knowledge and opportunities for contact, but also of how much knowledge individual i shares with everyone else and how many opportunities i has to contact everyone else, whereas the probability that individual j chooses to interact with i is a function of their shared knowledge and opportunities for contact and how much knowledge individual j shares with everyone else and how many opportunities j has to contact everyone else. Indeed, interaction probabilities are symmetric only when $Sim_{i-all}(t) = Sim_{j-all}(t)$.

Let us now consider some of the implications of this model for non-symmetric relations. Interaction non-symmetries between two people result under quite simple and frequent conditions. The individual who knows more, has a larger group of 'similar others', is more culturally integrated, or has opportunities to contact a wider range of people, will have a lower interaction probability. Differences in both the amount known and in what is known can result in non-symmetries. Similarly, differences in both the number of contact opportunities and whom can be contacted result in non-symmetries. In other words, interaction non-symmetries result from very real socio-cultural differences as cognitively perceived by the individuals. Inconsistencies at the Krackhardt style data level result in non-symmetries at the sociometric level.

4. Predictions

We will focus on only three of the predictions regarding interaction-based behavior that can be derived from the constructural perspective. The constructural theory is in terms of interaction and the probability of interaction. Interaction-based actions, such as friendship, that are by-products of actual interaction and functions of the probability of interaction will behave similarly to the probability of interaction.

4.1. *General Relationship*

The first prediction concerns the level of actual interaction without particular concern for whether the exchange is non-symmetric. This prediction follows directly from Eq. (3).

H1. The greater the relative similarity (knowledge and opportunities for contact) of two individuals the more likely they will interact and engage in interaction-based behaviors (such as friendship).

4.2. *Direction of actual non-symmetries*

Our second prediction concerns who sends friendship to whom, i.e. its non-symmetry. The basic idea is that, all else being equal, differences in relative similarity result in differences in the direction of friendships, such that for each dyad the one who is relatively more similar will be more likely to consider the other a friend.

H2. For a dyad, if there exists a non-symmetry in relative similarity (knowledge and opportunities for contact) the individual who is relatively more similar to the other will be more likely to choose to interact with the other and to send interaction-based behaviors (such as friendship).

4.3. *Differential perception of non-symmetries*

Where our second prediction dealt with initiation, our third deals with recall. In order to make this prediction we add to the constructural formulation the assumption that recall is a function of cognitive saliency. For example, extremely unusual or infrequent events may be salient and so recalled. However, generally events are recalled as they typically occur, as it is the typicality or the generality of the event that is the salient feature to the individual. Given this additional assumption, it follows from the constructural model that differences in perception should be attributable to differences in relative similarity such that for each dyad the one who is relatively more similar will remember interactions and interaction-based behaviors better. The individual with the higher relative similarity will be the individual for whom the interaction or interaction-based behavior, should it occur, will be the more expected or typical occurrence and so recalled better. This is a more detailed version of the argument that individuals who have fewer interaction partners remember (or perceive as stronger) the few ties they do have better than do those individuals who have many interaction partners. Similarly, individuals who have few friends recall them better than do those who have many friends.

H3. For a dyad, if there exists a non-symmetry in relative similarity (knowledge and opportunities for contact), the individual who is relatively more similar will be

more likely to recall the interaction or interaction-based behavior (such as friendship).

5. Data – social cognitions and interaction

The data used in this study are drawn from a more comprehensive study, conducted by Krackhardt and Kilduff (1990), on the cognitive social structures (Krackhardt 1987a) and an interaction-based behavior, friendship, in a distribution firm referred to as PACDIS.

5.1. Dependent variables

We utilize several dependent variables indicating the presence, initiation, and recall of an interaction-based behavior, friendship. These dependent variables are based on aggregations of the individual's cognitive social structures. According to Krackhardt and Porter (1985, 1986) and Krackhardt (1987a), a cognitive social structure is the internal mental model that the individual has of the extant social structures, i.e. the individual's map of who knows whom, who interacts with whom, and so on. In our case, there exists a single mental model for each individual: friendship – who does the individual think is friends with whom. That is, each individual k perceives a relation R (friendship) such that individual i is the sender of the relation (e.g. sends friendship to) and individual j is the receiver (e.g. receives friendship from). Then the underlying cognitive social structure can be denoted by R_{ijk} , such that

$$R_{ijk} = \begin{cases} 1 & k \text{ thinks that } i \text{ sends } R \text{ to } j \\ 0 & k \text{ thinks that } i \text{ does not send } R \text{ to } j \end{cases}$$

Thus, $Friend_{Katie, Steve, Cassi} = 1$ means that Cassi thinks that Katie sends friendship to Steve. In addition, we use R_{ij} to denote a measure of actual interaction-based behavior such that

$$R_{ij} = \begin{cases} 1 & i \text{ sends } R \text{ to } j \\ 0 & i \text{ does not send } R \text{ to } j \end{cases}$$

Through different aggregation schemes on the mental model (R_{ijk}) we define five different measures for friendship: interaction, weak; interaction, strong; initiation, weak; initiation, strong; recall. As noted by Antonucci and Israel (1986), there is no reason to assume the veridicality of a respondent's report about the relationship between the respondent and other individuals. Whether individuals are accurate in their reporting of their relations to others is a matter for empirical investigation and hinges on a variety of factors including, type of relation, recall, and available cues. Consequently we define both a weak and a strong measure of actual interaction-based behavior. In the weak measure we basically treat

friendship as occurring if either suggests this to be the case, whereas in the strong case the two respondents must agree.

FRIENDSHIP, WEAK: This variable is defined as ⁶

$$R_{ij} = \begin{cases} 1 & \text{iff } R_{iji} = 1 \vee R_{ijj} = 1 \\ 0 & \text{all other cases} \end{cases}$$

FRIENDSHIP, STRONG: This variable is defined as

$$\begin{aligned} & 0 \quad \text{iff } R_{iji} = R_{ijj} = 0 \\ R_{ij} = 1 & \quad \text{iff } (R_{iji} = 1 \vee R_{ijj} = 1) \wedge R_{jii} = R_{jij} = 0 \\ & 2 \quad \text{iff } ((R_{iji} = 1 \wedge R_{ijj} = 1) \cap R_{jii} = 1) \vee ((R_{iji} = 1 \vee R_{ijj} = 1) \wedge R_{jij} = 1) \end{aligned}$$

Similarly, we define both a weak and a strong measure of initiation. According to the weak measure friendship has been sent if both *i* and *j* agree that *i* sent the tie, otherwise there is no initiation. This is Krackhardt's (1987a) locally aggregated intersection rule. The strong case differs, not in when an initiation is said to occur, but in when it does not occur. In the strong case, we treat as missing information those cases in which the individuals disagree about whether *i* sent the tie. This is Krackhardt's (1987a) confirmed locally aggregated intersection rule.

INITIATED FRIENDSHIP, WEAK: This variable is defined as

$$R_{ij} = \begin{cases} 1 & \text{iff } R_{iji} = 1 \wedge R_{ijj} = 1 \\ 0 & \text{all other cases} \end{cases}$$

INITIATED FRIENDSHIP, STRONG: This variable is defined as

$$R_{ij} = \begin{cases} 1 & \text{iff } R_{iji} = 1 \wedge R_{ijj} = 1 \\ 0 & \text{iff } R_{iji} = 0 \wedge R_{ijj} = 0 \\ 999 & \text{all other cases} \end{cases}$$

In this case, $R_{ij} = 999$ is treated as a 'missing value'.

For recall we define a single measure. An individual is said to recall an interaction-based activity just in case the individual listed that either *i* sends friendship to *j* or *j* sends friendship to *i*.

RECALLED FRIENDSHIP: This variable is defined as

$$R_{ij} = \begin{cases} 1 & \text{iff } R_{iji} = 1 \vee R_{jii} = 1 \\ 0 & \text{all other cases} \end{cases}$$

⁶ The symbol \vee represents the logical 'or'.

5.2. Independent variables

Our main independent variable is RELATIVE COMBINED. However, to demonstrate that relative similarity cannot be accounted for by either shared knowledge or opportunities for contact, we also use the variables RELATIVE SHARED KNOWLEDGE and RELATIVE OPPORTUNITIES FOR CONTACT. Both RELATIVE SHARED KNOWLEDGE and RELATIVE COMBINED are calculated separately for each of seven dimensions, each of which measures a different piece of social work knowledge. We use the term social work knowledge to refer to the individual's knowledge about the style of organizational behavior exhibited by other members of the organization. Within PACDIS, according to Krackhardt and Kilduff (1990), there are seven dimensions for social work knowledge, i.e. there are seven dimensions which are used by members of PACDIS to distinguish or categorize the behavior of fellow work members.⁷ These seven dimensions are:

- (1) FLEX – inflexible, critical vs. flexible, tolerant;
- (2) BREATH – does the job and nothing more vs. eats, sleeps and breaths PACDIS;
- (3) CUTCOR – goes by the book vs. prepared to cut corners;
- (4) EFFIC – lets things slide vs. efficient, organized;
- (5) COMPET – easy-going, relaxed vs. aggressive, competitive;
- (6) STRTFD – tactful, diplomatic vs. straightforward, blunt;
- (7) TASK – people-oriented vs. task-oriented.

RELATIVE SHARED KNOWLEDGE: For each of the seven dimensions, a general shared knowledge matrix (SK) is calculated. Let DIM_{ih} be the rating individual i gives to individual h on the dimension. Then for that dimension, SK_{ij} is a count of the number of times $DIM_{ih} = DIM_{jh}$ for all h , including when $h = i$ and $h = j$. SK_{ij} ranges from 0 to N (the number of individuals in the sample)

⁷ In the questionnaire, each employee was asked to rate each other employee on a seven-point scale on each of these dimensions. The dimensions were not given names *per se*, but rather were defined for the respondent by the opposing anchors given above.

These dimensions were derived from a set of interviews with a cross-section of ten employees who were asked to describe their coworkers. To facilitate and focus their description, they were asked to talk about a set of nine employees in a structured manner. The person being interviewed was given 12 triads of names. For each triad, he or she was asked to pull out the one person who was most different from the other two and describe why that person was different. The critical variable was not whom they chose, but their reasons for differentiating the one individual. The reasons most commonly used by the ten respondents to discriminate their coworkers became the basis for the seven dimensions in the questionnaire (see Krackhardt and Kilduff 1990: 144-145, for a more complete description of the method employed).

divided by N . We then define the non-symmetric matrix of relative shared knowledge between individuals i and j , RSK_{ij} , as

$$RSK_{ij} = \frac{SK_{ij}}{\sum_{h=1}^N SK_{ih}}$$

In making this calculation we set $SK_{ii} = 0$, so the diagonal will not contribute to the row total. Since, prior to this, $SK_{ii} = N$ zeroing the diagonal results in the same constant offset in the denominator for each entry in RSK_{ij} .

Clearly this measure of shared knowledge is highly restrictive, as it requires the two individuals to place another individual at exactly the same point on a seven-point scale and it does not aggregate across the seven dimensions. Individual differences in the use of the scale, despite having a shared intent, will result in lower levels of shared knowledge. We use this measure, rather than a less restrictive one, so as to have the strongest test of shared knowledge. Given the restrictiveness of this measure, if we see any effect due to knowledge this is a strong indication that the results we see are real and support the theory.

RELATIVE OPPORTUNITIES FOR CONTACT: Within PACDIS the organizational and geographical structure constrain opportunities for contact. The formal organizational structure is defined by the organization chart (*OrgChart*). In this case, whether individual i works for individual j or the reverse we set $OrgChart_{i,j} = 1$. Since the PACDIS employees are spread throughout several cities, we defined physical location (*Location*) as being in the same building (hence same city). This is particularly important since many of the employees are part of a traveling sales force. Given the nature of PACDIS, there is no *a priori* reason to assume that the organizational or geographical constraints are more important, and thus we simply define opportunities for contact as

$$OC_{ij} = \left(\frac{OrgChart_{ij}}{\sum_{j=1}^I OrgChart_{ij}} + \frac{Location_{ij}}{\sum_{j=1}^I Location_{ij}} \right) / 2$$

We then define the relative opportunities for contact between individuals i and j , ROC_{ij} , as

$$ROC_{ij} = \frac{OC_{ij}}{\sum_{h=1}^I OC_{ih}}$$

In making this calculation we set $OC_{ii} = 0$ so the diagonal will not contribute to the row total.

