

Heider vs Simmel: Emergent Features in Dynamic Structures

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Abstract. Heider's balance theory is ubiquitous in the field of social networks as an explanation for why we so frequently observe symmetry and transitivity in social relations. We propose that Simmelian tie theory could explain the same phenomena without resorting to motivational tautologies that characterize psychological explanations. Further, while both theories predict the same equilibrium state, we argue that they suggest different processes by which this equilibrium is reached. We develop a dynamic exponential random graph model (ERGM) and apply it to the classic panel data collected by Newcomb to empirically explore these two theories. We find strong evidence that Simmelian triads exist and are stable beyond what would be expected through Heiderian tendencies in the data.

1 Heider's Balance Theory

One of the central questions in the field of network analysis is: How do networks form? A cornerstone to our understanding of this process from a structural point of view has been Heider's (1946) theory of balance[1]. According to this theory, a person is motivated to establish and maintain balance in their relationships. What constitutes balance has been the subject of some debate (e.g., [2, 3]), but the core principle has survived and underlies many of our attempts to model this process of network formation (see, for example, [4]).

Heider's (1946) original formulation of balance theory was broad, including people's attitudes towards objects and ideas, not just towards other people. The unifying argument was that people felt comfortable if they agreed with others whom they liked; they felt uncomfortable if they disagreed with others they liked. Moreover, people felt comfortable if they disagreed with others whom they disliked; and people felt uncomfortable if they agreed with others whom they liked.

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Heider noted we can represent like and agreement as positive sentiments, and dislike and disagreement as negative sentiments. Considering all combinations of such sentiments among "entities", be they people or objects, Heider simplified the predictions of the theory. "In the case of two entities, a balanced state exists if the relation between them is [mutually] positive (or [mutually] negative.... In the case of three entities, a balanced state exists if all three relations [among the three entities] are positive..., or if two are negative and one positive" (p. 110).

Even in his first paper, Heider noted that in the case where one was considering "entities" as people, then two special properties of balance emerge: symmetry and transitivity. In his terminology, positive affect from one person (p) to another (o) was indicated by " pLo ". As noted above, Heider affirms symmetry is basic to balance. His claim for transitivity was more qualified but nonetheless explicit: "Among the many possible cases [of relations among three people, p , o and q] we shall only consider one. (pLo) + (oLq) + (pLq)... This example shows ... the psychological transitivity of the L relation [under conditions of balance]" (p. 110).

The other critical tenet in Heider's original formulation was that balance predicted dynamics. Heider's claim was that balance was a state of equilibrium. Imbalance was a state of disequilibrium that would motivate an individual to change something (either a relation or an attitude) that would result in a move toward balance.

It was Cartwright and Harary[5] who first made explicit the connection between Heider's cognitive balance theory and mathematical graph theory. They demonstrated how the principles of balance could be represented by a signed directed graph. Further, by applying the principles of graph theory, they demonstrated how an entire digraph could be characterized as balanced or not depending on the product of the signs of each of its semicycles (or, equivalently, whether semicycles had an even number of negative ties). This extension became the seed for a series of papers and books, each building on Heider's original ideas to study social network structures.

In a series of papers by Leinhardt, Holland and Davis, two critical extensions to this work were developed (see [6], for a spirited review). First, there was the general recognition that most network data, if not actual relations among a set of individuals, were restricted to measurements of positive ties and not negative ties. Thus, they began to look at how balance could be re-thought of as a set of positive-only relations. The concept of transitivity became the dominant theme in these papers. Imbalance was viewed as represented by intransitive triples in the data (cases where $i \rightarrow j$ and $j \rightarrow k$ and not $i \rightarrow k$), rather than the number of negative ties in any semicycle. Balance was viewed as holding if the triple was transitive (or at least vacuously so).

Second, and equally important, they recognized that structures were hardly ever perfectly balanced. The question, they argued, is not whether structures were perfectly balanced but rather whether structures *tended toward* balance, beyond what one would expect by random chance given certain basic features of the graph. They developed a set of distributions and statistical tests for assessing these tendencies and discovered that, indeed, most observed structures

show very high degrees of transitivity, relative to chance [7, 8], 1981). This work has remained influential to this day, such that new analyses of balance in any network routinely look at the degree of transitivity (and reciprocity) as measures of balance [4, 9].

2 Simmelian Tie Theory

Simmel, writing at the very start of the 20th century, had a different view of the role of relationships in social settings. He began by noting that the dyad, the fundamental unit of analysis for anyone studying relationships, including social networkers, was *not* the best focus for understanding social behavior. Indeed, he argued that before making any predictions about how two people in a relationship might behave, it is important to understand their context. The context, Simmel continues, is determined by the set of third others who also engage in various relationships with the two focal parties. In other words, Simmel argued that the triad, not the dyad, is the fundamental social unit that needs to be studied.

At the turn of the last century, Simmel provides several theoretical rationales for proffering the triad as the basic social unit ([10]: p. 118-169). Primary among these is that the dyad in isolation has a different character, different set of expectations and demands on its participants, than the dyad embedded in a triad. The presence of a third person changes everything about the dyadic relationship. It is almost irrelevant, according to Simmel, what defines a relationship (marriage, friendship, colleague); Simmel (p. 126-127) even goes so far as to say that "intimacy [the strength or quality of a relationship] is not based on the *content* of the relationship" (emphasis his). Rather, it is based on the structure, the panoply of demands and social dynamics that impinge on that dyad. And those demands are best understood by locating the dyad within its larger context, by finding the groups of people (of at least three persons) that the dyadic members belong to.

Simmel articulates several features that differentiate what he terms the "isolated dyad" from the dyad embedded in a threesome. First, the presence of a third party changes the nature of the relationship itself. Members of a dyad experience an "intensification of relation by [the addition of] a third element, or by a social framework that transcends both members of the dyad" (p. 136). Similarly, members of a dyad are freer to retain their individuality than members of a group. "[A dyad by itself] favors a relatively greater individuality of its members.... [W]ithin a dyad, there can be no majority which could outvote the individual." (p. 137). Groups, on the other hand, develop norms of behavior; they develop rules of engagement. Individuality is less tolerated in a group, and conformity is more strongly enforced.

Conflict is more easily managed within a triad than in a dyad. Dyadic conflict often escalates out of control. The presence of a third party can ameliorate any conflict, perhaps through mediation, or perhaps simply through diffuse and indirect connection. "Discords between two parties which they themselves cannot

remedy are accommodated by the third or by absorption in a comprehensive whole" (p. 135).

Perhaps most central to Simmel's idea about triads is that groups develop an identity, a "super-individual unit" (p. 126). It is a social unit that is larger in meaning and scope than any of its individual components. A consequence of this super-individual identity is that it will outlast its members. That is, people may leave, they may even die, but the group is presumed to carry on. In a triad, the emergent "super-individual unit ... confronts the individual, while at the same time it makes him participate in it" (p. 126). In contrast, dyads by themselves do not reflect this transition to a larger-than-self unit. The dyad's existence is dependent on "the immediacy of interaction" of the two members of the dyad (p. 126). Once one person withdraws from the relationship, the dyad ceases to exist. "A dyad... depends on each of its two elements alone — in its death, though not in its life: for its life, it needs both, but for its death, only one" (p. 124). Thus, he argues, the presence of a third party creates a qualitatively different unit of identity, one that is more stable over time, and one that is more difficult to extricate oneself from.

Finally, Simmel also notes that, while triads are the smallest form of group, increasing group size does not significantly alter its critical features. "[T]he addition of a third person [to dyads] completely changes them, but ... the further expansion to four or more by no means correspondingly modifies the group any further" (p. 138).

Thus, a triad is substantively different from a dyad. The triad is the smallest form of a group. But its existence transforms the nature of all its dyadic constituencies in several important ways. It makes the relationships stronger; it makes them more stable; it makes them more controlling of the behavior of its members.

2.1 Simmelian Ties and Simmelian Decomposition

The foregoing line of Simmelian reasoning suggests that knowing the specific content, nature and strength of a relationship between pairs of people is insufficient to understand the dynamics that might emerge in a social system. Even at the dyadic level, it is critical to know whether any particular dyad is embedded in a group.

To explore the implications of Simmel's theory, Krackhardt [11] proposed using graph theoretic cliques [12] to identify groups. He then defined a Simmelian tie as a tie that was embedded in a clique. Formally, given a directed graph R such that $R_{i,j} = 1$ implies the directed arc $i \rightarrow j$ exists in R , then $R_{i,j}$ is defined as a **Simmelian tie** if and only if the following are all true:

$$R_{i,j} = 1$$

$$R_{j,i} = 1$$

$$\exists k \mid R_{i,k} = 1 \wedge R_{k,i} = 1 \wedge R_{j,k} = 1 \wedge R_{k,j} = 1$$

Gower [13] and more specifically Freeman [14] developed a method of decomposing networks into two components: asymmetric (or specifically "skew-symmetric" in their terminology) and symmetric. Freeman showed that by doing so one could capture more clearly the hierarchy that existed in the network data. Krackhardt extended Freeman's idea by proposing that a directed graph of network ties could be decomposed into three mutually exclusive and exhaustive types: asymmetric, sole-symmetric and Simmelian[11]. These types are defined on a directed graph R :

$$R_{i,j} = \begin{cases} \text{Asymmetric,} & \text{if } R_{i,j} = 1 \wedge R_{j,i} \neq 1; \\ \text{Sole-Symmetric,} & \text{if } R_{i,j} = 1 \wedge R_{j,i} = 1 \wedge R_{i,j} \text{ is not Simmelian;} \\ \text{Simmelian,} & \text{if } R_{i,j} \text{ meets definitional conditions above} \end{cases}$$

2.2 Evidence for the Strength of Simmelian Ties

Since this definition of Simmelian Tie was proposed, several studies have emerged testing various elements of Simmel's theory. Krackhardt [11] re-analyzed the data collected by Newcomb[15] to determine the stability of Simmelian ties relative to asymmetric and sole-symmetric ties. Newcomb had collected network data among a set of 17 college students assigned to live together in a fraternity house. In exchange for reimbursement for living expenses, each student filled out a questionnaire each week for 15 consecutive weeks (except for week 9, where the data were not collected). The network question asked each student to rank order all the remaining 16 students based on how much he liked the others.

For purposes of his analysis, Krackhardt[11] dichotomized these rankings at the median: a relatively high ranking of 1-8 was coded as a 1 (the tie exists); a relatively low ranking of 9-16 was coded as a 0 (the tie does not exist). He then asked the question, which ties have a higher survival rate: asymmetric ties, sole-symmetric ties, or Simmelian ties?

To address this question, he plotted the conditional probabilities that a tie would appear again after Δ weeks, where Δ ranged from 1 week to 14 weeks. That is, given that a tie of a particular type (asymmetric, sole-symmetric, Simmelian) existed at time t , what is the probability that a tie (of any type) will exist at time $t + \Delta$?

His results are reproduced in Figure 1. As can be seen in the graph, ties that were initially embedded in cliques (Simmelian ties) were substantially more likely to survive over time than either asymmetric or sole-symmetric ties. Simmelian ties survived at a rate hovering around .9 for up to 4 weeks, and decay to a rate of near .7 over a 14 week gap. In contrast, both asymmetric ties and sole-symmetric ties survived at a rate of .8 over 1 week's time, dropping quickly to a rate of .7 after 3-4 weeks, and continued down to about .5 after 14 weeks. Clearly, Simmelian contexts provided a substantial survival advantage for ties.

An interesting aside here was that over a large range of time lags (from about 6 to 12 weeks lag), asymmetric ties were considerably more durable than sole-symmetric ties. One possible interpretation of this is that reciprocity in ties, one

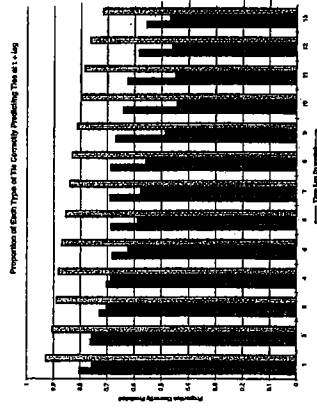


Fig. 1. Stability of Tie Types

of the key elements in Heider's balance theory, led to relative stability only when such ties were embedded in Simmelian triads.

However, Krackhardt did not provide any inferential tests for his results, a shortcoming we will return to later.

A second study[16] explored how much information was contained in Simmelian ties compared to raw ties (un-decomposed ties). The firm being studied

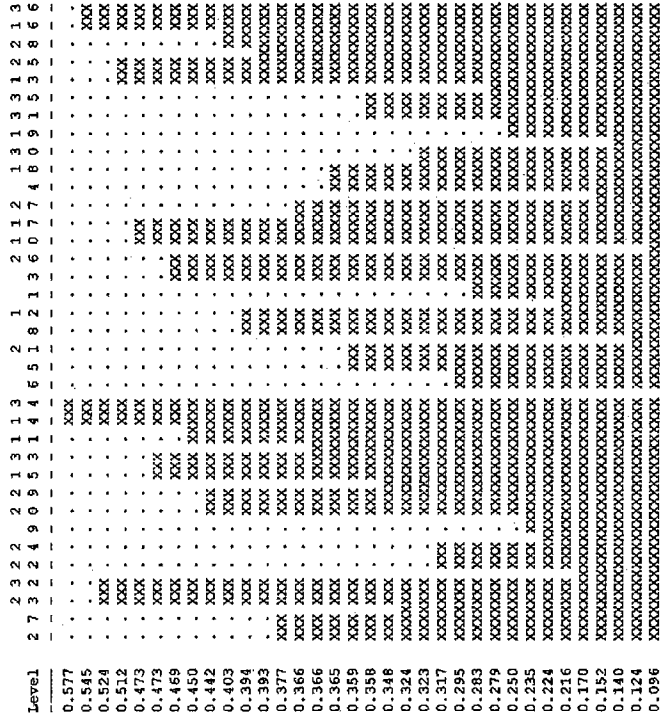


Fig. 2. Role Analysis Based on Raw Ties

had 36 employees, 15 of which were involved in a unionization effort. Some of the people involved were in favor of the union; some were against it. Some were vocal about their positions; some were quiet. He was able to demonstrate that the dynamics in the union drive and the subsequent defeat of the union was explained by observing how several key supporters of the union were "trapped" in Simmelian ties that kept them from freely expressing their views.

As part of this analysis, Krackhardt examined structurally equivalent role sets for the 36 employees [17]. He noted how clearly these roles emerged in the analysis of the Simmelian relations. What he failed to do was compare these roles to what would have been found had he analyzed the roles uncovered in the raw data.

We have re-analyzed his data (Figures 2 and 3) to make this comparison. Figure 2 provides the dendrogram for the role analysis for the raw data, as is typically done in role analysis in network data. The critical values on the left (vertical axis) represent correlations indicating how similar the roles are that people occupy at that particular cutoff level. What is clear from the analysis of the raw data in this figure is that roles are not coherent. To reach even a modest .3 correlation, the 36 people had to be divided up into 14 different roles. With an average of only a little over 2 people in each role, we learn very little about how role constraints based on the raw data may be playing a part in understanding the union dynamics here.

Figure 3 conducts the same analysis on the Simmelian ties. In contrast to the dendrogram in Figure 2, the correlations show a much better fit with fewer roles. Indeed, collapsing the 36 employees into 5 roles (an average of over 7 people per role) yields an average role similarity correlation of .42, a marked improvement over what was observed in the role analysis for raw data. A reasonable interpretation of these results suggests that raw data are noisy, making them difficult to see systematic patterns of roles and role constraints. Simmelian data appear much cleaner, crisper, suggesting that they could provide the informational backbone for structural analysis.

Thus, we have evidence that Simmelian ties are more stable, and that they provide a stronger, clearer picture of certain structural features in the network. However, again, these are descriptive measures. There is no stochastic model here, and hence no statistical framework within which we can assess the extent to which these results may be statistical artifacts or perhaps not different from what we would expect by chance. Moreover, these results tell us little about the dynamics of the process of network formation.

3 Dynamic Model Comparison of Heider and Simmel

We return to the central question we started with. What are the forces that seem to help us understand how networks form? We have presented two possible models, competing in their explanations of network dynamics. Both Heider and Simmel are similar in that they "predict" that one should observe many cliques (symmetric and transitive subgraphs). But their motivational underpinnings and their subtle dynamics are radically different.

Heider's model is a psychologically based one. People are motivated to right an imbalance (asymmetric pair or pre-transitive triple) to make it balanced (symmetric and transitive). Once balance is reached, people are said to have reached an equilibrium state and are motivated to maintain that balance. Simmel's theory, by contrast, rests in a sociological, structural explanation for the existence of symmetric and transitive triples. Cliques, once formed, become strong and stable; they resist change. However, there is no inherent motivation to form cliques. It's just that, once formed, the ties enter a phase that simultaneously increases their strength and reduces their propensity to decay over time. Thus, one could easily predict an equilibrium for each model that would be the same — dominance of symmetric pairs and transitive triples.

To see which model may better represent the real world, we re-analyzed the Newcomb data. These data provide an opportunity to not only see where the equilibrium might be headed but also to uncover what the actual dynamics are that form the pathway to that equilibrium.

We consider exponential random graph (ERG) models for the network. This class of models allow complex social structure to be represented in an interpretable and parsimonious manner [18, 19]. The model is a statistical exponential family for which the sufficient statistics are a set of functions $Z(\tau)$ of the

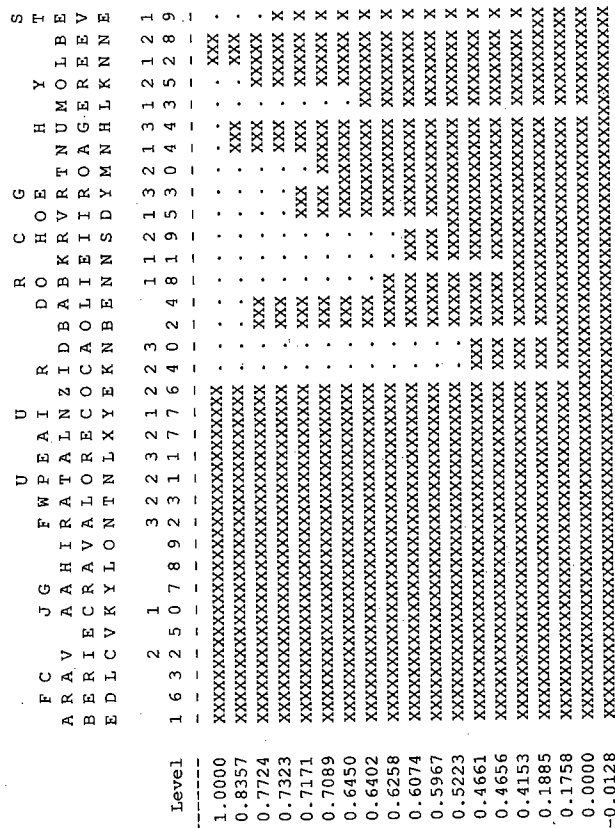


Fig. 3. Role Analysis Based on Simmelian Ties

network \mathcal{r} . The statistics $Z(\mathcal{r})$ are chosen to capture the hypothesized social structure of the network [20]. Models take the form:

$$P_{\theta}(\mathcal{R} = \mathcal{r}) = \frac{\exp(\theta \cdot Z(\mathcal{r}))}{\sum_{s \in \mathcal{R}} \exp(\theta \cdot Z(s))}, \quad (1)$$

where \mathcal{R} is the set of all possible networks and θ is our parameter vector. In this form, it is easy to see that $\sum_{s \in \mathcal{R}} \exp(\theta \cdot Z(s))$ normalizes our probabilities to ensure a valid distribution. Inference for the model parameter θ can be based on the likelihood function corresponding to the model (1). As the direct computation of the likelihood function is difficult, we approximate it via a MCMC algorithm [21].

The parameter corresponding to a statistic can be interpreted as the log-odds of a tie conditional on the other statistics in the model being fixed. It is also the logarithm of the ratio of the probability of a graph to a graph with a count one lower of the statistic (and all other statistics the same). Hence a positive parameter value indicates that the structural feature occurs with greater frequency than one would expect by chance (all else being fixed). A negative value indicates that the particular structural feature appears less than one would expect by chance.

The space of networks \mathcal{R} we consider for the Newcomb data are those that satisfy the definition of Section 2.2. Each student has exactly 8 out-ties. Hence the density and out-degree distribution of the network are fixed. To capture the propensity for a network to have Heiderian ties and triads we use two statistics:

$$Z_1(\mathcal{r}) = \text{number of symmetric dyads in } \mathcal{r} \quad (2)$$

Note that the number of edges in the graph is fixed at $17 \times 8 = 136$ and:

$$\text{number of edges} = Z_1 + \text{number of asymmetric dyads} \quad (3)$$

and the total number of dyads is $\binom{17}{2} = 136$ so

$$\text{number of asymmetric dyads} = 136 - Z_1$$

$$\text{number of null dyads} = \frac{1}{2} Z_1$$

$$\text{number of symmetric dyads} = Z_1$$

Hence Z_1 is sufficient to represent the Heiderian dyad census. To represent Heiderian triads we incorporate the statistic:

$$Z_2(\mathcal{r}) = \text{number of Heiderian (i.e., transitive) triads in } \mathcal{r}$$

To capture the propensity for the network to have Simmelian triads we incorporate the statistic:

$$Z_3(\mathcal{r}) = \text{number of Simmelian triads in } \mathcal{r},$$

that is, the number of complete sub-graphs of size three.

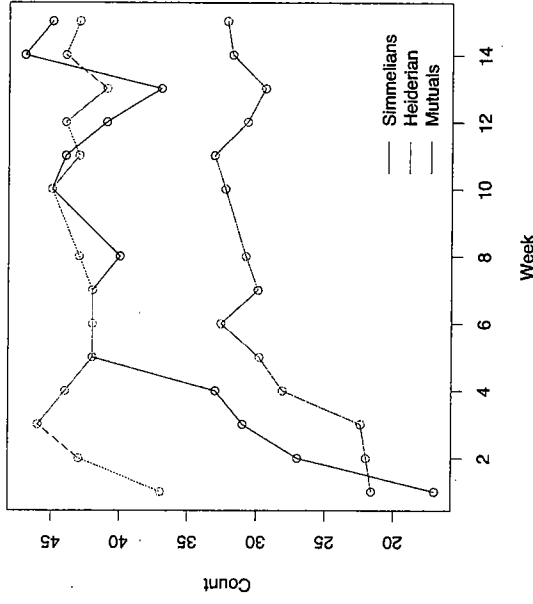


Fig. 4. Simmelian and Heiderian Statistics for the Newcomb Networks over Time. The number of Heiderian triads is divided by two to keep it on a common scale.

A model with this sample space \mathcal{R} controls for density and for individual out-degree patterns.

Figure 4 plots the three statistics for each of the 14 networks. The number of Heiderian triads generally increases over time with a larger rise in the initial weeks. The number of symmetric dyads jumps up, but is not generally increasing or decreasing. The number of Simmelian triads rapidly increases for the first five weeks and then is generally flat pattern. These descriptions can be supported by the confidence intervals for the parameters these statistics represent (not shown).

Traditional ERGM models use such parameters as static structural features. In our case, we are concerned about the transition from a state at time t and the subsequent state at time $t + 1$. Thus, we introduce a dynamic variant of the above model:

$$P_{\theta}(\mathcal{R}^{(t+1)} = \mathcal{r}^{(t+1)} | \mathcal{R}^{(t)} = \mathcal{r}^{(t)}) = \frac{\exp(\theta^{(t+1)} \cdot Z(\mathcal{r}^{(t+1)}; \mathcal{r}^{(t)}))}{\sum_{s \in \mathcal{R}} \exp(\theta^{(t+1)} \cdot Z(s; \mathcal{r}^{(t)}))} \quad t = 2, \dots, 15, \quad (4)$$

where \mathcal{R} is still the set of all possible networks with each student having out-degree four and $\theta^{(t+1)}$ is our parameter vector for the t to $t + 1$ transition. The network statistics $Z(\mathcal{r}^{(t+1)}; \mathcal{r}^{(t)})$ indicate how the network (statistics) at time $t + 1$ depend on the state at time t . This general model is adapted to the Newcomb data via two additional statistics dynamic statistics:

$Z_4(r^{(t+1)}; r^{(t)})$ = number of pre-Heiderian triads in $r^{(t)}$ that are Heiderian in $r^{(t+1)}$

$Z_5(r^{(t+1)}; r^{(t)})$ = number of Simmelian triads in $r^{(t)}$ that persist in $r^{(t+1)}$

The first follows the dynamics of pre-Heiderian (imbalanced) triads from time t to time $t + 1$. If there is a Heiderian process evolving we expect to see an increased propensity for the formation of Heiderian triads from their pre-Heiderian states (all else being equal). The second follows the dynamics of Simmelian (complete triples) triads from time t to time $t + 1$. If there is a Simmelian process evolving we expect to see persistence of Simmelian triads (all else being equal). Note that this allows a distinct process of Simmelian formation not controlled by this parameter. By including these statistics in the model, we can follow the dynamics in the Newcomb data to see how states transitioned from a non-balanced state and the stability of Simmelian state once formed.

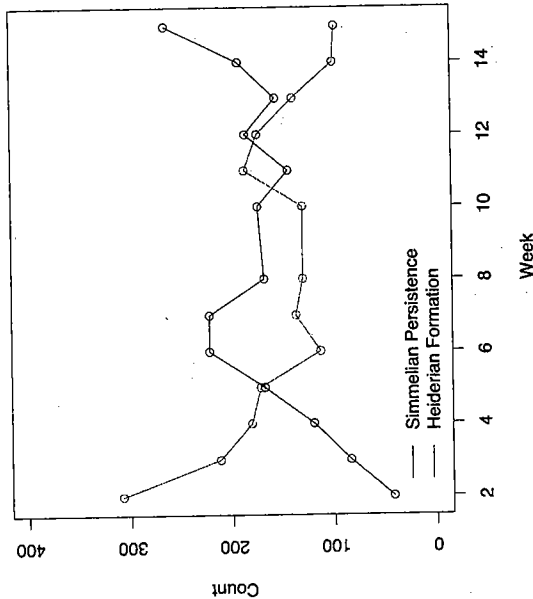


Fig. 5. The persistence of Simmelian triads and the formation of Heiderian triads for the Newcomb Networks over Time

Figure 5 plots the two dynamic statistics over the 14 weeks of data. We clearly see the increase persistence of Simmelian triads over time and the decreasing formation of Heiderian triads over time. Both these effects are strongest in the early weeks with a possible increase in the final weeks.

Both the cross-sectional and dynamic models and figures present overall Heiderian and Simmelian effects. To understand the interactions we consider the joint effects through the parameters of a dynamic model. Consider the model (4)

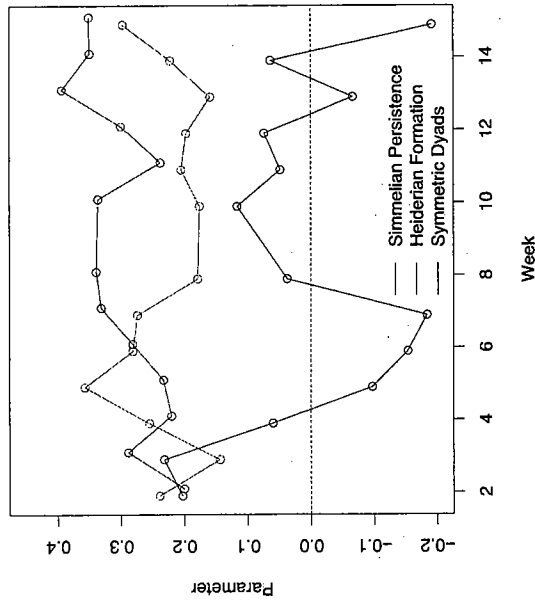


Fig. 6. The joint effects of the persistence of Simmelian triads, Heiderian dyadic balance, and the formation of Heiderian triads for the Newcomb networks over time. The values plotted are the maximum likelihood estimates of the parameters of model (4) for $t = 2, \dots, 15$.

with statistics Z_1, Z_4 and Z_5 . These measure the overall level of Heiderian dyads and the dynamics of the two processes. The maximum likelihood estimator of the parameter $\theta^{(t+1)}$ was estimated for $t = 2, \dots, 15$.

Figure 6 plots the parameters over the 14 weeks of data. It is important to note that these measure the simultaneous joint effect of the three factors. Consider the formation of Heiderian triads. We see that is positive for each time point indicating that Heiderian formation is substantively higher than due to chance. It is also modestly increasing over time indicating that the propensity for formation is modestly increasing even in the presence of the other structural factors. The pattern for the Simmelian persistence is also positive indicating substantially more persistence of Simmelian triads than expected due to chance *even adjusting for the Heiderian triadic and dyadic effects*. This has an early peak in the fifth week and appear to be increasing in the last weeks. Both these effects are confirmed by the confidence intervals for the parameters (not shown). Finally, the overall presence of Heiderian dyads is not significantly different from the random process. This is confirmed by the confidence intervals for the symmetric parameter, and also indicated by the point estimates arranged about zero.

All analyses in this section were implemented using the `statnet` package for network analysis [22]. This is written in the `R` language [23] due to its flexibility and power. `statnet` provides access to network analysis tools for the fitting, plotting, summarization, goodness-of-fit, and simulation of networks. Both `R` and the `statnet` package are publicly available (See websites in the references for details).

4 Conclusion

There have been reams of evidence for the frequent occurrence of symmetric and transitive structures in naturally occurring networks. Most of this work has been motivated by Heider's theory of balance. While Simmel's work is well-known among sociologists, little attention has been paid to his possible explanation of the same phenomena.

We have outlined how Simmel's theory, without resorting to any psychological motivations, can be used to predict the same structures as Heider's theory. Indeed, one would expect the same states from each model in equilibrium. But, the dynamics which reach these final states are substantially different. Statistical evidence from the Newcomb data suggest that Simmel's description of the evolution of these structures is a better fit with the data than Heider's.

The results of the dynamic modeling of the Newcomb data (Figure 6) indicate that Simmelian structures are important to the dynamics in the Newcomb data even when Heiderian dynamics and propensity have been accounted for. Thus the tendency to form Simmelian ties that persist most strongly and significantly throughout time is not just a by-product of a Heiderian process, but exists above and beyond that. The results also indicate that the overall level of Heiderian balance is a product of the dynamic formation of Heiderian triads from pre-Heiderian triads (above and beyond that naturally induced by the numbers of pre-Heiderian triads that exist at that point in time).

The results here are not conclusive. The Newcomb data are limited in their generalizability. But they are suggestive. Perhaps the dynamics that we have attributed all these years to Heider and balance theory are at least in part due to a completely different theory, a structural theory more consistent with Simmel's interpretation of structural dynamics.

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